Overview

- In this presentation I will shortly discuss the following topics (which follow the expansion of the universe):
  - neutrino decoupling
  - $e^+/e^-$ annihilation
  - big bang nucleosynthesis
  - recombination
  - photon decoupling
Neutrino decoupling

- The early universe was dominated by relativistic particles ($e^+/e^-/\nu_e/\gamma$), hot and dense and not opaque to even weakly interacting neutrinos.

- But as the universe expanded and cooled down, the distance between particles increased and the expansion rate of the universe ($H$) overtook the neutrino interaction rate ($\Gamma$)

- At this point the neutrino’s departed from equilibrium and decoupled

\[ \Gamma \gg H: \text{neutrino’s in equilibrium} \]
\[ \Gamma \ll H: \text{neutrino’s decoupled -- mean free path longer than horizon} \]
Neutrino decoupling

- The weak interaction rate is given by
  \[ \Gamma(T) = n \cdot \langle \sigma v \rangle \approx 2.54 \cdot G_F^2 \cdot T^6 \approx 3.48 \cdot 10^{-22} \cdot (T/\text{MeV})^5. \]
  
- The expansion rate of the universe
  \[ H(T) = \left( \frac{8\pi G}{3} \cdot \varrho \right)^{1/2} \text{ (Friedmann eq.)} \approx 4.46 \cdot 10^{-22} \cdot (T/\text{MeV})^3. \]

- Ratio:
  \[ \frac{\Gamma(T)}{H(T)} \approx 0.78 \cdot (T/\text{MeV})^3. \]

- Neutrino decoupling thus occurred at \( T \approx 1.09 \text{ MeV} \), only \( \sim 7 \) seconds after the creation of the universe.

Fermi coupling constant
\[ G_F = 1.17 \cdot 10^{-11} \text{ MeV}^{-2}. \]

Newton constant
\[ G = 6.707 \cdot 10^{-45} \text{ MeV}^{-2}. \]

Total energy density
\[ \varrho = T^4/30 \cdot \pi^2 \cdot g^*. \]

Energy degrees of freedom
\[ g^* = 2 + \frac{7}{8}(3 \cdot 2 + 2 \cdot 2) \]

\[ g^* = \sum_B g_B \cdot (T/T_\gamma)^4 + \frac{7}{8} \cdot \sum_F g_F \cdot (T/T_\gamma)^4 \]

- \( g_\gamma = 2 \): photon spin states
- \( g_e = 2 \cdot 2 \): \( e^+ / e^- \) spin states
- \( g_\nu = 3 \cdot 2 \): six neutrino’s
- \( T = T_\gamma = T_\nu \): coupled species
e\(^+\)/e\(^-\) annihilation

- Energy dropped below \(m_e = 0.5\) MeV after decoupling. The energy released by the annihilation of electrons and positrons heated the photons, but not the decoupled neutrinos.

The temperature of background photons is higher than of background neutrinos.

- For \(T_{\nu\text{dec}} > T > m_e\): \(g^*S = 2 + 7/8(2 \cdot 2) = 11/2\) (neutrino’s decoupled)
  For \(m_e > T\): \(g^*S = 2\) (electrons annihilated)
  So \((T_\gamma/T_\gamma)^3 = g^*S_{\text{before}}/g^*S_{\text{after}} = 11/4\)

\[T_\gamma = 1.4 T_\nu\]

Nowadays...

DMR 53 GHz Maps
\(T_\gamma = 2.728\) K

\(T_\nu\) expected to be \(~1.95\) K

Entropy density: \(s(T) \propto g^*S \cdot T^3\)
\[g^*S = \sum_B g_B \cdot (T/T_\gamma)^3 + 7/8 \cdot \sum_F g_F \cdot (T/T_\gamma)^3\]
Adiabatic expansion: \(d(sa^3) = 0 \Rightarrow (aT)^3 g^*S = \text{const.}\)
Cosmic neutrino background (CNB)

- But... current neutrino detectors cannot detect neutrinos with an energy as low as the neutrino background.

Measuring the CNB temperature can be done through measuring the ratio between Hydrogen and Helium.

- The proton/neutron ratio is dependent of T.

- For this we need to look at the nucleosynthesis process...
Nucleosynthesis

• Before neutrino decoupling all particles were in thermodynamic equilibrium, including the non-relativistic baryons.

The baryon number density is:
\[ n \approx g \cdot \left(\frac{mT}{2\pi}\right)^{3/2} \cdot \exp(-m/T) \]

The proton/neutron ratio gives:
\[ \frac{n_n}{n_p} \approx \exp\left(-\frac{(m_n-m_p)}{T}\right) \]

• At \( \sim 0.7 \text{ MeV} \) the ratio “freezes in” at \( \frac{n_n}{n_p} \approx 1/6 \)

• The only reaction that hereafter changes the number of neutrons is neutron decay, reduces the ratio to \( \frac{n_n}{n_p} \approx 1/7 \)

• Without further reactions to preserve neutrons in stable nuclei, the universe would be pure hydrogen.

\[ m_p \approx 938.3 \text{ MeV}, \quad m_n \approx 939.6 \text{ MeV} \]
\[ m_n - m_p \approx 1.3 \text{ MeV} \]
Nucleosynthesis

- The reaction that preserves neutrons is deuterium ($^2\text{H}$) formation.

$$X_i \approx g_i \cdot A^{3/2} \cdot \exp(B_i/T) \cdot 2^A \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot \left(\frac{m_n T}{2\pi}\right)^{3/2} \cdot (1-A)$$

- The number density can be written in terms of B:

$$n_i \approx g_i \cdot A^{3/2} \cdot \exp(B_i/T) \cdot 2^A \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot \left(\frac{m_n T}{2\pi}\right)^{3/2} \cdot (1-A)$$

- The mass fraction of a nucleus is $X_i = A_i n_i / n_B = A_i n_i / \sum A_i n_i$ ($\sum X_i = 1$) with $n_B$ the total Baryon density.

$$X_i \approx C_i \cdot \eta^{(A-1)} \cdot \exp(B_i/T) \cdot X_p^Z \cdot X_n^{(A-Z)} \cdot \left(\frac{T}{m_n}\right)^{3/2} (A-1)$$

**Binding energy (B):** $Z m_p + (Z-A) m_n - m$

(present) ratio

\[ \eta = \frac{n_B}{n_\gamma} = n_B \cdot \pi^2 \cdot (2\zeta(3)T^3)^{-1}. \]

$n_B = n_n + n_p$.
Nucleosynthesis

- $X_i$ shows that equilibrium abundances come later (lower $T$) for nuclei with small binding energies.

- He nuclei which has a larger binding energy and whose abundance would become large earlier, cannot be formed.

<table>
<thead>
<tr>
<th></th>
<th>B (MeV)</th>
<th>T (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^2\text{H}$</td>
<td>2.22</td>
<td>0.07</td>
</tr>
<tr>
<td>$^3\text{H}$</td>
<td>6.92</td>
<td>0.10</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>7.72</td>
<td>0.11</td>
</tr>
<tr>
<td>$^4\text{He}$</td>
<td>28.3</td>
<td>0.28</td>
</tr>
</tbody>
</table>
Nucleosynthesis

- Once D nuclei are created, the one way process of the creation of helium nuclei can begin.

- To good approximation all neutrons are now `captured' in $^4\text{He}$ nuclei, with a fraction of:

$$r = \frac{\#^4\text{He}}{(#\text{H} + \#^4\text{He})}$$

$$r = \frac{1}{2} \cdot \frac{n_n}{n_p - n_n + \frac{1}{2} \cdot n_n} = \frac{(n_n/n_p)/(2 - n_n/n_p)}{(1/7)/(2-1/7)} = 7.7\%$$

Mass fraction of:

$$m_{^4\text{He}}/m_B = \frac{2n_n m_p}{(n_p m_p + n_n m_p)} = \frac{2n_n}{(n_p + n_n)} = \frac{2(n_n/n_p)/(1+(n_n/n_p))}{2(1/7)/(1+1/7)} = 25\%$$

$$D + n \rightarrow ^3\text{H} + \gamma \Rightarrow ^3\text{H} + p \rightarrow ^4\text{He} + \gamma$$

$$D + p \rightarrow ^5\text{He} + \gamma \Rightarrow ^5\text{He} + n \rightarrow ^4\text{He} + \gamma$$
Helium/Hydrogen

- Measurements show the Helium/Hydrogen ratio is in the region of 25%.
- This number is consistent with our `back of the envelope’ calculation.
- In this calculation $T_\nu \sim 0.7$ MeV was used.
Radiation and matter in thermal equilibrium as long as there were lots of free electrons.

The density of free electrons reduces as T is low enough for electrons and protons form neutral hydrogen.

\[ n_H \text{ in terms of binding energy } (B = m_p + m_e - m_H = 13.6 \text{ eV}):\]
\[ n_H \approx \frac{g_H}{g_p g_e} \cdot n_p \cdot n_e \cdot \exp\left(\frac{B}{T}\right) \cdot \left(\frac{m_e T}{2\pi}\right)^{-3/2} \]
Photon decoupling

- As the number of free electrons drops, photons decouple at $\Gamma \sim H$
- We introduce fractional ionisation $X_e = n_e/n_B$:

$$(1-X_e)/X_e = n_H n_B/(n_e n_p) = 4\sqrt{2}\zeta(3)/\sqrt{\pi \cdot \eta} \cdot (T/m_e)^{3/2} \cdot \exp(B/T)$$

- When we assume recombination at $X_e \sim 0.1$ and $\eta \sim 10^{-10}$, then $T_{\text{rec}} \sim 0.31$ eV
- Go back to $\Gamma_\gamma$ and compare to matter dominated expansion then $T_{\text{dec}} \sim 0.26$ eV

Origin of CMB!
# Summary

<table>
<thead>
<tr>
<th>Energy ($\gamma$)</th>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MeV</td>
<td>7 sec</td>
<td>neutrino decoupling</td>
</tr>
<tr>
<td>0.5 MeV</td>
<td>10 sec</td>
<td>$e^+/e^-$ annihilation</td>
</tr>
<tr>
<td>70 keV</td>
<td>3 min</td>
<td>nucleosynthesis</td>
</tr>
<tr>
<td>0.31 eV</td>
<td>300,000 yr</td>
<td>recombination</td>
</tr>
<tr>
<td>0.26 eV</td>
<td>380,000 yr</td>
<td>photon decoupling</td>
</tr>
<tr>
<td>0.2 meV</td>
<td>14 Gyr</td>
<td>today</td>
</tr>
</tbody>
</table>
The End
Bose-Einstein/Fermi-Dirac distribution function:

- \( f_{FB}(k)d^3k = \frac{g}{(2\pi)^3} \cdot [\exp(E/T) \pm 1]^{-1} d^3k \)

Number density: \( n = \int f_{FB}(k) d^3k \)

- \( T >> m \) (\( E \sim k \)):
  - \( n_B \approx \frac{g\zeta(3)}{\pi^2} \cdot T^3 \)
  - \( n_F \approx 3\frac{g\zeta(3)}{4\pi^2} \cdot T^3 \)

- \( T << m \) (\( E \sim m+k^2/(2m), \text{ neglect } \pm 1 \)):
  - \( n_{BF} \approx g \cdot \left( \frac{mT}{2\pi} \right)^{3/2} \cdot \exp(-m/T) \)
back-up: number density

- \( n_B \approx g \cdot (mT/(2\pi))^{3/2} \cdot \exp(-m/T) \)

Binding energy (B): \( Zm_p + (Z-A)m_n - m \)

\[
\approx g \cdot (mT/(2\pi))^{3/2} \cdot \exp(B/T) \cdot \exp(-Zm_p/T) \cdot \exp(-(A-Z)m_n/T)
\]

\[
\approx g \cdot (mT/(2\pi))^{3/2} \cdot \exp(B/T) \cdot \exp(-m_p/T)^Z \cdot \exp(-m_n/T)^{A-Z}
\]

\[
\approx g \cdot (mT/(2\pi))^{3/2} \cdot \exp(B/T) \cdot 2^{-A} \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot (m_pT/(2\pi))^{-3/2} \cdot (m_nT/(2\pi))^{3/2} \cdot (A-Z)
\]

\[
\approx g \cdot (mT/(2\pi))^{3/2} \cdot \exp(B/T) \cdot 2^{-A} \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot (m_pT/(2\pi))^{-3/2} \cdot (m_nT/(2\pi))^{3/2} \cdot (A-Z)
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\]

\[
\approx g \cdot (Am_nT/(2\pi))^{3/2} \cdot \exp(B/T) \cdot 2^{-A} \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot (m_nT/(2\pi))^{-3/2} \cdot (m_nT/(2\pi))^{3/2} \cdot (A-Z)
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\approx g \cdot A^{3/2} \cdot \exp(B/T) \cdot 2^{-A} \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot (m_nT/(2\pi))^{3/2} \cdot (A-Z)
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\]
back-up: mass fraction

- \( n_i = C_0 \cdot \exp(B_i/T) \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot (m_nT)^{3/2}(1-A) \)
- \( X_i = C_1 \cdot n_B^{-1} \cdot \exp(B_i/T) \cdot n_p^Z \cdot n_n^{(A-Z)} \cdot (m_nT)^{3/2}(1-A) \)
  = \( C_1 \cdot n_B^{(A-Z)-1} \cdot \exp(B_i/T) \cdot n_p^Z \cdot X_n^{(A-Z)} \cdot (m_nT)^{3/2}(1-A) \)
  = \( C_1 \cdot n_B A^{-1} \cdot \exp(B_i/T) \cdot X_p^Z \cdot X_n^{(A-Z)} \cdot (m_nT)^{3/2}(1-A) \)

- \( \eta = n_B/n_f = n_B \cdot \pi^2 \cdot (2\zeta(3)T^3)^{-1} \)
  = \( C_2 \cdot \eta A^{-1} \cdot T^{3(A-1)} \cdot \exp(B_i/T) \cdot X_p^Z \cdot X_n^{(A-Z)} \cdot (m_nT)^{3/2}(1-A) \)
  = \( C_2 \cdot \eta A^{-1} \cdot \exp(B_i/T) \cdot X_p^Z \cdot X_n^{(A-Z)} \cdot (T/m_n)^{-3/2}(1-A) \)

\( C_0, C_1, C_2 \) constants