Searching for CP violation in differential distributions of multibody decays

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with Yuval Grossman (Cornell), spin-0 multibody decays

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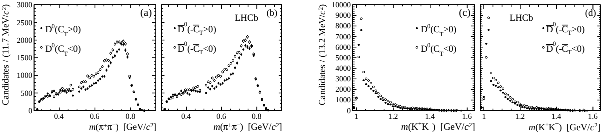
spin-1/2 multibody decays

21 November 2016
LPHE seminar, EPFL, Lausanne

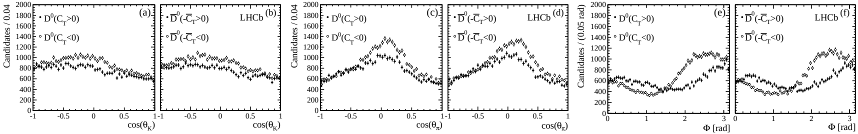
Multibody hadronic decays

- Multidimensional phase space

  e.g. 5d in $D^0 \to K^+K^-\pi^+\pi^-$:



[1408.1299]



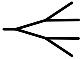
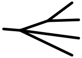
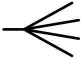
- Large statistics

$B_s^0 \to \phi\phi \to K^+K^-K^+K^-$	3950 ± 67 candidates	[1407.2222]
$D^0 \to K^+K^-\pi^+\pi^-$	$171\,300 \pm 600$	[1408.1299]
$\Lambda_b \to p\pi^-\pi^+\pi^-$	$6\,646 \pm 105$	[1609.05216]
$\Lambda_b \to \Lambda\phi \to p\pi^-K^+K^-$	89 ± 13	[1603.02870]
$\Lambda_b \to pK^-J/\psi \to pK^-\mu^+\mu^-$	$28\,834 \pm 204$	[1603.06961]

…

The paradox of richness and complexity

- Rich variety of interfering contributions

	Intermediate states in $D^0 \to K^+K^-\pi^+\pi^-$	Br $/10^{-4}$
	$(\phi\rho^0)_S, \quad \phi \to K^+K^-, \quad \rho^0 \to \pi^+\pi^-$	9.3 ± 1.2
	D	0.83 ± 0.23
	$(K^{*0}\overline{K}^{*0})_S, \quad K^{*0} \to K^\pm\pi^\mp$	1.48 ± 0.30
	$\phi(\pi^+\pi^-)_S, \quad \phi \to K^+K^-$	2.50 ± 0.33
	$(K^-\pi^+)_P(K^+\pi^-)_S$	2.6 ± 0.5
	$K_1^+K^-, \quad K_1^+ \to K^{*0}\pi^+$	1.8 ± 0.5
	$K_1^-K^+, \quad K_1^- \to \overline{K}^{*0}\pi^-$	0.22 ± 0.12
	$K_1^+K^-, \quad K_1^+ \to \rho^0 K^+$	1.14 ± 0.26
	$K_1^-K^+, \quad K_1^- \to \rho^0 K^-$	1.46 ± 0.25
	$K^*(1410)^+K^-, \; K^*(1410)^+ \to K^{*0}\pi^+$	1.02 ± 0.26
	$K^*(1410)^-K^+, \; K^*(1410)^- \to \overline{K}^{*0}\pi^-$	1.14 ± 0.25

[CLEO '12]

$\Longrightarrow$ Opportunities for CP violation searches
but also modelling challenges!

Searching for CP violation in differential distributions of multibody decays

CP violation in differential distributions

- With or without strong phases
- With untagged samples / self-conjugate states

Spinless case: systematic modelling-independent analysis

Spinful case: resolving ambiguities with modellisation

Motion reversal $\hat{\mathrm{T}}$

$\hat{\mathrm{T}}$ flips $\vec{p}$ and $\vec{s}$.

(often called *naive time reversal*)

$\hat{\mathrm{T}}$-oddity arises from $\epsilon_{\mu\nu\rho\sigma}\ p^\mu q^\nu r^\rho s^\sigma$ contractions ...

- ϵ from the Lagrangian: $i\tilde{F}^{\mu\nu} \equiv \frac{i}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$
- ϵ from chiral fermions: $\gamma^5 \equiv \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$

... of four independent momenta or spin vectors.

$\rightarrow$ minimal multiplicity

e.g. spinless four-body decays

In the p restframe,

$$\epsilon_{\mu\nu\rho\sigma}\ p^\mu q^\nu r^\rho s^\sigma \quad \propto \quad \vec{q}\cdot(\vec{r}\times\vec{s})$$

is a scalar *triple product*.

Differential CP violation

Compare the CP-conjugate amplitudes (squared)

$$\mathcal{M}(\{\vec{p}_i, \sigma_i\}) \quad \text{and} \quad \bar{\mathcal{M}}(\{-\vec{p}_{\bar{\imath}}, -\sigma_{\bar{\imath}}\})\Big|_{\vec{p}_{\bar{\imath}}=\vec{p}_i, \sigma_{\bar{\imath}}=\sigma_i}$$

phase-space point by phase-space point.

Contributions of definite
- *strong* δ and *weak* φ phases
- $\hat{\mathrm{T}}$ transformation properties

$$\mathcal{M}(\{\vec{p}_i, \sigma_i\}) = +a(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_a+\varphi_a)} +b(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_b+\varphi_b)} +c(\{\vec{p}_i, \sigma_i\})\, e^{i(\delta_c+[\varphi_c+\pi/2])} + \cdots$$

$$\bar{\mathcal{M}}(\{-\vec{p}_i, -\sigma_i\}) = +a(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_a-\varphi_a)} +b(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_b-\varphi_b)} +c(\{-\vec{p}_i, -\sigma_i\})\, e^{i(\delta_c-[\varphi_c+\pi/2])} + \cdots$$

with
$a(\{-\vec{p}_i, -\sigma_i\}) = +a(\{\vec{p}_i, \sigma_i\})$ $\hat{\mathrm{T}}$-even
$b(\{-\vec{p}_i, -\sigma_i\}) = +b(\{\vec{p}_i, \sigma_i\})$ $\hat{\mathrm{T}}$-even
$c(\{-\vec{p}_i, -\sigma_i\}) = -c(\{\vec{p}_i, \sigma_i\})$ $\hat{\mathrm{T}}$-odd
$\cdots$

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with

$$\begin{aligned}a(\{-\vec{p}_i,-\sigma_i\}) &= +a(\{\vec{p}_i,\sigma_i\}) \quad \hat{\mathrm{T}}\text{-even} \\ b(\{-\vec{p}_i,-\sigma_i\}) &= +b(\{\vec{p}_i,\sigma_i\}) \quad \hat{\mathrm{T}}\text{-even} \\ c(\{-\vec{p}_i,-\sigma_i\}) &= -c(\{\vec{p}_i,\sigma_i\}) \quad \hat{\mathrm{T}}\text{-odd} \\ &\cdots\end{aligned}$$

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with
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$c(\{-\vec{p}_i,-\sigma_i\}) = -c(\{\vec{p}_i,\sigma_i\})$ $\hat{\mathrm{T}}$-odd
$\cdots$

$\Longrightarrow$ The φ phases are defined to contain all ‘CP-oddity’.

CP violation and strong phases

Distributions of definite CP and $\hat{\text{T}}$ transformation properties

$$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-}^{\text{even}}_{\text{odd}}}_{CP\text{-}^{\text{even}}_{\text{odd}}} \equiv \frac{\mathbb{I} \pm \hat{T}}{2}\,\frac{\mathbb{I} \pm CP}{2}\,\frac{d\Gamma}{d\Phi}$$

- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-even}}_{CP\text{-even}} \propto \begin{array}{l} a\ a + b\ b + c\ c \\ +2\,a\,b\ \cos(\delta_a - \delta_b)\cos(\varphi_a - \varphi_b) \end{array}$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-odd}}_{CP\text{-even}} \propto \begin{array}{l} 2\,a\,c\ \sin(\delta_a - \delta_c)\ \cos(\varphi_a - \varphi_c) \\ +2\,b\,c\ \sin(\delta_b - \delta_c)\ \cos(\varphi_b - \varphi_c) \end{array}$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-even}}_{CP\text{-odd}} \propto -2\,a\,b\ \sin(\delta_a - \delta_b)\ \sin(\varphi_a - \varphi_b)$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-odd}}_{CP\text{-odd}} \propto \begin{array}{l} 2\,a\,c\ \cos(\delta_a - \delta_c)\sin(\varphi_a - \varphi_c) \\ +2\,b\,c\ \cos(\delta_b - \delta_c)\sin(\varphi_b - \varphi_c) \end{array}$

$\Longrightarrow$ Four different sensitivities to strong and weak phases.

CP violation and strong phases

Distributions of definite CP and $\hat{T}$ transformation properties

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- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-even}}_{CP\text{-even}} \propto \; a\,a + b\,b + c\,c$
  $+2\,a\,b\,\cos(\delta_a-\delta_b)\cos(\varphi_a-\varphi_b)$
- $\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{T}\text{-odd}}_{CP\text{-even}} \propto \; 2\,a\,c\,\sin(\delta_a-\delta_c)\,\cos(\varphi_a-\varphi_c)$
  $+2\,b\,c\,\sin(\delta_b-\delta_c)\,\cos(\varphi_b-\varphi_c)$

$\left.\frac{d\Gamma}{d\Phi}\right\|^{\hat{T}\text{-even}}_{CP\text{-odd}} \propto$ ‘$\sin\delta\sin\varphi$’	Sensitivity to small differences of CP-odd phases between decay amplitudes of different CP-even phases.

$\left.\frac{d\Gamma}{d\Phi}\right\|^{\hat{T}\text{-odd}}_{CP\text{-odd}} \propto$ ‘$\cos\delta\sin\varphi$’	Sensitivity to small differences of CP-odd phases between decay amplitudes of identical —or vanishing— CP-even phases.

$\Longrightarrow$ Four different sensitivities to strong and weak phases.

Untagged samples / self-conjugate states

- Tagging CP-conjugate processes may cost efficiency, and is N/A with self-conjugate initial and final states.

- An untagged sample is: [as in 1503.05362]

e.g. $$\begin{cases} B_s^0 \to K^+(+\vec{p}_1) \quad \pi^-(+\vec{p}_2) \quad K^-(+\vec{p}_3) \quad \pi^+(+\vec{p}_4) \\ \bar{B}_s^0 \to K^-(-\vec{p}_1) \quad \pi^+(-\vec{p}_2) \quad K^+(-\vec{p}_3) \quad \pi^-(-\vec{p}_4) \end{cases}$$

$$\frac{\mathbb{I} + \mathrm{CP}}{2} \; \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}}}{2} \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}E}^*}{2} \, \frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}$$

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$$\frac{\mathbb{I} + \mathrm{CP\hat{T}E^*}}{2} \, \frac{d\Gamma}{d\Phi}$$

It has two CP-odd distributions, $\hat{\mathrm{T}}$-odd or E^*-odd:

$$\frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \, \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \left(\frac{\mathbb{I} + \mathrm{CP\hat{T}E^*}}{2} \, \frac{d\Gamma}{d\Phi} \right) = \frac{\mathbb{I} \pm \hat{\mathrm{T}}}{2} \, \frac{\mathbb{I} \mp \mathrm{E}^*}{2} \, \frac{\mathbb{I} - \mathrm{CP}}{2} \, \frac{d\Gamma}{d\Phi}$$

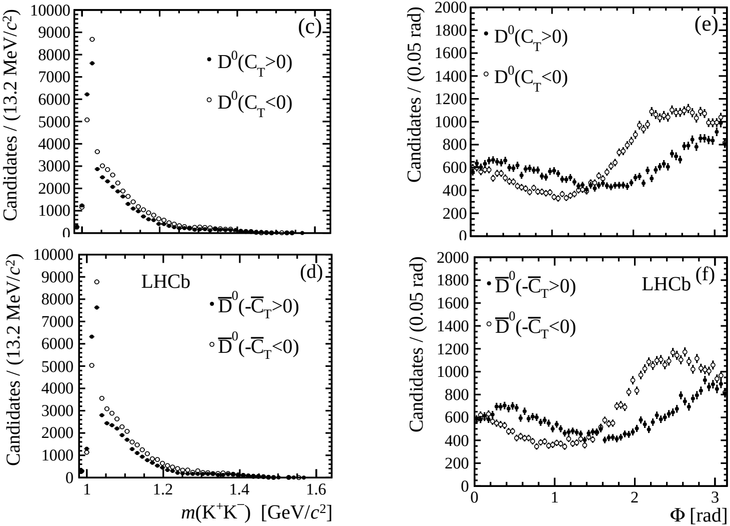
Searching for CP violation in differential distributions of multibody decays

- CP violation in differential distributions
  - With or without strong phases
  - With untagged samples / self-conjugate states
- Spinless case: systematic modelling-independent analysis
- Spinful case: resolving ambiguities with modellisation

Spinless case: $\hat{T}$-folding of the phase space

$$\frac{d\Gamma}{d\Phi}(D^0 \to K^+K^-\pi^+\pi^-)$$

[1408.1299]



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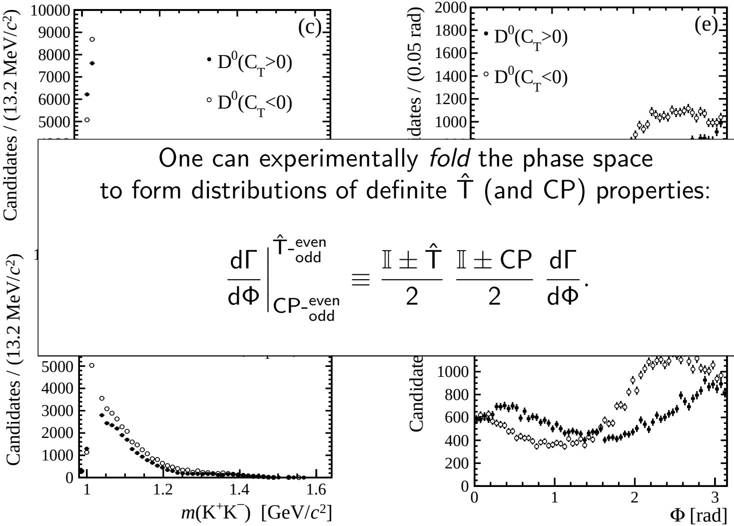

One can experimentally *fold* the phase space
to form distributions of definite $\hat{\mathrm{T}}$ (and CP) properties:

$$\left.\frac{d\Gamma}{d\Phi}\right|^{\hat{\mathrm{T}}\text{-}^{\text{even}}_{\text{odd}}}_{\mathrm{CP}\text{-}^{\text{even}}_{\text{odd}}} \equiv \frac{\mathbb{I}\pm\hat{\mathrm{T}}}{2}\,\frac{\mathbb{I}\pm\mathrm{CP}}{2}\,\frac{d\Gamma}{d\Phi}.$$

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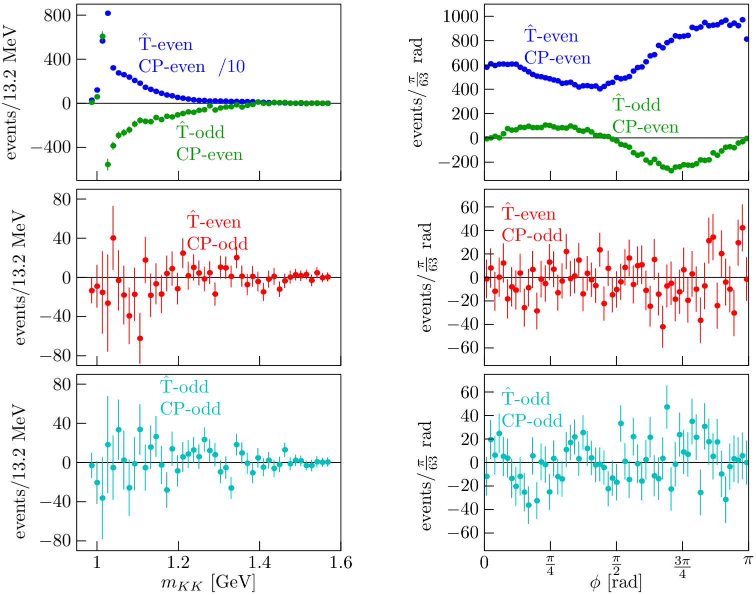
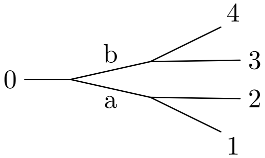
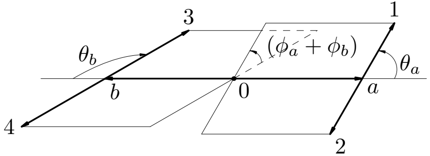


Spinless case: modelling-independent analysis

1. Fix a phase-space parametrisation

   A set of anlges and invariant masses biasing the analysis sensitivity





2. Define CP-odd asymmetries (or moments) systematically

   Exploiting the full phase space

$$\mathcal{A}^{kl}_{no} \equiv \int d\Omega \left(\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} - \frac{1}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\Omega} \right) \text{sign}\left\{ f_k(\cos\theta_a)\, f_l(\cos\theta_b)\, \sin\left(n\phi_a + n\phi_b + o\frac{\pi}{2} \right) \right\}$$

with $o = 0$: $\hat{\text{T}}$-odd

$o = 1$: $\hat{\text{T}}$-even

3. Back to 1., with another parametrisation

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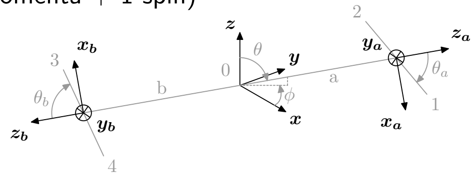
The richness of polarization

$\hat{\text{T}}$-oddity with lower multiplicity

in the three-body decay of a polarized particle
($\epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu r^\rho s^\sigma$ with 3 momenta + 1 spin)

e.g. $\Lambda_b \to \Lambda^* \gamma \to pK\gamma$
$\Lambda_b \to N^* K \to p\pi K$



More angles

One polarization component breaks two rotation symmetries.

New $\hat{\text{T}}$-odd variables

- one polarization component

  P_z: P-even–$\hat{\text{T}}$-odd ($\perp$ production plane)
  $P_{x,y}$: P-odd–T-even

- angular variables: $\cos\theta$, $\cos\phi_a$, $\cos\phi_b$

The ambiguity of polarization

The initial polarization has a double status:

- kinematic variables in the decay
- fixed by production amplitudes

And specific values values cannot be selected

without considering production kinematics.

From the decay perspective:

– $\hat{\mathrm{T}}$-folding along P_z cannot be performed.
– Distributions of definite $\hat{\mathrm{T}}$ properties cannot be defined.
– The sensitivity of angular observables to decay amplitude phases can only be obtained through additional modelling:
    - from helicity formalism [Jacob, Wick 59']
    - with resonance+spin structure specified

Spinful case: resolving ambiguities

e.g. for $\frac{1}{2} \to \frac{1}{2}\, 1 \to (\frac{1}{2}\, 0)\, (0\, 0)$ like $\Lambda_b \to N^* \rho \to p\pi\pi\pi$, $\Lambda_b \to \Lambda\, \phi \to p\pi KK$

$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$					$\sin^2\theta_b$	
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a			$\cos\theta_a$	$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a			$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$		P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$		P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\cos^2\theta_b$	
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-6	$\mathrm{Re}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\cos\phi_a$
$+3$	$\mathrm{Re}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\cos(\phi_a+2\phi_b)$
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\sin\phi_b$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} - \mathrm{Im}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\sin\phi_b$
-6	$\mathrm{Im}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\sin\phi_a$
$+3$	$\mathrm{Im}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\sin(\phi_a+2\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$

Spinful case: resolving ambiguities

e.g. for $\frac{1}{2} \to \frac{1}{2}\,1 \to (\frac{1}{2}\,0)\,(0\,0)$ like $\Lambda_b \to N^* \rho \to p\pi\pi\pi$
$\Lambda_b \to \Lambda\,\phi \to p\pi KK$

$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$					$\sin^2\theta_b$	
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a			$\cos\theta_a$	$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a			$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$		P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$		P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\cos^2\theta_b$	
$-3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$	α_a	P_z	$\cos\theta$	$\cos\theta_a$	$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} + \mathrm{Re}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} + \mathrm{Re}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\cos\phi_b$
-6	$\mathrm{Re}\{A_+^* A_-\}$						
$+3$	$\mathrm{Re}\{B_+^* B_-\}$						
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$						
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$						
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} - \mathrm{Im}\{A_-^* B_-\}$						
-6	$\mathrm{Im}\{A_+^* A_-\}$						
$+3$	$\mathrm{Im}\{B_+^* B_-\}$						
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$						

$$P_{x,y} = 0$$

$$A_\pm \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}1}(\pm\tfrac{1}{2}, 0)$$

$$B_\pm \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}1}(\pm\tfrac{1}{2}, \pm 1)$$

$$1 \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(+\tfrac{1}{2}, 0) + \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(-\tfrac{1}{2}, 0)$$

$$\alpha_a \equiv \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(+\tfrac{1}{2}, 0) - \mathcal{M}_{\frac{1}{2}\to\frac{1}{2}0}(-\tfrac{1}{2}, 0)$$

$$1 \equiv \mathcal{M}_{1\to 00}(0, 0)$$

Spinful case: resolving ambiguities

e.g. for $\frac{1}{2} \to \frac{1}{2}\, 1 \to (\frac{1}{2}\, 0)\,(0\, 0)$ like $\Lambda_b \to N^* \rho \to p\pi\pi\pi$

$\Lambda_b \to \Lambda\, \phi \to p\pi KK$

$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$					$\cos^2\theta_b$	
$+3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$						
$+3$	$\lvert A_+\rvert^2 - \lvert A_-\rvert^2$	α_a					
$+3/2$	$\lvert B_+\rvert^2 - \lvert B_-\rvert^2$	α_a					
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} - \mathrm{Re}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3$			P_z	$\cos\theta$		$\cos^2\theta_b$	
$-3/2$			P_z	$\cos\theta$		$\sin^2\theta_b$	
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} - \mathrm{Re}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\cos\phi_b$
$+3$	$\lvert A_+\rvert^2 + \lvert A_-\rvert^2$	α_a	P_z				
$-3/2$	$\lvert B_+\rvert^2 + \lvert B_-\rvert^2$	α_a	P_z				
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_-\} + \mathrm{Re}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\cos(\phi_a+\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Re}\{A_+^* B_+\} + \mathrm{Re}\{A_-^* B_-\}$	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\cos\phi_b$
-6	$\mathrm{Re}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\cos\phi_a$
$+3$	$\mathrm{Re}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\cos(\phi_a+2\phi_b)$
$-3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_+\} + \mathrm{Im}\{A_-^* B_-\}$		P_z	$\sin\theta$		$\sin 2\theta_b$	$\sin\phi_b$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} - \mathrm{Im}\{A_-^* B_+\}$	α_a	P_z	$\cos\theta$	$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$
$-3/\sqrt{2}$	Im	α_a	P_z	$\sin\theta$	$\cos\theta_a$	$\sin 2\theta_b$	$\sin\phi_b$
-6	$\mathrm{Im}\{A_+^* A_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\cos^2\theta_b$	$\sin\phi_a$
$+3$	$\mathrm{Im}\{B_+^* B_-\}$	α_a	P_z	$\sin\theta$	$\sin\theta_a$	$\sin^2\theta_b$	$\sin(\phi_a+2\phi_b)$
$+3/\sqrt{2}$	$\mathrm{Im}\{A_+^* B_-\} + \mathrm{Im}\{A_-^* B_+\}$	α_a			$\sin\theta_a$	$\sin 2\theta_b$	$\sin(\phi_a+\phi_b)$

$\hat{\mathrm{T}}$-even angular distrib.

$\to$ '$\sin\delta \sin\varphi$'

$\hat{\mathrm{T}}$-odd angular distrib.

$\to$ '$\cos\delta \sin\varphi$'

Spinful case: resolving ambiguities

With a resonance structure isolated, the helicity formalism determines the sensitivity to amplitude phases that each CP-odd angular asymmetry has.

$$\mathcal{A}^{jkl}_{mno} \equiv \int d\Omega \left(\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} - \frac{1}{\bar{\Gamma}}\frac{d\bar{\Gamma}}{d\Omega} \right) \text{sign}\left\{ f_j(\cos\theta)\, f_k(\cos\theta_a)\, f_l(\cos\theta_b)\, \sin\left(m\phi_a + n\phi_b + o\frac{\pi}{2} \right) \right\}$$

with $o \in \{0,1\}$, $j+m+n+o \in 2\mathbb{Z} \to \hat{\mathrm{T}}$-odd

$j+m+n+o \in (2\mathbb{Z}+1) \to \hat{\mathrm{T}}$-even

+ further understanding gained:

e.g., in strongly-produced $\frac{1}{2} \to \frac{1}{2}\ 1 \to (\frac{1}{2}\ 0)\ (0\ 0)$,

- vanishing 'classic' $\sin(\phi_a + \phi_b)$ asymmetry (integrated $a_{CP}^{\hat{T}\text{-odd}}$)
- vanishing asymmetries based on 'special' angles [hep-ph/0602043]

$$\cos\Phi_a = \frac{\cos\theta\cos\phi\sin\phi_a + \sin\phi\cos\phi_a}{\sqrt{1-\sin^2\phi_a\sin^2\theta}}, \quad \sin\Phi_a = \frac{\cos\theta\sin\phi\sin\phi_a - \cos\phi\cos\phi_a}{\sqrt{1-\sin^2\phi_a\sin^2\theta}},$$

$$\cos\Phi_b = \frac{\cos\theta\cos\phi\sin\phi_b - \sin\phi\cos\phi_b}{\sqrt{1-\sin^2\phi_b\sin^2\theta}}, \quad \sin\Phi_b = \frac{\cos\theta\sin\phi\sin\phi_a + \cos\phi\cos\phi_b}{\sqrt{1-\sin^2\phi_b\sin^2\theta}}.$$

measured in $\Lambda_b \to \Lambda\phi \to p\pi KK$ [1603.02870]

Decay analysis strategies

Systematic and modelling-independent

- straightforward in spinless decays
- potentially using production kinematics, in polarized decays

$\longrightarrow$ may spot unexpected signals

Helicity amplitude decomposition

- select a specific resonance+spin structure
- target allowed asymmetries

$\longrightarrow$ relates dynamics and kinematics

Searching for CP violation in differential distributions of multibody decays

Differential distributions are rich of opportunities
to search for CP violation.

In spinless decays, systematic search procedures
are free of modelling limitations.

In spinful decays, the helicity formalism indicates
the sensitivities to amplitude phases
that each angular distributions provides.