Observation of the decay

\[ B^+ \rightarrow K^+ \pi^- \pi^+ \gamma \]

at LHCb
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1 Introduction

The Standard Model (SM) is a set of theories that constitute the most fundamental description of elementary particles up to now. Relativistic Quantum Field Theory is the base on which the SM is built; it combines the dynamics of quantum fields with the space-time of special relativity. The SM has 12 elementary particles/anti-particles divided into bosons and fermions according to their spin and introduces different interaction fields: the weak interaction, mediated by neutral $Z^0$ and charged $W^\pm$ bosons, the electromagnetic interaction, associated with the photon $\gamma$, and finally the strong interaction, carried by the gluons. A large number of experiments have validated the SM consistency, however the SM has some difficulties to describe phenomena such as quantum gravitation, dark energy/matter, neutrino oscillation...

1.1 Theory

Flavour-Changing Neutral Current (FCNC) interactions change the flavour of a quark but not its charge. They arise from SM through loop processes known as penguin or box diagrams and are forbidden at tree level. Virtual particles circulate in such loops and may be particles not described in the SM. Hence FCNC decays are a good place to look for new physics beyond the SM. Radiative decays are a specific type of FCNC. In the $B^+ \rightarrow K_1(1270)^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ decay modes, a $\bar{b}$ goes to $\bar{s}$ ejecting a photon. The loop is generated by a $W^+$ boson coupling to a $\bar{t}$ (or $\bar{u}$, or $\bar{c}$) quark. The $b \rightarrow s \gamma$ transition can be described by the penguin Feynman diagram shown in Figure 1.1.

![Feynman diagram of the $b \rightarrow s \gamma$ transition in the Standard Model description.](image)

Processes such as $b \rightarrow s \gamma$ cannot be directly measured at quark level due to the hadronization, caused by the confinement property of QCD. Theoretical predictions of branching fractions are then difficult. However, other observables can be found which are not so sensitive to hadronization factors such as CP, forward-backward and isospin asymmetries.

For example, photon polarization can be determined by measuring the polarization of the axial-vector meson $K_1$ in the $B \rightarrow K_1(1270)\gamma$ decay through its decay to a three-body final state ($K\pi\pi$). This is possible since helicity is conserved, because angular momentum is conserved and the $B$ meson is a pseudoscalar. Also it is necessary to have a three-body decay since the photon helicity is odd under parity and since we only measure the momenta of final decay products (spin information cannot be obtained from a two-body decay). Then a parity-odd triple product can be calculated $\vec{p}_\gamma \cdot (\vec{p}_1 \wedge \vec{p}_2)$, where $\vec{p}_\gamma$ is the photon momentum, and $\vec{p}_1$ and $\vec{p}_2$ are the momenta of two of the final-state hadrons measured in the $K\pi\pi$ resonance rest frame. The sign of this product will indicate if the photon is right- or left-handed. The Standard Model predicts that photons are mostly left-handed in $b \rightarrow s \gamma$ and right-handed in $\bar{b} \rightarrow \bar{s} \gamma$ [1].

$^1$A small right-handed component of order $(m_s/m_b)^2 \approx 0.1\%$ is expected (up to 1 – 10% with QCD corrections) [2].
1.2 Experimental status

In 1993, the first exclusive radiative decay mode \( B \to K^* \gamma \) was observed by the CLEO collaboration [3], followed by \( B \to K^*_2(1430) \gamma \) reported by Belle [4]. Using a data sample of 140 fb\(^{-1}\) taken at the \( \Upsilon(4S) \) resonance, the Belle detector at the KEKB \( e^+e^- \) collider observed the \( B^+ \to K_1(1270)^+ (K^+\pi^+\pi^-) \gamma \) radiative decay in 2005 [7] and measured its branching fraction (see Table 1). In 2008, using a 210.9 fb\(^{-1}\) dataset collected at the \( \Upsilon(4S) \) resonance, the Babar experiment at the PEP-II asymmetric-energy storage ring measured the inclusive \( B^+ \to K^+\pi^-\pi^+\gamma \) branching fraction, \( B(B^+ \to K^+\pi^-\pi^+\gamma) = (2.95 \pm 0.13 \pm 0.20) \times 10^{-5} \) in the \( m_{K\pi\pi\gamma} < 1.8 \text{ GeV}/c^2 \) mass range [8]. The latest \( B^0 \to K^{*0} \gamma \), \( B^+ \to K_1(1270)^+ \gamma \) and \( B^+ \to K^+\pi^-\pi^+\gamma \) branching fractions measured by these experiments are given in Table 1.

In LHCb, the first radiative decays to be observed were \( B^0 \to K^{*0} \gamma \) and \( B^0_s \to \phi \gamma \), and the ratio of their branching fractions was measured, in addition to the \( B^0 \to K^{*0} \gamma \) direct CP asymmetry [11].

<table>
<thead>
<tr>
<th></th>
<th>( B(B^0 \to K^{*0} \gamma) )</th>
<th>( B(B^+ \to K_1(1270)^+ \gamma) )</th>
<th>( B(B^+ \to K^+\pi^-\pi^+\gamma) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO [5]</td>
<td>((4.55^{+0.72}_{-0.68} \pm 0.34) \times 10^{-5})</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Babar [9]</td>
<td>((4.47 \pm 0.1 \pm 0.16) \times 10^{-5})</td>
<td>(2.95 \pm 0.13 \pm 0.2) \times 10^{-5})</td>
<td></td>
</tr>
<tr>
<td>Belle [6, 7]</td>
<td>((4.01 \pm 0.21 \pm 0.17) \times 10^{-5})</td>
<td>((4.3 \pm 0.9 \pm 0.9) \times 10^{-5})</td>
<td>((2.5 \pm 0.18 \pm 0.22) \times 10^{-5})</td>
</tr>
<tr>
<td>PDG 2012 [10]</td>
<td>((4.33 \pm 0.15) \times 10^{-5})</td>
<td>((4.3 \pm 1.3) \times 10^{-5})</td>
<td>((2.76 \pm 0.22) \times 10^{-5})</td>
</tr>
</tbody>
</table>

Table 1: Current experimental results of the branching fractions of some radiative decays from the CLEO, Babar and Belle collaborations, and their averages.

1.3 Objectives

The main objective of this work is to observe the inclusive \( B^+ \to K^+\pi^-\pi^+\gamma \) decay mode. The \( B \to K\pi\pi\gamma \) signal will be analysed from a Monte Carlo (MC) sample to determine its shape and its selection efficiency. Then various backgrounds will be studied to determine their typical shape and their contribution, thereby it will be possible to construct a model to fit the data taken by LHCb in 2012. The \( B^+ \to K^+\pi^-\pi^+\gamma \) signal yield will be extracted and compared to the already observed \( B^0 \to K^{*0} \gamma \) mode to determine its branching fraction. This study is a first step towards a possible photon polarization study using \( B \to K\pi\pi\gamma \) [1].
2 LHCb

LHCb is one of the four main experiments at the CERN’s Large Hadron Collider (LHC). It is dedicated to heavy flavour physics, more precisely to the measurement of CP violation and rare decays of beauty and charm hadrons. It is located at Interaction Point 8 of the LHC accelerator.

2.1 LHC and data run 2012

LHC is a superconducting hadron accelerator and collider of 26.7 km circumference. It consists of two rings in which protons beams are accelerated in opposite directions to enable the pp collisions. Each protons beam reaches 4 TeV energy which implies a 8 TeV centre-of-mass energy. To reach this energy, protons follow a certain path through different accelerators (Figure 2.1): first, protons are produced at 100 keV by an ion source, then they are accelerated to 50 MeV by a small linear accelerator (LINAC). They reach 1 GeV energy thanks to the Booster synchrotron. Afterwards, the Proton Synchrotron PS boosts them to 26 GeV. Finally, they are injected into the Super Proton Synchrotron SPS which accelerates them up to 450 GeV. At this moment, the proton beam is split into two beams which are injected in opposite directions in the two LHC rings; there, they reach 4 TeV energy. LHC is made of straight sections and arcs where 8.33 T field magnets are used to bent the beam trajectories. These magnets are superconducting and cooled down to 1.9 K [12, 13].

LHCb started running in 2009. It has recorded a luminosity of 2.2 fb$^{-1}$ in 2012, as shown in Figure 2.2, of which 1.85 fb$^{-1}$ are used in the analysis described in this report.

![CERN Accelerator Complex](image)

**Figure 2.1:** The CERN accelerator complex, showing the chain of accelerators (LINAC, Booster, PS, SPS) needed to inject beams in the LHC.
2.2 The LHCb detector

The LHCb detector [20] is a single-arm spectrometer designed to take into account the $b\bar{b}$-pair boost in the direction of the most energetic parton, which is along the beam direction (either forwards or backwards). It is composed of several specific detectors: VELO (Vertex Locator), RICH (Cherenkov detector), Trackers, CALorimeters and Muon system, as shown in Figure 2.3.

![Figure 2.3: Schematic view of the LHCb experiment.](image)

**Figure 2.2**: LHCb yearly recorded luminosity.

![LHCb Integrated Luminosity](image)

**Figure 2.2**: LHCb yearly recorded luminosity.

![LHCb Integrated Luminosity](image)
**VErtex LOcator** The VErtex LOcator (VELO) measures with precision the tracks and vertices close to the interaction point. The VELO detector is composed of twenty-one stations, each of which has a $r$-sensor and a $\phi$-sensor. The $r$-sensors have concentric strips centered around the beam pipe. The $\phi$-sensors are subdivided in two regions: inner and outer. The inner region counts 683 strips whereas the outer counts 1365.

**Dipole magnet** A magnet of 4 Tm integrated magnetic field allows the measurement of the momentum of charged particles from their curvature radius (charged particles are bent in a magnetic field).

**Tracker Turicensis** The Tracker Turicensis is a silicon tracker located upstream of the dipole magnet which covers the full LHCb acceptance. It is made of four layers of microstrip sensors arranged in a $x-u-v-x$ geometry. The $x$ layers are oriented vertically and the $u$ and $v$ layers are tilted with an angle of $\pm5^\circ$ respectively.

**Inner Tracker** The Inner Tracker is a silicon tracker too, and is located close to the beam pipe downstream of the magnet at the centre of the three tracking stations T1-T3. Its microstrip sensors are arranged with the same geometry as the Turicensis Tracker.

**Outer tracker** The Outer Tracker is part of the T1-T3 tracking stations and surrounds the Inner Tracker stations. It is a drift-time detector and measures charged particle tracks over a large fraction of the LHCb acceptance in the outer region. It is made of drift tubes filled with a mixture of Argon (70%) and CO$_2$ (30%).

**RICH detectors (Ring Imaging Cherenkov detectors)** A RICH detector uses the Cherenkov effect to mainly distinguish pions from kaons of a given momentum, since the aperture angle of the Cherenkov cone depends on the velocity. Two RICH detectors are used with different radiators ($C_4F_{10}$ and $CF_4$) to cover the entire momentum spectrum.

**Calorimeters** Calorimeters detect particles through their energy deposition in the detector. The characteristic shape of the “shower” (length and radius) enables particle identification. The penetration length of the particle give also information on its energy.

- **SPD/PS (Scintillating Pad/Preshower detector):** The SPD/PS system is used for particle identification. It consists of a lead converter sandwiched between two almost identical scintillator pads ($2.5X_0$ thick). The scintillator pads are divided in three regions (inner, middle and outer with respectively 1536, 1792 and 2688 cells) and can distinguish a charged particle from a neutral particle. On the other hand the PS distinguishes electrons and photons from hadrons, mainly thanks to their electromagnetic shower’s longitudinal profile.

- **ECAL (Electromagnetic calorimeter):** The ECAL is $25X_0$ thick to contain the whole electromagnetic shower. It is made with a scintillator and lead structure and the readout is ensured by plastic WLS (Wave-Length Shifting) fibers. It is used for particle identification and energy measurement.

- **HCAL (Hadronic calorimeter):** The HCAL is mainly used for trigger and particle identification. It is made of iron and scintillating plates which are oriented parallel to the beam axis. The HCAL is only $5.6X_0$ thick and cannot contain the full hadronic shower, but still allows the particle to be identified and its energy to be estimated.
Muon detector  Muon chambers are put at the end of the detector because the iron and lead from the calorimeters are used as a “muon filter”, i.e., to stop all the detectable particles except muons. The muon detector is composed of five stations M1 to M5. They are Multi-Wire Proportional Chambers placed perpendicular to the beam axis. M1 is placed before the calorimeters and is used for trigger purposes. The other muon chambers are placed downstream of the calorimeters and interleaved with iron absorbers (HCAL). The M1-M5 thickness is $\approx 20$ interaction lengths ($\lambda$).

Trigger  The trigger is used to select a small fraction of the events to be stored for offline analysis. The selected events should be those of higher physical interest for the experiment.

- **Level-0 Trigger**: The Level-0 Trigger is the first level trigger. It is used to reduce the nominal bunch-crossing rate ($\sim 40$ MHz) to 1 MHz and is implemented using custom hardware parts. This trigger is composed of two main parts: the L0 calorimeter trigger and the L0 muon trigger. They each send the information from their detector to the L0 Decision Unit (L0 DU) which takes the global decision.

  - The L0 calorimeter trigger selects high $E_T$ electrons, photons, neutral pions or hadrons because $B$ meson decay products are usually high transverse momentum and energy particles.
  - The L0 muon trigger looks for high $p_T$ muons in the muon chambers (from decays like $B^0 \rightarrow J/\psi(\mu^+\mu^-)K_S^0$ or $B^0_s \rightarrow J/\psi(\mu^+\mu^-)\phi$).

- **HLT (High Level Trigger)**: The High Level Trigger uses software algorithms to filter events. It is composed of two stages: HLT1 and HLT2.

  - The HLT1 reduces the event rate from 1 MHz to 50 kHz by selecting single tracks with high momentum, large impact parameter (with respect to all primary vertices) and a good track quality.
  - The HLT2 reduces the event rate from 50 kHz to 3 kHz by reconstructing all the tracks with $p_T > 500$ MeV/c and $p > 5000$ MeV/c. The HLT2 can do a better selection because it has fully reconstructed events. The HLT2 is made of inclusive and exclusive trigger lines which search for typical $B$ decay features (displaced vertices for example) and select specific decays with requirements close to the ones used in the offline analyses.
3 Event selection

Our aim is to study both $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ events with data recorded by the LHCb experiment. To do so we will need to reconstruct and select candidates in these data and also in MC samples of both signal and possible backgrounds.

3.1 Samples used

The data used correspond to a luminosity of $1.85\text{ fb}^{-1}$ recorded in 2012 by the LHCb detector. These events need to have passed the $K_1\gamma$ or $K^*\gamma$ lines in the Radiative Stripping stream. No specific requirements on the trigger are applied.

As for the MC samples, the different decays used as well as the sample sizes are summarized in Table 2. They all have been generated using the MC11 Monte Carlo configuration.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Number of events</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow K_1(1270)^+\gamma$</td>
<td>173500</td>
</tr>
<tr>
<td>$B^+ \rightarrow K_2^{*+}\gamma$</td>
<td>50999</td>
</tr>
<tr>
<td>$B^0 \rightarrow K^{*0}\gamma$</td>
<td>6190472</td>
</tr>
<tr>
<td>$B^+ \rightarrow D_0^{0}\pi^+$</td>
<td>109499</td>
</tr>
<tr>
<td>$B^+ \rightarrow D^{*0}\pi^+$</td>
<td>502996</td>
</tr>
<tr>
<td>$B^+ \rightarrow K^{*+}\pi^+\pi^-$</td>
<td>1041996</td>
</tr>
<tr>
<td>$B^+ \rightarrow K^{*0}\pi^+\gamma$</td>
<td>1930989</td>
</tr>
</tbody>
</table>

Table 2: MC samples used in this work.

3.2 $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ reconstruction and selection cuts

A charged $B$ meson candidate is reconstructed from a selected hadronic system combined with a photon. This hadronic system is composed of a charged kaon $K^+$ and two oppositely charged $\pi^+$ and $\pi^-$ pions\footnote{Charge conjugated states are included unless specifically stated.}. The $K\pi\pi$ combinations forming a good vertex and with an invariant mass in the 1–2 GeV/$c^2$ range are kept for building the $B$ candidates.

A neutral $B$ meson candidate is reconstructed from an excited kaon $K^{*0}$ combined with a photon. $K^*$ candidates are built from a $K^+$ and a $\pi^-$ and required to have an invariant mass in the 792–992 MeV/$c^2$ range.

The selection criteria applied to the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ candidates are shown in Table 3. The cuts are similar to those of a previous LHCb study of $B^0 \rightarrow K^{*0}\gamma$ \cite{11} but slightly looser in terms of particle identification (PID). We use almost the same cuts for both $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$.

- Track IP $\chi^2$: The impact parameter $\chi^2$, which tests the compatibility of a track with the primary vertex, has to be large for the $K^+$, $\pi^+$ and $\pi^-$ candidates because we want to make sure they do not come from primary vertex. See Figure 3.1.

- Track $p_T$: The transverse momentum of each track has to be large. This requirement excludes combinatorial background made of soft tracks.

- PID: The PID variables allow the separation of $\pi$ from $K$ or $K$ from $p$ or $\pi$... There are provided by the RICH system and are computed as the difference in logarithms of the likelihoods (DLL) between two particle hypotheses.
Table 3: Criteria applied to select $B \to K\pi\pi\gamma$ and $B \to K^*\gamma$ decays.

- $\gamma E_T$: High transverse momentum photons are selected to only keep the photons coming from $B$ decays (rather than from sub-decays), and to avoid soft combinatorial photons.
- $\gamma$ CL (photon Confidence Level): The aim of the variable is to distinguish a $\gamma$ from an electron. The CL is calculated using calorimeter clusters and matching them to tracks. If there is no nearby track, there is a high confidence that it is a neutral electromagnetic particle, otherwise it is a charged one.
- $\gamma/\pi^0$ separation: This variable is used to distinguish $\pi^0$ from $\gamma$. At high $p_T$, the two photons from a $\pi^0 \to \gamma\gamma$ decay are really close to each other and it is difficult to distinguish them because they are seen as a single cluster in the calorimeter. However, this cluster has a different shower shape in ECAL, and this fact is exploited to distinguish between these two types of particles.
- Hadronic system vertex $\chi^2$: This variable ensures the quality of the three- or two-track vertex.
- Hadronic system isolation $\chi^2$: This variable checks if there are not any other compatible tracks in the event for a given three- or two-body vertex. If an extra track is found and compatible with the $K\pi\pi$ or $K\pi$ vertex, the candidate is rejected. It can help eliminate some partially reconstructed background in which the missed particle is charged.
- $B$ FD $\chi^2$: A minimum requirement on the flight distance of the $B$ candidate removes combinations of tracks produced at the primary vertex. This cut is efficient on the $B$ signal because of the long $B$ lifetime.
- $B$ IP $\chi^2$: The $B$ impact parameter with respect to the primary vertex has to be small to make sure the $B$ candidate comes from the primary $pp$ vertex.

Figure 3.2 shows the $B^+ \to K^+\pi^-\pi^+\gamma$ mass spectrum after the selection criteria have been applied on the data sample.
3 Event selection

Figure 3.1: Explanation of the impact parameter for the kaon track.

\[ K^+ \pi^- \pi^+ \gamma \text{ mass distribution} \]

Figure 3.2: Mass distribution of the $B^+ \to K^+\pi^-\pi^+\gamma$ candidates selected in the 1.85 fb$^{-1}$ data sample. The selection criteria are those of Table 3.

3.3 Additional mass cuts for the $B^+ \to K^+\pi^-\pi^+\gamma$ selection

The $B^+ \to \bar{D}^0\rho^+$ background is a partially reconstructed background with $\bar{D}^0 \to K^+\pi^-\pi^0$ (or possibly $\bar{D}^0 \to K^+\rho^-$ followed by $\rho^- \to \pi^-\pi^0$) and $\rho^+ \to \pi^+\pi^0$. In this section, the goal is to show how further mass cuts can help in removing the $B^+ \to \bar{D}^0\rho^+$ background, which has a potentially large contamination to the $B^+ \to K^+\pi^-\pi^+\gamma$ signal.

\[
\begin{align*}
\pi^+\pi^0 \text{ mass [MeV}/c^2] &> 1100 \\
K^+\pi^-\pi^0 \text{ mass [MeV}/c^2] &> 2000
\end{align*}
\]

(3.1) (3.2)
We compute the invariant masses of the $\pi^+\gamma$ and $K^+\pi^−\gamma$ systems, where we assign the $\pi^0$ mass to the photon candidate, assuming it is a true $\pi^0$ misidentified as a photon. We then apply cuts on these $\pi^+\pi^0$ and $K^+\pi^−\pi^0$ masses. The $\pi^+\pi^0$ mass is required to be sufficiently above the $\rho$ mass (taking into account the $\rho$ width and resolution) to reject $\rho^+ \to \pi^+\pi^0$ background, Equation 3.1. Then the $K^+\pi^−\pi^0$ mass is required to be sufficiently above the $D^0$ mass (taking into account the resolution) to reject $\bar{D}^0 \to K^+\pi^−\pi^0$ background, Equation 3.2.

Two cases are possible:

- **Case 1**: we miss the $\pi^0$ from the $\bar{D}^0$ and misidentify the $\pi^0$ from the $\rho^+$ as a $\gamma$. Then no $D$ meson is observed in the $K^+\pi^−\pi^0$ mass spectrum but a $\rho$ is visible in the $\pi^+\pi^0$ mass spectrum.

- **Case 2**: we miss the $\pi^0$ from the $\rho^+$ and misidentify the $\pi^0$ from the $\bar{D}^0$ as a $\gamma$. Then, the $D$ meson is observed in the $K^+\pi^−\pi^0$ mass spectrum. In addition, some $\rho^−$ may be observed in $\pi^−\pi^0$ mass spectrum.

To understand this contamination, we compare $B^+ \to K^+\pi^−\pi^+\gamma$ MC and real data, after all cuts of Table 3. Figures 3.3, 3.5 and 3.4, show the distributions of the $K^+\pi^−\pi^+\gamma$, $\pi^+\pi^0$, $K^+\pi^−\pi^0$ and $\pi^−\pi^0$ mass spectrum.

More specifically, Figure 3.3 compares the two-dimensional distribution of $M(K^+\pi^−\pi^+\gamma)$ and $M(\pi^+\pi^0)$ in data and in signal MC. We see that the peak corresponding to a $\rho^+$ in the $\pi^+\pi^0$ mass distribution can be removed by cutting on the $\pi^+\pi^0$ mass without removing any signal.

Figure 3.4 shows the two-dimensional distribution of $M(K^+\pi^−\pi^+\gamma)$ and $M(K^+\pi^−\pi^0)$ in data and signal MC. We observe a $\bar{D}^0$ mass peak in the $K^+\pi^−\pi^0$ mass spectrum of the data and see that this peaking background can be removed with a cut on the $K^+\pi^−\pi^0$ mass without losing any signal.

It is interesting to note that the $\bar{D}^0$ peak in the data is correlated with a $\rho^−$ peak in the $\pi^−\pi^0$ mass spectrum (see Figure 3.5). The cut on the $K^+\pi^−\pi^0$ mass therefore also removes the $\rho^−$ peak.

In the Figures 3.3 and 3.4, we can see that the $\rho^+$ and $\bar{D}^0$ are peaking in the region corresponding to the combinatorial background of the $B$ mass data.
Figure 3.3: Case 1: 2D mass plot ($M(K^+\pi^0\pi^+\gamma), M(K^+\pi^0\gamma)$).
3.3 Additional mass cuts for the $B^+ \to K^+ \pi^- \pi^+ \gamma$ selection

![Mass distributions for $B^+ \to K^+ \pi^- \pi^+ \gamma$]

(a) Data

(b) $B^+ \to K^+ \pi^- \pi^+ \gamma$ signal Monte Carlo

Figure 3.4: Case 2: 2D mass plot ($M(K^+ \pi^- \pi^+) , M(K^+ \pi^- \pi^0)$)
Figure 3.5: Case 2: 2D mass plot \(M(K^+\pi^-\pi^0), M(\pi^-\pi^0)\) from data.
3.3 Additional mass cuts for the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ selection

$K^+\pi^-\pi^+\gamma$ mass distribution

Figure 3.6: $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ mass distribution (2012 data 1.85 fb$^{-1}$): the blue histogram shows all events before the $\pi^+\pi^0$ and $K^+\pi^-\pi^0$ mass cuts, and the cyan histogram shows the events removed by these cuts.

We show the shape of the data removed by the $\pi^+\pi^0$ and $K^+\pi^-\pi^0$ mass cuts as the cyan part in Figure 3.6. In fact, cut (3.2) removes the $B^0$ resonance we can observe in Figure 3.7a and cut (3.1) the $\rho^+$ one in Figure 3.7b.

We check that a large fraction of the signal passes these mass cuts, and that the removed events have no “bump” in the signal region (cyan histogram of Figure 3.6). Therefore, these cuts are helpful to remove a significant part of low mass partially reconstructed background corresponding to $B^+ \rightarrow \bar{D}^0\rho^+$.

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PDM, winter semester 2012
4 MC signal studies

MC samples of $B^+ \rightarrow K_1(1270)^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ decays are studied to determine the shape of the signal mass spectra and the signal efficiencies after all selection cuts. We use $B^+ \rightarrow K_1(1270)^+\gamma$ MC events as a proxy for $B^+ \rightarrow K^{+}\pi^{-}\pi^{+}\gamma$. All reconstructed and selected candidates are used (no “MC truth matching”).

4.1 Shape and resolution

The mass distributions of MC $B^+ \rightarrow K_1(1270)^+\gamma$ signal and $B^0 \rightarrow K^{*0}\gamma$ signal candidates are fitted using unbinned extended maximum likelihood fit, as shown in Figures 4.1a and 4.1b, with a double Crystal Ball function. The fit values are given in Table 4. The double Crystal Ball probability density function (pdf) has the following expression:

$$f(x; \alpha_L, n_L, \alpha_R, n_R, \bar{x}, \sigma) = \begin{cases} 
N \cdot A_L \cdot (B_L - \frac{x - \bar{x}}{\sigma})^{-n_L}, & \text{for } \frac{x - \bar{x}}{\sigma} \leq -\alpha_L \\
N \cdot \exp\left(-\frac{(x - \bar{x})^2}{2\sigma^2}\right), & \text{for } +\alpha_R > \frac{x - \bar{x}}{\sigma} > -\alpha_L \\
N \cdot A_R \cdot (B_R + \frac{x - \bar{x}}{\sigma})^{-n_R}, & \text{for } \frac{x - \bar{x}}{\sigma} \geq -\alpha_R
\end{cases}$$

(4.1)

where $\alpha_L > 0$ and $\alpha_R < 0$, $N$ is a normalisation factor and

$$A_i = \left(\frac{n_i}{|\alpha_i|}\right)^{n_i} \cdot \exp\left(-\frac{|\alpha_i|^2}{2}\right)$$

$$B_i = \frac{n_i}{|\alpha_i|} - |\alpha_i|.$$ 

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$B^+ \rightarrow K_1(1270)^+\gamma$</th>
<th>$B^0 \rightarrow K^{*0}\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$ [MeV/$c^2$]</td>
<td>$5287 \pm 3.8$</td>
<td>$5281.26 \pm 0.48$</td>
</tr>
<tr>
<td>$\sigma$ [MeV/$c^2$]</td>
<td>$95.7 \pm 2.7$</td>
<td>$95.96 \pm 0.88$</td>
</tr>
<tr>
<td>$\alpha_L$</td>
<td>$2.25 \pm 0.12$</td>
<td>$2.24 \pm 0.09$</td>
</tr>
<tr>
<td>$n_L$</td>
<td>$1.33 \pm 0.32$</td>
<td>$0.97 \pm 0.14$</td>
</tr>
<tr>
<td>$\alpha_R$</td>
<td>$-2.21 \pm 1.6$</td>
<td>$-1.795 \pm 0.13$</td>
</tr>
<tr>
<td>$n_R$</td>
<td>$1.9 \pm 5.3$</td>
<td>$4.77 \pm 1.04$</td>
</tr>
<tr>
<td>$\chi^2$/ndf</td>
<td>$0.543$</td>
<td>$1.523$</td>
</tr>
</tbody>
</table>

Table 4: Fitted parameters of the mass distributions of $B \rightarrow K_1(1270)^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ MC signal events.

The $\alpha$ and $n$ values obtained from the $B^+ \rightarrow K_1(1270)^+\gamma$ MC fit will be used to fix the signal shape in the data fit.

The $\chi^2$ residuals located at the bottom of the mass plot in Figure 4.1 give a hint about the pdf suitability. We can have an idea on the “goodness” of the fit if almost all of them lie within the $\pm 2\sigma$ band. They are calculated as the multinomial likelihood chi-squared [14]:

$$\chi^2 = \sum_i 2 \cdot \text{data}_i \cdot \ln \left(\frac{\text{data}_i}{\text{pdf}_i}\right),$$

(4.2)

where $i$ is the bin number.
4.2 Efficiency

We measure the number of events selected from the MC samples for the $B^+ \rightarrow K_1(1270)^+\gamma$ channel: $1185 \pm 34$ events over a total number of events of 173500 were fitted after applying the selection criteria.
This represents a selection efficiency of \((0.68 \pm 0.02) \times 10^{-2}\).

To check the previous result, the \(B^+ \rightarrow K_2^*(1430)^+\gamma\) channel is also studied. Applying the same selection criteria on the MC sample for this channel, only \(318 \pm 18\) events were selected over a total number of 50999 events. The obtained selection efficiency \((0.624 \pm 0.035) \times 10^{-2}\), is of the same order as for the \(B^+ \rightarrow K_1(1270)^+\gamma\) channel. We can see that the different resonances have compatible efficiencies. The number of generated events in MC sample is low and so the number of selected events too, but we can consider it is sufficient to estimate the efficiencies.

The efficiency \((\varepsilon_{\text{gen}} \times \varepsilon_{\text{sel}})\) for the \(B^+ \rightarrow K_1(1270)^+\gamma\) channel is given by:

\[
\varepsilon_{\text{gen}} \times \varepsilon_{\text{sel}} = (21.48 \pm 0.06) \times 10^{-2} \times (0.68 \pm 0.02) \times 10^{-2} = (14.6 \pm 0.4) \times 10^{-4},
\]

where the value of \(\varepsilon_{\text{gen}}\) is taken from Ref. [15]. \(\varepsilon_{\text{gen}}\) represents the efficiency of the 400 mrad aperture cut (due to the detector geometry) applied when producing the MC sample. As \(B\) production is not isotropic, this efficiency must be estimated using the MC generator.

Concerning the \(B^0 \rightarrow K^{*0}\gamma\) decay, the number of fitted events from the MC sample is \(108374 \pm 329\) over a total number of events of 7482967 after applying the selection criteria. This is corresponding to a selection efficiency of \((1.448 \pm 0.004) \times 10^{-2}\) and the efficiency \((\varepsilon_{\text{gen}} \times \varepsilon_{\text{sel}})\) for this decay is given by

\[
\varepsilon_{\text{gen}} \times \varepsilon_{\text{sel}} = (23.41 \pm 0.09) \cdot 10^{-2} \times (1.448 \pm 0.004) \cdot 10^{-2} = (3.394 \pm 0.016) \times 10^{-3}
\]
5 MC background studies for $B \to K\pi\pi\gamma$

We perform a MC study to learn about some features of the possible backgrounds.

Using different MC samples of potential backgrounds, we apply the offline cuts and calculate efficiencies. In the case of backgrounds in which no event is selected, we calculate an upper limit for the yield. Several background types have to be considered in the $B \to K\pi\pi\gamma$ study and they are detailed in the following sections.

5.1 Peaking background

The peaking backgrounds, in which the final state is the same as the signal except for a possible $\pi^0/\gamma$ misidentification, taken into account in the $K\pi\pi\gamma$ study are:

- $B^+ \to D^0(K^+\pi^0)\pi^+$
- $B^+ \to D^{*0}(\bar{D}^0(K^+\pi^-)\gamma)\pi^+$
- $B^+ \to K^{*+}(K^+\pi^0)\pi^+\pi^-$

According to the analysis done in Section 3.3, the two $D^0$ backgrounds should not pass the selection, but since they have a large branching fraction we decide to study them in more detail to know their possible contribution.

Given the fact that none or very few events in the MC samples pass the selection criteria, we use the so-called Neyman construction [16] (further explanations are given in Annexe A) to estimate or give an upper limit to the selection efficiencies.

![Graph](image)

**Figure 5.1:** $(N_{sel}, \varepsilon)$ plot from the Neyman construction for the $B \to D^0\pi$ peaking background. The solid line represents the 95% confidence belt and the dashed line the 95% upper limit.
Table 5 gives the conclusions of the Neyman study. The number of selected MC events for the $B^+ \to D^{0} \pi^+$ and $B^+ \to D^{*0} \pi^+$ backgrounds is 0. As there is a statistical uncertainty on this number, this does not mean the applied cuts can remove the two $D^0$ backgrounds but that the efficiency has an upper limit. The calculation of this upper limit is important to have an idea on how really this background can contribute, i.e., be present in a significant proportion.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\varepsilon_{\text{gen}}$ [%]</th>
<th>$\varepsilon_{\text{sel}}$ (# selected/# MC generated)</th>
<th>$\varepsilon_{\text{sel}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \to D^{0} \pi^+$</td>
<td>$14.57 \pm 0.04$</td>
<td>$0/109499$</td>
<td>$&lt; 2.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>$B^+ \to D^{*0} \pi^+$</td>
<td>$14.79 \pm 0.03$</td>
<td>$0/502996$</td>
<td>$&lt; 4 \times 10^{-6}$</td>
</tr>
<tr>
<td>$B^+ \to K^{*+} \pi^+ \pi^-$</td>
<td>$15 \pm 5$</td>
<td>$2/1041996$</td>
<td>$(1.92 \pm 4.68) \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 5: Efficiencies for some backgrounds. The value of $\varepsilon_{\text{gen}}$ for the $B^+ \to K^{*+} \pi^+ \pi^-$ background was not available at the time of writing, we assume it is similar to the other backgrounds and given by $(15 \pm 5)$%.
To see if the previous study is consistent, we compare our results to the ones predicted by a Poisson distribution [17]. The upper limit on efficiencies is calculated from the $\nu_{\text{eff}}$, upper limit on mean value $\nu$, divided by the total number of events. The efficiencies upper limits obtained with the Neyman study are taken from the Graphs 5.1b, 5.2b and 5.3b. They are summarized in Table 6.

The values are compatible according to the low precision in the measurement of the Neyman upper limit values (large interval in efficiency). More precisely, in the Neyman study, we are making steps in efficiency of the order of $2 \cdot 10^{-6}$ given by the size of the studied MC sample.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Upper limit on $\varepsilon_{\text{sel}}$ at 95% CL</th>
<th>Poisson</th>
<th>Neyman</th>
<th>Relative difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \to D^0\pi^+$</td>
<td>0</td>
<td>$2.74 \cdot 10^{-5}$</td>
<td>$2.6 \cdot 10^{-5}$</td>
<td>5.1</td>
</tr>
<tr>
<td>$B^+ \to D^*0\pi^+$</td>
<td>0</td>
<td>$5.96 \cdot 10^{-6}$</td>
<td>$4 \cdot 10^{-6}$</td>
<td>32</td>
</tr>
<tr>
<td>$B^+ \to K^{*+}\pi^+\pi^-$</td>
<td>2</td>
<td>$6.05 \cdot 10^{-6}$</td>
<td>$6.6 \cdot 10^{-6}$</td>
<td>9.1</td>
</tr>
</tbody>
</table>

Table 6: Comparison between Poisson and Neyman $\varepsilon_{\text{sel}}$ upper limits.

The contamination from a given background is given by

$$B/S = \frac{N_{bkg}}{N_{sig}} = \frac{f_{bkg}}{f_{sig}} \cdot \frac{B(bkg)}{B(K\pi\gamma)} \cdot \frac{\varepsilon_{bkg}}{\varepsilon_{sig}} \cdot \frac{\varepsilon_{gen}}{\varepsilon_{trig}} \cdot \frac{\varepsilon_{sel}}{\varepsilon_{sel}}.$$  

We assume that $\varepsilon_{\text{trig}}$ is the same for signal and background. The selection and generation efficiencies for $B^+ \to K^+\pi^+\pi^+\gamma$ are assumed to be those of $B^+ \to K_1(1270)^+\gamma$: 

$$\varepsilon_{\text{sig}} = (21.48 \pm 0.06) \times 10^{-2}$$  

$$\varepsilon_{\text{sel}} = (6.8 \pm 0.4) \times 10^{-3}$$  

The HFAG branching fraction for the signal is $B(B \to K^+\pi^-\pi^+\gamma) = (27.6 \pm 2.2) \times 10^{-6}$ [10]. The branching fraction for the backgrounds are calculated in Annexe B. The different values of contamination for each background are presented in Table 7.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$B/S$ at 95% CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \to D^0(K^+\pi^-\pi^0)\pi^+$</td>
<td>$&lt; 6.16 \times 10^{-2}$</td>
</tr>
<tr>
<td>$B^+ \to D^*0(D^0(K^+\pi^-)\gamma)\pi^+$</td>
<td>$&lt; 8.01 \times 10^{-4}$</td>
</tr>
<tr>
<td>$B^+ \to K^{*+}(K^+\pi^0)\pi^+\pi^-$</td>
<td>$(1.79^{+4.42}_{-1.48}) \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 7: Peaking background contaminations using the branching fractions calculated in Annexe B and the efficiencies measured in Table 5.

Even if the contamination of $B^+ \to \bar{D}^0\pi^+$ is potentially large, the mass cuts can make us confident about the real contribution of this background in the $B^+ \to K^+\pi^-\pi^+\gamma$ mass spectrum.

### 5.2 Partially reconstructed background: $K^*\pi\gamma$ reconstructed as $K^*\gamma$

Decays which looks like the signal by missing one or several particles are called partially reconstructed backgrounds. The fit of Figure 5.4 gives an idea of the typical shape for a “missing $\pi$” background. This is the fit of $B^0 \to K^{*0}\gamma$ candidates reconstructed in a $B^+ \to K^{*0}\pi^+\gamma$ MC sample. The fitting function is an Argus function convoluted with a Gaussian function: an Argus function is chosen because of the shape of the endpoint given by the missing pion and the Gaussian function models the mass resolution. The Argus probability density function is given by

$$f(x; c, m_0, p) = \frac{2^{-p} \Gamma(p+1)}{\Gamma(p+1) - \Gamma(p+1, \frac{1}{2}c^2)} \cdot \frac{x}{m_0^2} \left(1 - \frac{x^2}{m_0^2}\right)^{p} \exp\left(-\frac{1}{2}c^2 \left(1 - \frac{x^2}{m_0^2}\right)\right).$$  

$$f(x; c, m_0, p) = \frac{2^{-p} \Gamma(p+1)}{\Gamma(p+1) - \Gamma(p+1, \frac{1}{2}c^2)} \cdot \frac{x}{m_0^2} \left(1 - \frac{x^2}{m_0^2}\right)^{p} \exp\left(-\frac{1}{2}c^2 \left(1 - \frac{x^2}{m_0^2}\right)\right).$$  

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PDM, winter semester 2012
for $0 \leq x < m_0$, where $m_0$ is the endpoint, $c$ the curvature, and $p$ the power. $\Gamma(-)$ is the gamma function and $\Gamma(-,-)$ is the upper incomplete gamma function. The Gaussian probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right).$$

(5.4)

In the case of the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ selection, the “missing $\pi$” background would be for example $K^*\pi\gamma$ reconstructed as a $K\pi\gamma$. Since such MC sample is not available, we assume that the “missing $\pi$” shape of Figure 5.4 is also valid for the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ selection.

![A RooPlot of "M(k*gamma)"

Figure 5.4: Fitting of MC background: $K^*\pi\gamma$ reconstructed as $K^*\gamma$. The $\xi$ parameter is the curvature of the Argus function, we will call it $c$ in further sections. The endpoint is fixed to $m_0 = 5147$ MeV/c$^2$ (equal to the MC $\mu$ value for $B^+ \rightarrow K_1(1270)^+\gamma$ obtained from the fit in Figure 4.1a minus the pion mass), the resolution is fixed to the MC signal value $\sigma = 95.7$ MeV/c$^2 = \sigma_{sig}$ and the power is set to $p = 0.5$. 

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6 Fit

The real data, corresponding to 1.85 fb\(^{-1}\) taken in 2012, can be now fitted by adding all the previously studied components: signal, combinatorial background, “missing \(\pi\)” background and other partially reconstructed background. We separate the “missing \(\pi\)” background, which is a partially reconstructed background we have studied in the previous section, from the others partially reconstructed backgrounds.

6.1 Components

The following components are included in the fit:

- the signal is fitted by a Double Crystal Ball shape, with \(\alpha\) and \(n\) parameters fixed from previous MC fits (see Figure 4.1), and \(\mu, \sigma\) free;
- the combinatorial background is fitted with an exponential, with its decay constant \((c_{comb})\) as a free parameter;
- the “missing \(\pi\)” background is fitted with a convolution of an Argus function with a Gaussian function, with the Argus endpoint \(m_0\) fixed to the signal mean minus the mass of the pion, \(\sigma\) the same as the signal, and the \(c_{miss\pi}\) parameter fixed to the obtained fit value in Figure 5.4;
- the partially reconstructed background is fitted with an Argus function with all its parameters \((m_{0,partial}, c_{partial}, p_{partial})\) free.

The fractions of these components, \(f_{\text{sig}}\), \(f_{\text{comb}}\), \(f_{\text{miss}\pi}\) and \(f_{\text{partial}} = 1 - f_{\text{sig}} - f_{\text{comb}} - f_{\text{miss}\pi}\), are free.

6.2 Results

The results of the unbinned extended maximum likelihood fits for \(B^+ \to K^+\pi^-\pi^+\gamma\) and \(B^0 \to K^{*0}\gamma\) are shown in Figures 6.1a and 6.1b. The fitted parameters are given in Table 8 and the correlations between the fitted fractions are given in Annexe D.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value for (B^+ \to K^+\pi^-\pi^+\gamma)</th>
<th>value for (B^0 \to K^{*0}\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu) [MeV/c(^2)]</td>
<td>5286.2 ± 1.5</td>
<td>5297.90 ± 0.90</td>
</tr>
<tr>
<td>(\sigma) [MeV/c(^2)]</td>
<td>93.2 ± 1.6</td>
<td>92 ± 1</td>
</tr>
<tr>
<td>(c_{comb})</td>
<td>((-5.7 \pm 0.5) \times 10^{-4})</td>
<td>((-6.09 \pm 0.17) \times 10^{-4})</td>
</tr>
<tr>
<td>(m_{0,partial}) [MeV/c(^2)]</td>
<td>4829 ± 8</td>
<td>4971 ± 159</td>
</tr>
<tr>
<td>(c_{partial})</td>
<td>(-0.0012 \pm 6.4)</td>
<td>(-1.7 \pm 0.4)</td>
</tr>
<tr>
<td>(p_{partial})</td>
<td>1.31 ± 0.27</td>
<td>1.55 ± 0.23</td>
</tr>
<tr>
<td>(N_{\text{evt}})</td>
<td>((5.607 \pm 0.024) \times 10^4)</td>
<td>((8.831 \pm 0.030) \times 10^4)</td>
</tr>
<tr>
<td>(f_{\text{sig}}) [%]</td>
<td>16.25 ± 0.26</td>
<td>23.65 ± 0.18</td>
</tr>
<tr>
<td>(f_{\text{comb}}) [%]</td>
<td>20.8 ± 1.1</td>
<td>44.5 ± 0.9</td>
</tr>
<tr>
<td>(f_{\text{miss}\pi}) [%]</td>
<td>20.0 ± 0.5</td>
<td>10.5 ± 0.8</td>
</tr>
<tr>
<td>(\chi^2/\text{ndf})</td>
<td>1.320</td>
<td>1.038</td>
</tr>
</tbody>
</table>

Table 8: Fitted parameters of the mass distribution of \(B^+ \to K^+\pi^-\pi^+\gamma\) and \(B^0 \to K^{*0}\gamma\) signal events. \(N_{\text{evt}}\) is the total number of events.

The combinatorial background in the \(B^+ \to K^+\pi^-\pi^+\gamma\) mass fit is smaller than in the \(B^0 \to K^{*0}\gamma\) mass fit. As an hypothesis, we can think that there are less three-body than two-body combinations because it is less probable to form a good vertex with three random particles rather than only with two.
6.3 Cross-checks

We check the compatibility of the fit by drawing the pull distribution, i.e., the distribution of the $\chi^2$ residuals defined in Equation 4.2. For each of the two mass fits we obtain, within statistical uncertainties, a Gaussian distribution centred at zero and with a unity width, as shown in Figure 6.2.
Figure 6.2: Distribution of the $\chi^2$ residuals for the mass fits shown in Figure 6.1a.

To be sure that no forgotten background is peaking under the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ signal, we compare the fitted $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ mass resolution, $\sigma_{K\pi\pi\gamma}^{\text{data}}$, with that in the MC, $\sigma_{K\pi\pi\gamma}^{\text{MC}}$. Their ratio may not be equal to 1 if the MC is not perfectly reproducing the data. The same ratio is calculated for the $B^0 \rightarrow K^{*0}\gamma$ decay which has already been studied in detail in LHCb [11]. The ratios for $K^{*}\gamma$ and $K\pi\pi\gamma$ will be compared and, if there is no forgotten peaking background in the $K\pi\pi\gamma$ mass range, they should be equal. If some background were to be forgotten, it could artificially increase $\sigma_{K\pi\pi\gamma}^{\text{data}}$.

From the results of the previous MC and real data fits of the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ mass spectra (Tables 4 and 8), the MC/data resolution ratios are

$$\frac{\sigma_{K\pi\pi\gamma}^{\text{MC}}}{\sigma_{K\pi\pi\gamma}^{\text{data}}} = \frac{95.7 \pm 2.7}{93.2 \pm 1.6} = 1.027 \pm 0.034,$$

$$\frac{\sigma_{K^{*}\gamma}^{\text{MC}}}{\sigma_{K^{*}\gamma}^{\text{data}}} = \frac{95.96 \pm 0.88}{91.56 \pm 0.96} = 1.048 \pm 0.015,$$

leading to a double ratio

$$\frac{\sigma_{K\pi\pi\gamma}^{\text{MC}}/\sigma_{K\pi\pi\gamma}^{\text{data}}}{\sigma_{K^{*}\gamma}^{\text{MC}}/\sigma_{K^{*}\gamma}^{\text{data}}} = 0.980 \pm 0.035$$

compatible with 1.
7 Results and conclusions

The number of $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ signal events are given in Table 8. Then the branching fraction for the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ decay can be calculated.

7.1 Yields

From Table 8, we obtain the values for $N_{evt}$ and $f_{sig}$ for the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B \rightarrow K^*\gamma$ decays. Then the $N_{K\pi\pi\gamma}$ and $N_{K\gamma\gamma}$ yields can be calculated according to the relation $N = f_{sig} \cdot N_{evt}$:

\[
N_{K\pi\pi\gamma} = (9.11 \pm 0.15) \cdot 10^3, \quad (7.1)
\]

\[
N_{K\gamma\gamma} = (20.89 \pm 0.17) \cdot 10^3. \quad (7.2)
\]

7.2 Branching fraction estimation

The branching fraction for the studied radiative decays can be calculated thanks to the relation

\[
N = \mathcal{L} \cdot \sigma_{bb} \cdot 2f \cdot B(B \rightarrow X) \cdot \varepsilon , \quad (7.3)
\]

where $\mathcal{L}$ is the integrated luminosity, $\sigma_{bb}$ is the cross-section of $bb$-pair production, $f$ is the hadronization fraction, $B(B \rightarrow X)$ is the branching fraction of the $B \rightarrow X$ decay and $\varepsilon$ is the total efficiency. The branching fraction for $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ can be directly calculated as done in Annexe E. But, in order to reduce the contribution of statistical uncertainty on luminosity or cross-section and because branching fraction for $B^0 \rightarrow K^{*0}\gamma$ is well measured, we calculate the ratio of branching fractions. Therefore, the ratio of branching fractions between $B^0 \rightarrow K^{*0}\gamma$ and $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ is

\[
\frac{B(B^+ \rightarrow K^+\pi^-\pi^+\gamma)}{B(B^0 \rightarrow K^{*0}\gamma) \cdot B(K^{*0} \rightarrow K^+\pi^-)} = \frac{N_{K\pi\pi\gamma} \cdot \varepsilon_{B \rightarrow K^{*}\gamma}}{N_{K\gamma\gamma} \cdot \varepsilon_{B \rightarrow K\pi\pi\gamma} \cdot f_{B^0}} \cdot \frac{f_{B^0}}{f_{B^+}},
\]

where we assume $\frac{f_{B^0}}{f_{B^+}} = 1$. We calculate the ratio $N_{K\pi\pi\gamma}/N_{K\gamma\gamma}$ from the fitted yields:

\[
\frac{N_{K\pi\pi\gamma}}{N_{K\gamma\gamma}} = \frac{(9.11 \pm 0.15) \cdot 10^3}{(20.89 \pm 0.17) \cdot 10^3} = (4.36 \pm 0.08) \cdot 10^{-1}
\]

We calculate the efficiencies for the two decays using the results of Section 4.2:

\[
\varepsilon_{B \rightarrow K\pi\pi\gamma} = (\varepsilon_{gen} \times \varepsilon_{sed}) \times \varepsilon_{trig} = (1.46 \pm 0.043) \cdot 10^{-3} \times \varepsilon_{trig},
\]

\[
\varepsilon_{B \rightarrow K^{*}\gamma} = (3.4 \pm 0.016) \cdot 10^{-3} \times \varepsilon_{trig}.
\]

Then, assuming the trigger efficiencies are the same for the two decays,

\[
\frac{B(B^+ \rightarrow K^+\pi^-\pi^+\gamma)}{B(B^0 \rightarrow K^{*0}\gamma) \cdot B(K^{*0} \rightarrow K^+\pi^-)} = (4.36 \pm 0.08) \cdot 10^{-1} \cdot (3.4 \pm 0.016) \cdot 10^{-3} \cdot (1.46 \pm 0.043) \cdot 10^{-3}
\]

\[
= (1.02 \pm 0.04).
\]

(7.4)

Since $B(B^0 \rightarrow K^{*0}\gamma) \cdot B(K^{*0} \rightarrow K^+\pi^-)$ is well measured $B(B^0 \rightarrow K^{*0}\gamma) \cdot B(K^{*0} \rightarrow K^+\pi^-) = (4.33 \pm 0.15) \cdot 10^{-5}$ \cite{HFAG}, we use it to find the branching fraction for $B^+ \rightarrow K^+\pi^-\pi^+\gamma$:

\[
\Rightarrow \frac{B(B^+ \rightarrow K^+\pi^-\pi^+\gamma)}{B(B^0 \rightarrow K^{*0}\gamma) \cdot B(K^{*0} \rightarrow K^+\pi^-)} = (29.3 \pm 1.4) \cdot 10^{-6}
\]

(7.5)

This result is compatible with the HFAG value for the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ branching fraction.

\footnote{We consider the correlations in the error calculation, the correlation matrix is given in Annexe D.}

\footnote{The error calculated here is statistical only.}
7.3 Conclusion

The study of radiative decays, which are an example of Flavour-Changing Neutral Currents, can reveal new physics beyond the Standard Model by proving the existence of new heavy particles in the $b \rightarrow s\gamma$ loop. The first step in radiative decays studies is the selection. Since the selection criteria are not removing all the possible backgrounds, we had to look for possible backgrounds that can pass the selection. Then a MC study is needed to associate a shape to the signal and each background types which would be contributing to the data. This MC study enable to have an idea of the shape and size of the backgrounds. We obtained a pdf for the data which combines signal and backgrounds shapes, and used it to fit the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ mass spectra. The consistency of the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ fit is verified by a few cross-checks such as residuals projection, resolution of signal and branching ratio calculation. The $B^0 \rightarrow K^{*0}\gamma$ radiative decay is used as a reference in those cross-checks.

Finally, the branching fraction for the ratio between $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ and $B^0 \rightarrow K^{*0}\gamma$ radiative decays is estimated from data fit yields and MC efficiencies:

\[
\frac{\mathcal{B}(B^+ \rightarrow K^+\pi^-\pi^+\gamma)}{\mathcal{B}(B^0 \rightarrow K^{*0}\gamma)} \cdot \mathcal{B}(K^{*0} \rightarrow K^+\pi^-) = 1.02 \pm 0.04. \tag{7.6}
\]

The branching ratio for $B \rightarrow K\pi\pi\gamma$ is then found to be

\[
\mathcal{B}(B^+ \rightarrow K^+\pi^-\pi^+\gamma) = (29.3 \pm 1.4) \times 10^{-6}, \tag{7.7}
\]

where the quoted error is statistical only. This result is compatible with the measured values in previous studies made by the Babar and Belle collaborations.

Systematic errors have not been considered in this work. An additional study on systematics can be performed, since systematic errors would appear both in the background description and in the efficiencies calculation. The choice of the shapes and components for the data fitting is adding a systematic error as in the MC simulation, some MC/data discrepancies can be observed which affect the selection efficiencies. Moreover the assumption of equal trigger efficiencies hides a non-negligible systematic error.
ANNEXE

A Neyman construction and \((N_{sel}, \varepsilon)\) plot

Let \(f(t|\theta)\) be a pdf with \(t(X)\) some function of the data and \(\theta\) a parameter. We consider the probability given by [16]:

\[
\beta = \int_{t_1}^{t_2} f(t|\theta)dt.
\]

The solution of this equation, i.e. the interval \([t_1(\theta), t_2(\theta)]\), is unique if we impose

\[
\int_{-\infty}^{t_1} f(t|\theta)dt = \frac{1 - \beta}{2} = \int_{t_2}^{\infty} f(t|\theta)dt.
\]

For each value of \(\theta\) we can find the \([t_1(\theta), t_2(\theta)]\) interval and then plot the \(t_1(\theta)\) and \(t_2(\theta)\) functions. The region between them is the confidence belt. When it is constructed, we can determine from experimental data \(t\), the confidence interval for \(\theta\), \([\theta_L, \theta_U]\). Here \(t\) is known as \(N_{sel}\) (number of events selected) and \(\theta\) as \(\varepsilon\) the efficiency of selection. \(t_i(\theta)\) functions are not continuous but discrete so the previous integral becomes a sum.

Concretely our statistical study is the following: the selection efficiency and the number of selected events are related as \(N_{in} \cdot \varepsilon_{sel} = N_{sel}\). For each value of \(\varepsilon_{sel}\), we generate a value for \(N_{sel}\) (in a loop over \(N_{in}\), a random number between 0 and 1 is chosen, if this one is lower than the efficiency value then the event is “selected” and we add +1 to the \(N_{sel}\) value). This operation is repeated 10000 times to obtain 10000 values for \(N_{sel}\) and then a distribution is drawn (see an example in Figure A.1a). From this distribution, we can look for the \(N_{sel}\) value for which 95% of the events are selected. For a confidence belt, we want both upper and lower limits on \(N_{sel}\) that contains 95% of the events: so we ask for the \(N_{up/down}\) value such that the number of events with \(N_{sel} < N_{up}\) (respectively \(N_{sel} > N_{down}\)) is equal to 10000 \times 2.5%. For an upper limit, we are looking for a \(N_{up}\) value such that the number of events is equal to 10000 \times 5% for \(N_{sel} < N_{up}\). Then the procedure is repeated for several values of \(\varepsilon\). Figure A.1c can be drawn “horizontally” (Figure A.1b): for one value of \(\varepsilon\) we have 2 points (confidence belt) or 1 point (upper limit).
B Calculation of resonance and background branching fractions

We try here to estimate the branching fraction value for some resonances or backgrounds for the $B^+ \rightarrow K^+\pi^-\pi^+\gamma$ decay by looking at every possible channel leading to a $K^+\pi^-\pi^+\gamma$ final state (or a final state which can be detected as $K^+\pi^-\pi^+\gamma$).

Resonances:

1. The $B^+ \rightarrow K_1(1270)^+\gamma$ decay has a branching fraction given by $B(B^+ \rightarrow K_1(1270)^+\gamma) = (4.3 \pm 1.3) \times 10^{-5}$.

Figure A.1: Example of the Neyman construction.
So visible branching fraction is \((1.54 \pm 0.50) \cdot 10^{-5}\).

2. The \(B^+ \rightarrow K_1(1400)^+\gamma\) decay has a branching fraction given by \(\mathcal{B}(B^+ \rightarrow K_1(1400)^+\gamma) < 1.5 \times 10^{-5}\)

So visible branching fraction is \(< (3.66 \pm 3.02) \cdot 10^{-7}\).

3. The \(B^+ \rightarrow K_2^*(1430)^+\gamma\) decay has a branching fraction given by \(\mathcal{B}(B^+ \rightarrow K_2^*(1430)^+\gamma) = (1.4 \pm 0.4) \times 10^{-5}\)

So visible branching fraction is \((3.39 \pm 0.99) \cdot 10^{-6}\).
C Isospin conservation in strong decays

To determine the branching fractions for some strong decays $X^0 \to Y^+ Z^-$ (or $Y^0 Z^0$), we use isospin conservation. The final results are given in Table 9. Here is the isospin state $|I I_3\rangle$ of each particle:

- $|K^0\rangle = |K^{*0}\rangle = |d\bar{s}\rangle = |\frac{1}{2} - \frac{1}{2}\rangle$
- $|K^+\rangle = |K^{*+}\rangle = |u\bar{s}\rangle = |\frac{1}{2} \frac{1}{2}\rangle$
- $|\pi^+\rangle = |u\bar{d}\rangle = |1 1\rangle$

### Backgrounds:

1. The $B^+ \to \bar{D}^0 \pi^+$ decay has a branching fraction given by $\mathcal{B}(B^+ \to \bar{D}^0 \pi^+) = (4.84 \pm 0.15) \times 10^{-3}$

<table>
<thead>
<tr>
<th>Decay $\bar{D}^0 \to K^+ \pi^- \pi^0,\bar{\pi}^0$</th>
<th>Branching fraction $(13.9 \pm 0.5) \cdot 10^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Rightarrow (6.73 \pm 0.32) \cdot 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>
So visible branching fraction is $(6.73 \pm 0.32) \cdot 10^{-4}$.

2. The $B^+ \to D^0 \pi^+$ decay has a branching fraction given by $\mathcal{B}(B^+ \to D^0 \pi^+) = (2.9 \pm 0.7) \times 10^{-3}$

<table>
<thead>
<tr>
<th>Decay $D^0(2007)\to D^0 \gamma,$</th>
<th>Branching fraction $(38.1 \pm 2.9) \cdot 10^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Rightarrow (4.30 \pm 1.08) \cdot 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
So visible branching fraction is $(4.30 \pm 1.08) \cdot 10^{-5}$.

3. The $B^+ \to K^{*}(892)^+ \pi^+ \pi^-$ decay has a branching fraction given by $\mathcal{B}(B^+ \to K^{*}(892)^+ \pi^+ \pi^-) = (7.5 \pm 1.0) \times 10^{-5}$

<table>
<thead>
<tr>
<th>Decay $K^{*}(892)^+ \to K^+ \pi^+ \pi^0,\bar{\pi}^0$</th>
<th>Branching fraction $&lt; 7 \cdot 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Rightarrow (2.51 \pm 0.33) \cdot 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
So visible branching fraction is $(2.51 \pm 0.33) \cdot 10^{-5}$.

4. The $B^0 \to K^{*}(892)^0 \gamma$ decay has a branching fraction given by $\mathcal{B}(B^0 \to K^{*}(892)^0 \gamma) = (4.33 \pm 0.15) \times 10^{-5}$

<table>
<thead>
<tr>
<th>Decay $K^{*}(892)^0 \to K^+ \pi^-,$</th>
<th>Branching fraction $\frac{7}{10} \cdot (100%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Rightarrow (2.9 \pm 0.1) \cdot 10^{-5}$</td>
<td></td>
</tr>
</tbody>
</table>
So visible branching fraction is $(2.9 \pm 0.1) \cdot 10^{-5}$.

5. The $B^0 \to \rho^0 \gamma$ decay has a branching fraction given by $\mathcal{B}(B^0 \to \rho^0 \gamma) = (8.6 \pm 1.5) \times 10^{-7}$

<table>
<thead>
<tr>
<th>Decay $\rho^0 \to \pi^+ \pi^-,$</th>
<th>Branching fraction $\frac{7}{10} \cdot (100%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Rightarrow (4.3 \pm 0.75) \cdot 10^{-7}$</td>
<td></td>
</tr>
</tbody>
</table>
So visible branching fraction is $(4.3 \pm 0.75) \cdot 10^{-7}$.
\[ |\pi^0\rangle = \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle = |1\ 0\rangle \]

\[ |\pi^-\rangle = |d\bar{u}\rangle = |1\ -1\rangle \]

\[ |\rho^0\rangle = |1\ 0\rangle \]

\[ |\omega^0\rangle = |f_0\rangle = |\eta^0\rangle = |0\ 0\rangle \]

The calculations for several decays (needed in the previous section for the branching fraction calculation) are given here:

- **\( K^*0 \to K^+\pi^- \)**
  \[
  |K^+\pi^-\rangle = \sqrt{\frac{3}{3}} |\frac{3}{2} - \frac{1}{2}\rangle - \sqrt{\frac{2}{3}} |\frac{1}{2} - \frac{1}{2}\rangle \\
  \Rightarrow |\langle K^*0|K^+\pi^-\rangle|^2 = \frac{2}{3}
  \]

- **\( K^*0 \to K^0\pi^0 \)**
  \[
  |K^0\pi^0\rangle = \sqrt{\frac{3}{3}} |\frac{3}{2} - \frac{1}{2}\rangle + \sqrt{\frac{2}{3}} |\frac{1}{2} - \frac{1}{2}\rangle \\
  \Rightarrow |\langle K^*0|K^0\pi^0\rangle|^2 = \frac{1}{3}
  \]

- **\( K^*+ \to K^+\pi^0 \)**
  \[
  |K^+\pi^0\rangle = \sqrt{\frac{2}{3}} |\frac{3}{2} - \frac{1}{2}\rangle - \sqrt{\frac{2}{3}} |\frac{1}{2} - \frac{1}{2}\rangle \\
  \Rightarrow |\langle K^*+|K^+\pi^0\rangle|^2 = \frac{1}{3}
  \]

- **\( K^*+ \to K^0\pi^+ \)**
  \[
  |K^0\pi^+\rangle = \sqrt{\frac{1}{2}} |\frac{3}{2} + \frac{1}{2}\rangle + \frac{1}{2} |\frac{1}{2} - \frac{1}{2}\rangle \\
  \Rightarrow |\langle K^*+|K^0\pi^+\rangle|^2 = \frac{2}{3}
  \]

- **\( K^*- \to K^-\pi^0 \)**
  \[
  |K^-\pi^0\rangle = \sqrt{\frac{2}{3}} |\frac{3}{2} - \frac{1}{2}\rangle + \frac{1}{2} |\frac{1}{2} - \frac{1}{2}\rangle \\
  \Rightarrow |\langle K^-|K^-\pi^0\rangle|^2 = \frac{1}{3}
  \]

- **\( K^*- \to K^0\pi^- \)**
  \[
  \bar{|K^0\pi^-\rangle} = \sqrt{\frac{1}{2}} |\frac{3}{2} - \frac{1}{2}\rangle - \frac{1}{2} |\frac{1}{2} - \frac{1}{2}\rangle \\
  \Rightarrow |\langle K^-|\bar{K^0\pi^-}\rangle|^2 = \frac{2}{3}
  \]

- **\( \rho^0 \to \pi^+\pi^- \)**
  \[
  |\pi^+\pi^-\rangle = \sqrt{\frac{1}{6}} |2\ 0\rangle + \sqrt{\frac{1}{2}} |1\ 0\rangle + \sqrt{\frac{1}{3}} |0\ 0\rangle \\
  \Rightarrow |\langle \rho^0|\pi^+\pi^-\rangle|^2 = \frac{1}{2}
  \]

(same calculation for \( \pi^-\pi^+ \))
**•** $K_1^+ \rightarrow K^+ \rho^0$

\[
|K^+ \rho^0\rangle = \sqrt{\frac{2}{3}}|\frac{1}{2} \ 1\rangle - \sqrt{\frac{1}{3}}|\frac{1}{2} \ 0\rangle
\]

\[\Rightarrow |\langle K_1^+ | K^+ \rho^0 \rangle|^2 = \frac{1}{3}\]

**•** $K_2^{*+} \rightarrow K^+ \omega^0$ (on $f_0$ or $\eta^0$)

\[|K^+ \omega^0\rangle = |K^+\rangle \]

\[\Rightarrow |\langle K^{*+} | K^+ \omega^0 \rangle|^2 = 1\]

---

**Summary table**

| Branching Fraction | $|\langle K^{*0} | K^+ \pi^- \rangle|^2$ | $|\langle K^{*+} | K^0 \pi^0 \rangle|^2$ | $|\langle K^{*+} | K^0 \pi^+ \rangle|^2$ | $|\langle K^{*0} | K^0 \pi^- \rangle|^2$ | $|\langle K^0 \pi^+ \pi^- \rangle|^2$ | $|\langle K_1^+ | K^+ \rho^0 \rangle|^2$ | $|\langle K^{*+} | K^+ \omega^0 \rangle|^2$ |
|-------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $|\langle K^{*0} | K^+ \pi^- \rangle|^2$ | $\frac{2}{3}$ | | | | | | |
| $|\langle K^{*+} | K^0 \pi^0 \rangle|^2$ | $\frac{1}{3}$ | | | | | | |
| $|\langle K^{*+} | K^0 \pi^+ \rangle|^2$ | $\frac{1}{3}$ | | | | | | |
| $|\langle K^{*0} | K^0 \pi^- \rangle|^2$ | | $\frac{2}{3}$ | | | | | |
| $|\langle K^0 \pi^+ \pi^- \rangle|^2$ | | | $\frac{1}{3}$ | | | | |
| $|\langle K_1^+ | K^+ \rho^0 \rangle|^2$ | | | | $\frac{1}{3}$ | | | |
| $|\langle K^{*+} | K^+ \omega^0 \rangle|^2$ | | | | | $\frac{1}{3}$ | | |

Table 9: $K^*$ and $\rho^0$ branching fractions computed from isospin conservation.

---

**D Correlation matrix for the yields**

<table>
<thead>
<tr>
<th>$f_{\text{comb}}$</th>
<th>$f_{\text{miss}\pi}$</th>
<th>$f_{\text{sig}}$</th>
<th>$N_{\text{evt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{comb}}$</td>
<td>1.000</td>
<td>-0.412</td>
<td>-0.365</td>
</tr>
<tr>
<td>$f_{\text{miss}\pi}$</td>
<td>-0.412</td>
<td>1.000</td>
<td>-0.150</td>
</tr>
<tr>
<td>$f_{\text{sig}}$</td>
<td>-0.365</td>
<td>-0.150</td>
<td>1.000</td>
</tr>
<tr>
<td>$N_{\text{evt}}$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 10: Correlation matrix for the fractions of the $B^+ \rightarrow K^+ \pi^- \pi^+ \gamma$ mass fit.

<table>
<thead>
<tr>
<th>$f_{\text{comb}}$</th>
<th>$f_{\text{miss}\pi}$</th>
<th>$f_{\text{sig}}$</th>
<th>$N_{\text{evt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{comb}}$</td>
<td>1.000</td>
<td>-0.539</td>
<td>-0.481</td>
</tr>
<tr>
<td>$f_{\text{miss}\pi}$</td>
<td>-0.539</td>
<td>1.000</td>
<td>-0.061</td>
</tr>
<tr>
<td>$f_{\text{sig}}$</td>
<td>-0.481</td>
<td>-0.061</td>
<td>1.000</td>
</tr>
<tr>
<td>$N_{\text{evt}}$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 11: Correlation matrix for the fractions of the $B^0 \rightarrow K^{*0} \gamma$ mass fit.

We can see that there is no correlation between $N_{\text{evt}}$ and the coefficients for the two decays.
E Branching fraction calculation

We start from the following formula :

\[ N_{K\pi\pi\gamma} = \mathcal{L} \cdot \sigma_{b\bar{b}} \cdot 2f_u \cdot B(B^+ \rightarrow K^+\pi^-\pi^+\gamma) \cdot \varepsilon \]

\[ \Rightarrow B(B^+ \rightarrow K^+\pi^-\pi^+\gamma) = \frac{N_{K\pi\pi\gamma}}{\mathcal{L} \cdot \sigma_{b\bar{b}} \cdot 2f_u \cdot \varepsilon} \]

- The signal yield \( N_{K\pi\pi\gamma} = N_{\text{evt}} \cdot f_{\text{sig}} \) is computed from Table 8, where correlations from Table 10 are considered.

\[ N_{K\pi\pi\gamma} = (5.607 \pm 0.024) \cdot 10^4 \times (16.25 \pm 0.26) \cdot 10^{-2} = (9.11 \pm 0.15) \cdot 10^3 \]

- The LHC luminosity is given by \( \mathcal{L} \approx 1.85 \pm 0.19 \text{ fb}^{-1} = (1.85 \pm 0.19) \cdot 10^{15} \text{ b}^{-1} \).

- The \( b\bar{b} \) cross section is given by \( \sigma_{b\bar{b}} \approx 284 \pm 20 \pm 49 \) for \( \sqrt{s} = 7 \text{ TeV} \) [21]. Assuming the relation between cross-section and energy is linear, we calculate the \( b\bar{b} \) cross-section for \( \sqrt{s} = 8 \text{ TeV} \):

\[ \sigma_{b\bar{b}} \approx 325 \pm 23 \pm 56 \approx 325 \pm 60.48 \mu\text{b}. \]

- \( f_u \approx 0.401 \pm 0.007 \) [10].

- \( \varepsilon = \varepsilon_{\text{sel}} \cdot \varepsilon_{\text{gen}} \cdot \varepsilon_{\text{trig}} = (21.48 \pm 0.06) \cdot 10^{-2} \cdot (0.68 \pm 0.02) \cdot 10^{-2} \cdot \varepsilon_{\text{trig}} \approx (14.61 \pm 0.43) \cdot 10^{-4} \cdot \varepsilon_{\text{trig}} \) (we assume the trigger efficiency is \( 0.6 \pm 0.1 \) \([19]\)).

Then

\[ B(B^+ \rightarrow K^+\pi^-\pi^+\gamma) = (21.55 \pm 5.88) \cdot 10^{-6} \]

This value is compatible with the tabulated value \((27.6 \pm 2.2) \cdot 10^{-6} \) [10].
References


[11] LHCb collaboration, R. Aaij et al., Measurement of the ratio of branching fractions $B(B^0 \rightarrow K^{*0}\gamma)/B(B^0_s \rightarrow \phi\gamma)$ and the direct CP asymmetry in $B^0 \rightarrow K^{*0}\gamma$, Nucl. Phys. B867 (2013) 1.


