Branching fractions and charge asymmetry in $B^+ \rightarrow \eta'K^+$ decays at LHCb

Swiss Federal Institute of Technology of Lausanne (EPFL)
High Energy Physics Laboratory (LPHE)

Under the supervision of:

**Dr. Frédéric Blanc**
fred.blanc@epfl.ch

Expert:

**Dr. Ivan Belyaev**
ivan.belyaev@cern.ch

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This work presents a study of the charmless decay $B^+ \rightarrow K^+ \eta'(\pi^+\pi^-\gamma)$ using 2011-Stripping 17b data recorded at LHCb, the Large Hadron Collider experiment dedicated to the study of the bottom quark and CP-violation. We report on branching fraction and charge asymmetry measurements.

1.1 Motivation and goals

Decays involving the production of $\eta$ and $\eta'$ mesons are of interest since their mechanism of production can constructively or destructively interfere leading $B^+ \rightarrow \eta K^+$ to be suppressed compared with $B^+ \rightarrow \eta' K^+$, as reported by HFAG (Heavy Flavour Averaging Group) [1] (Fig. 1.2).

Within the framework of the Standard Model, the $B^+ \rightarrow \eta' K^+$ decay is dominated by penguin and tree amplitudes. These two contributions are sketched in Fig. 1.1. Comparing the decays $B^+ \rightarrow \eta K^+$ and $B^+ \rightarrow \eta' K^+$ may give information on the relative contribution of these two diagrams by and their possible interference power[2] (see Sec. 1.2).

Figure 1.1: Feynman diagrams for $B^+ \rightarrow \eta'K^+$

Furthermore, in the Standard Model, the elements of the Cabibbo-Kobayashi-Maskawa matrix (CKM) may be complex and have an imaginary part. The interference terms between $b \rightarrow u$
tree amplitudes and $b \rightarrow s$ penguin amplitudes in the decay are governed by the phase factor of the CKM matrix, possibly leading to large CP violation in the decay (direct CP violation).

$$B(B \rightarrow (\eta, \eta')(K^{(*)}, \pi, \rho))$$

![Figure 1.2: 2010 HFAG compilation of measurements for $B(B \rightarrow (\eta, \eta')(K^{(*)}, \pi, \rho))$ decays.](image)

### 1.2 Branching Fraction

The branching fraction measurement provides information on the relative importance of tree and penguin contributions. This is therefore an essential input for the understanding of the underlying processes.

The dominant tree diagram can only produce a final $K^+\eta(\prime)$ via the non-strange component $u\bar{u}$ through the weak $b$ decay transition:

$$b \rightarrow \bar{u} + W^+ \rightarrow \bar{u} + u + \bar{s}$$  \hspace{1cm} (1.1)

If the production mechanism of $\eta$ and $\eta'$ is happening via a non-strange or a strange transition only, there would be no interference and we would observe equal contributions to the branching fractions for creation of $\eta$ and $\eta'$.

The penguin diagram is achieved with either strange or non-strange component of $\eta(\prime)$:

$$\bar{b} \rightarrow \bar{s} + g \rightarrow \bar{s} + q + \bar{q}$$  \hspace{1cm} (1.2)
A mechanism producing $\eta$ or $\eta'$ via both strange and non-strange-components can interfere constructively or destructively. In this case, we observe a contribution to the branching fraction about 8 times greater for the $B^+ \rightarrow \eta' K^+$ channel than for $B^+ \rightarrow \eta K^+ [2]$. 

The 2010 HFAG average value for the branching fraction $\mathcal{B}(B \rightarrow K \eta')$ computed from 2009 BABAR [3], 2006 Belle [4] and 2000 CLEO [5] measurements, is:

$$\mathcal{B}(B^+ \rightarrow K^+ \eta') = (70.0 \pm 2.5) \cdot 10^{-6}$$

In this work, as sketched on Fig. 1.3, the reconstruction is performed with the channel $\eta' \rightarrow \pi^+ \pi^- \gamma$ whose PDG [6] branching fraction is known to be:

$$\mathcal{B}(\eta' \rightarrow \pi^+ \pi^- \gamma) = (29.3 \pm 0.6) \cdot 10^{-2}, \text{ including resonant } \eta' \rightarrow \rho^0 \gamma$$

![Figure 1.3: Channels considered in the study of $B^+ \rightarrow \eta' K^+$](image)

An integrated luminosity $\mathcal{L} = (1017.0 \pm 36) \text{ pb}^{-1}$ collected at LHCb is used to reconstruct $B^+ \rightarrow \eta' K^+$ candidates through a well chosen set of selection criteria. The measured number of $B^+ \rightarrow \eta' K^+$ decays, $N_S$, depends on the integrated luminosity, on the cross section for formation of $b$ quark $\sigma_{bb}$, on the fraction of $b$ quarks hadronizing into $B^+$ $f_u$, on the efficiency of the selection procedure $\epsilon_{\text{sel}}$, of the stripping $\epsilon_{\text{strip}}$ and of the trigger $\epsilon_{\text{trig}}$, on the geometric acceptance of the detector $\epsilon_{\text{geom}}$ and on the respective branching fractions for $B^+ \rightarrow \eta' K^+$ and $\eta' \rightarrow \pi \pi \gamma$.

$$N_S = 2 \cdot \mathcal{L} \cdot \sigma_{bb} \cdot f_u \cdot \mathcal{B}(B^+ \rightarrow K^+ \eta') \cdot \mathcal{B}(\eta' \rightarrow \pi \pi \gamma) \cdot \epsilon_{\text{geom}} \cdot \epsilon_{\text{trig}} \cdot \epsilon_{\text{strip}} \cdot \epsilon_{\text{sel}}$$

where:

- $\mathcal{L} = (1017.0 \pm 36) \text{ pb}^{-1}$ Integrated luminosity at LHCb
- $\sigma_{bb} = (284 \pm 9 \pm 20 \pm 49) \mu \text{b}$ [7] Cross section for formation of $b$-quark pair
- $f_u = (40.1 \pm 1.3) \%$ [6] Fraction of $b$ quarks hadronizing into $B^+$
- $\mathcal{B}(B^+ \rightarrow K^+ \eta') = (70.0 \pm 2.5) \cdot 10^{-6}$ [1] Branching ratio of $B^+ \rightarrow K^+ \eta'$
- $\mathcal{B}(\eta' \rightarrow \pi \pi \gamma) = (29.3 \pm 0.6) \cdot 10^{-2}$ [6] Branching ratio of $\eta' \rightarrow \pi \pi \gamma$
- $\epsilon_{\text{tot}}$ Total efficiency

We will use the PDG [6] values for $\mathcal{B}(\eta' \rightarrow \pi \pi \gamma)$ and $f_u$. The number of signal will be retrieved from a fit performed on real data and the efficiencies computed from a Monte-Carlo simulation.
1.3 Discrete symmetries and CP violation

The Standard model has three associated natural near-symmetries:

- The symmetry under \textbf{charge-conjugation} transformation, called \textit{C}-symmetry: the evolution of a system remains unchanged when replacing the charges by their opposite value (particle $\rightarrow$ antiparticle),

- The symmetry under \textbf{space-inversion}, called \textit{parity} or \textit{P}-symmetry: a system is equivalent to its reflexion in a mirror,

- The symmetry under \textbf{time-reversal}, called \textit{T}-symmetry: the description of the evolution of a system is symmetric under time inversion.

These symmetries are broken in the current Universe. However, the Standard Model predicts that the combination of \textit{C}, \textit{P}, and \textit{T} must be a symmetry, that is the combination of these three transformations must keep the system properties unchanged.

CP-violation plays a crucial role explaining the presence of the non-negligible amount of baryonic matter in our Universe. It is also a key mechanism involved in the decays governed by weak interaction. As previously stated, it is incorporated in the Standard Model via a complex phase in the CKM matrix which describes the mixing of quarks. The necessary condition for the appearance of this complex term, and hence, for the \textit{CP} violation, is the existence of a minimum of three generations of quarks.

1.3.1 Direct CP-violation

The relevant interaction for \textit{CP} violation is the weak force which occurs through a $W^\pm$ mediator. The exchange boson can, for instance, turn a quark $b$ into a quark $u$ with a given coupling constant $V_{ub}$. The complex nature of this coupling constant and its interference with amplitudes of different phases gives rise to \textit{CP} violation.

Using the Wolfenstein parameterization, the quark-mixing matrix is:

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & \lambda A^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A^2 \lambda^2 \\ A\lambda^2(1 - \rho - i\eta) & -A^2 \lambda & 1 \end{bmatrix}$$  \hspace{1cm} (1.4)

where $\lambda$, $A$, $\rho$, and $\eta$ are currently known to be [6]:

\begin{align*}
\lambda &= 0.22535 \pm 0.00065 \\
A &= 0.811^{+0.022}_{-0.012} \\
\rho &= 0.131^{+0.026}_{-0.013} \\
\eta &= 0.345^{+0.013}_{-0.014}
\end{align*}

The term $i\eta$ is the one contributing to the complex phase inducing \textit{CP} violation when different from zero.

The CKM matrix can be depicted by triangles in the complex plane whose geometry is governed by the unitarity properties of the matrix. For instance:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$  \hspace{1cm} (1.5)
giving the following angles:

\[
\alpha = \arg \left( \frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right) \quad \beta = \arg \left( \frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right) \quad \gamma = \arg \left( \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)
\] (1.6)

Figure 1.4: Example of CKM triangle using unitarity condition 1.5

One can write the partial width of the \( B^+ \rightarrow \eta' K^+ \) and \( B^- \rightarrow \eta' K^- \) as functions of amplitudes of the allowed tree and loop Feynman diagrams (see Fig. 1.1):

\[
A_1 = |A_1|e^{i\delta_1}e^{i\phi_1} \text{ (tree)}
\]
\[
A_2 = |A_2|e^{i\delta_2}e^{i\phi_2} \text{ (loop)}
\]

where \( \phi_i \) is the weak CKM phase and \( \delta_i \) is the strong phase of the process \( i \).

The weak and strong phases appear in addition to another conventional phase, called spurious phase, which comes from an arbitrary choice of phase and that we fix to zero for simplicity.

The strong phase origin is due to the possible contribution from intermediate on-shell states in the decay process and is, hence, very difficult to measure. Since it is coming from \( CP \)-invariant interactions, the strong phases are the same in the amplitude for a process and its conjugate. The difference between strong phases in two different terms of the amplitude \( (\delta_1 - \delta_2) \) is convention-independent. The weak phase occurs only in the coupling of \( W \) bosons and may vary for the conjugate state. Similarly, since the phase is convention-dependent, only the relative weak phases \( (\phi_1 - \phi_2) \) has a physical meaning.

The Fig. 1.5 shows strong and weak phases for a given state and its \( CP \)-conjugate, representing the first amplitude horizontally by convention.

Figure 1.5: Phase representation of the amplitudes of a process (left) and its \( CP \)-conjugate (right). \( \gamma \) is the weak CKM phase and \( \delta \) the strong phase.
Unfortunately, since the determination of strong phases is very challenging, measurements of $CP$ violation do not yield clean information about the weak phases.

The decay rates are written:

\[
\Gamma(B^+ \to \eta' K^+) = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos[(\phi_1 - \phi_2) + (\delta_1 - \delta_2)] \tag{1.7}
\]

\[
\Gamma(B^- \to \eta' K^-) = |A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos[-(\phi_1 - \phi_2) + (\delta_1 - \delta_2)] \tag{1.8}
\]

with $\phi_1 - \phi_2$ is a function of $\gamma$ since $V_{ub}$ is the only complex term of the $CKM$ matrix involved in these two processes. The rate difference between the positively charged state and its $CP$-conjugated state is therefore dependent on the presence of different strong and weak phases in each amplitudes.

**Charge asymmetry**

The direct $CP$ violation can be detected as a charge asymmetry from the decay rates differences between the two charged modes:

\[
A = \frac{\Gamma(B^- \to \eta' K^-) - \Gamma(B^+ \to \eta' K^+)}{\Gamma(B^- \to \eta' K^-) + \Gamma(B^+ \to \eta' K^+)} \tag{1.9}
\]

which can be written, using eqs 1.7 and 1.8:

\[
A = \frac{2|A_1||A_2|\sin(\phi_1 - \phi_2)\sin(\delta_1 - \delta_2)}{|A_1|^2 + |A_2|^2 + 2|A_1||A_2|\cos(\phi_1 - \phi_2)\cos(\delta_1 - \delta_2)} \tag{1.10}
\]

If the difference between the phase terms are non zero, the term of asymmetry would be different from zero as well. $A_1$ and $A_2$ must have the same order of magnitude for this asymmetry to be noticeable.

### 1.4 The LHCb experiment

The LHCb experiment, detailed on Fig. 1.6, consists in a single-arm spectrometer built towards the direction of propagation of one of the protons in the LHC beam. It is essentially composed of:

- a vertex locator (VELO) in the vicinity of the collision point. The VELO is made of silicon-strip detectors used to determine the B-mesons vertices,
- a dipole magnet used to bend the charged particles trajectories and deduce their momentum,
- a tracking system: Trigger Tracker (TT) and tracking stations (T1, T2, T3) on both sides of the magnet. They are composed of silicon and streamer tubes and used to determine particles trajectories and momenta,
- two Ring Imaging CHerenkov counters (RICH 1 & 2) to identify particles by measuring their velocity using the Cherenkov light cone they produce and combining this with the momentum information given by the tracking system,
- a set of Hadronic and Electromagnetic CALorimeters (HCAL & ECAL) to identify the particles with high transverse momentum, by measuring the energy deposit of hadronic and electronic showers. The photons of interest will hence be detected with the ECAL.
• five muons detection stations (M1, ... M5) made of multi-wire proportional chambers.

The VErtex LOcator and the tracking system are the main sources of information used for the analysis of charged particles. The electromagnetic calorimeter is used to detect photons. The PID information from the two Cherenkov detectors is also needed in order to differentiate kaons from pions.

1.5 The software environment

The DaVinci v30r0p1 framework, composed of a large collection of C++ libraries, is the environment to produce the data for the analysis. After having been computed and collected with DaVinci algorithms, the information coming from Monte-Carlo simulation or real LHCb data, can be stored in an nTuple which basically consists in an array containing all the variables of interest.

The ROOT [8] application is a C++ interpreter developed at CERN which allows to treat this nTuple and produce all the graphs and fits presented in this report.

The RooFit [9] packages have also been used for modeling the expected distributions of events. The RooFit tools are integrated with the object-oriented and interactive ROOT graphical environment.

Figure 1.6: YZ view of LHCb detector
Chapter 2

Event selection

In this section, we detail the reconstruction and selection methods used to extract the $B^+ \rightarrow \eta' K^+$ signal.

2.1 Reconstruction procedures

The principle of reconstruction is to combine charged tracks and electromagnetic showers identified by the detector to form the particle candidate of interest. We considered two options for reconstruction:

1. A partial reconstruction method using only the information coming from the final state charged particles. In this case, the photon energy and momentum measurement is avoided: these quantities are determined from kinematics and geometric constraints.

2. A full reconstruction, where all the particles, including the photon, are reconstructed. In this case, the photon is detected by the electromagnetic calorimeter.

The first method allows to avoid the uncertainty on the measured photon energy coming from the calorimeter imprecision. However, the $B$ mass expressed in terms of the direction of propagation of the $B$ meson. Hence, the resolution on the $B$ mass measurement depends on the resolution on the direction of propagation. The best angular resolution will be achieved when the $B$ meson is flying a long distance before decaying. To cut on the $B$ meson distance of flight is then required leading our statistical sample to shrink.

An additional condition is also required beforehand, since the natural width for $\eta'$ is well known and much smaller than the resolution, it is constrained to its nominal mass when combining it with a $K^+$ to build a $B^+$ candidate.

Fig. 2.1 gives the $\eta'$ mass distribution versus the $B^+$ mass distribution for a sample with (right) and without (left) constraint on $\eta'$ mass. A linear correlation between $B^+$ mass and $\eta'$ can be observed on the left figure.
CHAPTER 2. EVENT SELECTION

Figure 2.1: Correlation plots for $B^+$ mass versus $\eta'$ mass without $\eta'$ mass constraint (left) and with $\eta'$ mass constraint (right) with a sample satisfying requirements from Tables C.1 and C.2. The linear correlation clearly appears in the left plot.

Nevertheless, it completely vanishes when applying the $\eta'$ constraint. Since the correlations are avoided, the simultaneous fit can be performed by multiplying the two probability distributions functions for $B$ and $\eta'$ signal components and background components as assumed in the Likelihood formula (Eq. 3.1).

We chose the method that gives the most significant signal yield. For this purpose, we define the significance to be:

$$S = \frac{N_S}{\sigma_{N_S}}$$

where $N_S$ is the number of signal events extracted from the fit and $\sigma_{N_S}$ its uncertainty.

The partial reconstruction provided the significance, $S = 4.8$ (see Appendix A), and the full reconstruction a significance of 39 (see Chapter 3). This is why we decided to value the full reconstruction over the partial reconstruction method.

### 2.2 Selection requirements

The $B$ candidates are all $K\pi\pi\gamma$ combinations from Stripping 17b fulfilling the selection conditions given in Table 2.1.

These conditions are chosen after having optimized the selection on real data. To reach the best significance, variables with low correlation are chosen and each constraint is varied independently. We report on previous selection criteria for Stripping 17 in appendix C.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of $B$ meson</td>
<td>$5000 &lt; m_B &lt; 5600$ MeV/c²</td>
</tr>
<tr>
<td>Transverse momentum of $B$</td>
<td>$p_T(B) &gt; 2000$ MeV/c</td>
</tr>
<tr>
<td>Angle btw $p_B$ and primary vertex direction</td>
<td>$\cos(\alpha(p_B)) &gt; 0.99997$</td>
</tr>
<tr>
<td>Distance of flight for the $B$ meson</td>
<td>$D_f(B) &gt; 2.0$ mm</td>
</tr>
</tbody>
</table>

Table 2.1: Constraints on the mother particle. The conditions in grey are required in the event reconstruction, before producing the nTuple

For each reconstruction, the daughter particles are also required to satisfy a set of conditions (given on Table 2.2) which are chosen in order to maximize the significance.
### Table 2.2: Constraints on the daughter particles.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Variable</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta'$</td>
<td>$\eta'$ mass</td>
<td>$</td>
</tr>
<tr>
<td>$K$</td>
<td>Total momentum of the kaon</td>
<td>$p(K^\pm) &gt; 10000 \text{ MeV}/c$</td>
</tr>
<tr>
<td></td>
<td>Transverse momentum of the kaon</td>
<td>$p_T(K^\pm) &gt; 1000 \text{ MeV}/c$</td>
</tr>
<tr>
<td></td>
<td>Particle identification (PID)</td>
<td>Prob($K$) &gt; 0.1</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Total momentum of the pion</td>
<td>$p(\pi^\pm) &gt; 5000 \text{ MeV}/c$</td>
</tr>
<tr>
<td></td>
<td>Transverse momentum of the pions</td>
<td>$p_T(\pi^\pm) &gt; 500 \text{ MeV}/c$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Energy of the photon</td>
<td>$E_\gamma &gt; 2500 \text{ MeV}$</td>
</tr>
<tr>
<td></td>
<td>Calorimeter Confidence Level</td>
<td>$CL(\gamma) &gt; 0.05$</td>
</tr>
</tbody>
</table>

As $B$ and $\eta'$ masses will be used in a fit, the requirements imposed on them are loose to allow fitting the sidebands.
Chapter 3

Yield measurement

3.1 Maximum Likelihood Fit method

The $m_B$ and $m_{\eta'}$ distributions are fitted with signal and background functions using an extended maximum likelihood fit. RooFit allows to fit two variables simultaneously [9]. This two-dimensional optimization provides more information which results in a significance gain.

In order to perform such an analysis, the two variables of interest must be uncorrelated, such that the distribution function of the two variables is the product of each variable marginal distribution. Hence, the extended likelihood can be written as:

$$
L_{\text{tot}}(N_S, N_B) = \frac{e^{-(N_S+N_B)}}{N!} \prod_{i=1}^{N} \left[ N_S P_{S_i}^{\text{tot}}(m_B, m_{\eta'}) + N_B P_{B_i}^{\text{tot}}(m_B, m_{\eta'}) \right]
$$

(3.1)

where the total probability distribution functions (p.d.fs) for signal $S$ and background $B$ are the product of each marginal p.d.f as detailed above:

$$
P_{S_i}^{\text{tot}}(m_B, m_{\eta'}) = P_{S_i}(m_B) \cdot P_{S_i}(m_{\eta'})
$$

$$
P_{B_i}^{\text{tot}}(m_B, m_{\eta'}) = P_{B_i}(m_B) \cdot P_{B_i}(m_{\eta'})
$$

In the extended maximum likelihood fit, the extracted yields for signal $N_S$ and for background $N_B$ are considered to follow the Poisson statistics and the sum of $N_S$ and $N_B$ equals the number of input candidates $N_{\text{tot}}$ when the likelihood is maximized. The first term in the likelihood definition describes this poissonian distribution.

3.2 Monte-Carlo study

Fig. 3.1 shows the $B^+ \rightarrow \eta'K^+$ Monte-Carlo data selected with the criteria given in Tables 2.1 and 2.2. The fit exhibits two main contributions for $B$ and $\eta'$:

- The signal components are both described using the sum of two Gaussian distribution functions.
- The background is modeled with a first order Chebyshev function [11] defined in eq. B.4 in Appendix B and is composed of misreconstructed tracks.

The value of the parameters used in the fit are reported in Table 3.1. $\sigma_{G1_B}$ is the width of the first Gaussian modeling $B$ signal (in orange on fig. 3.1), $f_r(G1)$ is the ratio between the width...
of the two Gaussian for $B$ signal, $f_{\text{events}}(G1_B) \text{ and } f_{\text{events}}(G1_{\eta'})$ are the respective fraction of events associated to the first Gaussian compared with the total number of events associated to signal for $B$ and $\eta'$ distributions, $a_0(B) \text{ and } a_0(\eta')$ give the value of the first order coefficient of the Chebyshev function to model $B$ and $\eta'$ respective backgrounds.

![Figure 3.1: Simultaneous fit of the distributions of $m_B$ and $m_{\eta'}$ for a sample of Monte-Carlo events generated from Stripping 17b. For $m_B$, the first gaussian shaped component is orange dashed, the second Gaussian is red dashed. For $m_{\eta'}$, the main Gaussian is blue dashed and the other one violet dashed. For both, the plain blue lines are the resulting shapes and the black dashed lines correspond to the Chebyshev background. The final values of the parameters varied in the Log Likelihood maximization are reported in Table 3.1.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Final value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{m}_B$ [MeV/$c^2$]</td>
<td>5280.51 ± 0.61</td>
</tr>
<tr>
<td>$\bar{m}_{\eta'}$ [MeV/$c^2$]</td>
<td>959.53 ± 0.44</td>
</tr>
<tr>
<td>$\sigma_{G1_B}$ [MeV/$c^2$]</td>
<td>16.24 ± 0.23</td>
</tr>
<tr>
<td>$\sigma_{G1_{\eta'}}$ [MeV/$c^2$]</td>
<td>10.66 ± 0.74</td>
</tr>
<tr>
<td>$f_{\text{e}}(G1)$</td>
<td>(41.33 ± 3.80)%</td>
</tr>
<tr>
<td>$f_{\text{events}}(G1_B)$</td>
<td>(34.7809 %)</td>
</tr>
<tr>
<td>$f_{\text{events}}(G1_{\eta'})$</td>
<td>(76.1341)%</td>
</tr>
<tr>
<td>$a_0(B)$</td>
<td>0.29 ± 0.13</td>
</tr>
<tr>
<td>$a_0(\eta')$</td>
<td>0.29 ± 0.11</td>
</tr>
</tbody>
</table>

Table 3.1: Final values of the fitted parameters from the Monte-Carlo data.
CHAPTER 3. YIELD MEASUREMENT

3.3 Real data study

We apply the same selections procedure on real data acquired through Stripping 17b.

3.3.1 Possible sources of background

Background may come from different sources whose expected p.d.f. features are given in Table 3.2.

- **Background 1** arises from random combinations of particles in continuum.
- **Background 2** is due to real \( \eta' \) particles which are coming from other decays and random kaons.
- **Background 3** corresponds to real \( B \) mesons decays with the same final states as the signal \((\pi^+\pi^-\gamma K^+)\).
- **Background 4** is the feed-down contribution for the particular case where there is no reconstructed \( \eta' \). This could be essentially due to channels like:
  \[ \eta' \rightarrow \gamma \omega(\pi^+\pi^-\pi^0) \]
  whose total branching fraction is \((2.45 \pm 0.20)\%\).
  
  For the \( \eta' \rightarrow \gamma \omega(\pi^+\pi^-\pi^0) \) channel, the \( \eta' \) is misreconstructed coupling the two charges pions from the \( \omega \) decay with the photon. The uncharged extra pion may produce a shifted feed-down starting below \([m_B - m_{\pi^\pm}]\).
- **Background 5** is the feed-down component in the case where \( \eta' \) is truly reconstructed.
  The main contribution to this particular component is:
  \[ B \rightarrow \eta'K^*(K\pi) \]
  whose total branching fraction is \((3.8 \pm 1.2) \cdot 10^{-6}\).

The extra pion leads the \( B \) meson mass distribution to be shifted from the mass of one charged pion \([m_B - m_{\pi^\pm}]\).

<table>
<thead>
<tr>
<th></th>
<th>( B )</th>
<th>( \eta' )</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>flat</td>
<td>flat</td>
<td>strong</td>
</tr>
<tr>
<td>2</td>
<td>flat</td>
<td>peaked</td>
<td>medium</td>
</tr>
<tr>
<td>3</td>
<td>peaked</td>
<td>flat</td>
<td>negligible</td>
</tr>
<tr>
<td>4</td>
<td>feed-down</td>
<td>flat</td>
<td>weak</td>
</tr>
<tr>
<td>5</td>
<td>feed-down</td>
<td>peaked</td>
<td>weak</td>
</tr>
</tbody>
</table>

Table 3.2: Expected contributions to background for real data

3.3.2 Signal and background PDFs

The fit on the real data has been done using the combination of each contribution to signal and background stated above. These were modelled using a probability distribution function defined at appendix B.

Similarly to Monte-Carlo case, \( m_B \) and \( m_{\eta'} \) signals have been fitted using the sum of two Gaussian distributions. We chose Chebyshev probability distribution functions to describe the combinatorial background. We tried to fit the background containing real \( \eta' \) particles...
which are coming from other decays with a Chebyshev component for $m_B$ and a Gaussian for $m_{\eta'}$. The feed-down background behaviour has been modeled using phase-space background components (ARGUS [12]) with the end-point fixed at $m_B - m_{\pi} \approx 5140$ MeV. The associated background component for $m_{\eta'}$ may exhibit a peak at the effective $m_{\eta'}$ that we modeled with a Gaussian, but it may also be flat and fitted with a Chebyshev polynomial. Both feed-down contributions are convoluted with the resolution model.

3.3.3 Results of the fit

The resulting fit exhibits a significant level of combinatorial background which overcomes all the other non-significant contributions to background. So, we choose to neglect these contributions in favor of a unique probability distribution function. Since kinematic cuts are contributing at low $B$ mass leading the left sideband of the background to be depreciated, a second order Chebyshev polynomial distribution is appropriate to used the background.

![Simultaneous fit of the distributions of $m_B$ and $m_{\eta'}$ for real events from stripping 17b.](image)

(a) Linear scale

(b) Logarithmic scale

Figure 3.2: Simultaneous fit of the distributions of $m_B$ and $m_{\eta'}$ for real events from stripping 17b. For $m_B$, the first gaussian shaped component is orange dashed, the second gaussian is red dashed. For $m_{\eta'}$, the main gaussian is blue dashed and the other one violet dashed. For both, the plain blue lines are the resulting shapes and the black dashed lines correspond to the Chebyshev background. The final values of the parameters varied in the Log Likelihood maximization are reported on Table 3.3.
Fig. 3.2 shows the result of the final fit for the real data sample. The convergence values of the free parameters are reported in Table 3.3. With this process, we measure \( 2624 \pm 73 \) signal events with a significance of:

\[
S_{17b} = 39.07
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Final value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{m}_B ) [MeV/c^2]</td>
<td>5282.11 ± 0.58</td>
</tr>
<tr>
<td>( \bar{m}_{\eta'} ) [MeV/c^2]</td>
<td>959.58 ± 0.44</td>
</tr>
<tr>
<td>( \sigma_{G1_B} ) [MeV/c^2]</td>
<td>20.78 ± 0.91</td>
</tr>
<tr>
<td>( \sigma_{G1_{\eta'}} ) [MeV/c^2]</td>
<td>11.10 ± 0.73</td>
</tr>
<tr>
<td>( a_0(B) )</td>
<td>-0.43 ± 0.02</td>
</tr>
<tr>
<td>( b_0(B) )</td>
<td>-0.009 ± 0.023</td>
</tr>
<tr>
<td>( a_0(\eta') )</td>
<td>0.097 ± 0.023</td>
</tr>
<tr>
<td>( N_S )</td>
<td>2624 ± 73</td>
</tr>
<tr>
<td>( N_B )</td>
<td>6373 ± 95</td>
</tr>
</tbody>
</table>

Table 3.3: Values of parameters after the fit on real data from Stripping 17b. Fixed parameters are not reported here. \( b_0(B) \) is the second order coefficient of the Chebyshev polynomial.

\( \sigma_{G1_B} \) is the width of the first Gaussian modeling \( B \) signal, \( \sigma_{G1_{\eta'}} \) is the width of the first Gaussian modeling \( \eta' \) signal, \( a_0(B) \) and \( a_0(\eta') \) give the value of the first order coefficient of the Chebyshev function to model \( B \) and \( \eta' \) respective backgrounds. \( b_0 \) is the second order coefficient used to model \( B \) background and has been found to be compatible with zero. In the end, the \( B \) background could be considered as linear.
Chapter 4

Crosschecks

The consistency of the result is confirmed with the few crosschecks listed there after.

4.1 \( B^+ \) lifetime

We extracted the sPlot \cite{13} for the \( B \) meson proper time using the results of the fit for \( B^+ \to \eta'K^+ \) (Sec. 3.3.3). The sPlot is a particular histogram of the time of flight whose events have been weighted with regard to the \( B^+ \) distribution of mass fit, such that one allocate a more significant weighting to events affiliated to the signal in the \( B^+ \) distribution of mass than to the events coming from background.

Using Monte-Carlo data, we computed the deviation between the lifetime distribution data with and without having required our selection criteria. This gave us the correction to apply to our real data in order to take into account the acceptance effect of our selection cuts.

The weighted and corrected distribution of the \( B \) meson time of flight is then fitted using an exponential \( \chi^2 \) minimization whose theoretical distribution is implemented in ROOT:

\[
P(t) = A_0 e^{-\frac{t}{\tau}}
\]

where \( \tau \) is the characteristic lifetime of the \( B \) meson.

![Figure 4.1: Acceptance-corrected sPlot of the time of flight for the \( B \) meson for data from Stripping 17b. The distribution is plotted with a logarithmic scale. The blue line is the fit function. Due to resolution limits, the two first bins are ignored in the fit.](image)

Fig. 4.1 shows the distribution of the time of flight. We measured \( \tau = 1.66 \pm 0.06 \) ps, which is well-compatible with the PDG \cite{6} value (1.641 \pm 0.008 ps).
4.2 Invariant mass of the $\pi\pi$ system

The dipion system mass distribution for the decay of interest: $\eta' \rightarrow \pi^+\pi^-\gamma$ has been measured several times, as listed in this reference [14]. The most precise measurement has been performed by the Crystal Barrel Collaboration [15] at CERN in 1997.

The amplitude $\Gamma(\eta' \rightarrow \pi\pi\gamma)$ has two main components:

1. A dominant component due to the $\rho^0$ meson resonance visible in $e^+e^- \rightarrow \pi^+\pi^-$.  

2. An additional term predicted by QCD [16], which modifies the usual distribution and is called the box anomaly. According to the Crystal Barrel experiment, this term shifts the $\rho$ peak towards lower energies (about 20 MeV$/c^2$ shift with respect to $m_\rho = 770$ MeV$/c^2$) and increases the damping factor at high energies, making the $\pi\pi$ invariant mass spectrum an identifiable signature for $\eta'$ decays.

The resulting distribution behaviour can be described using the analytical formula [14]:

$$\frac{d\Gamma(\eta' \rightarrow \pi\pi\gamma)}{dm^{\text{inv}}_{\pi\pi}} = \frac{C^2 \alpha_{\text{em}}}{36 \left(2\pi f_\pi \right)^6} m^{\text{inv}}_{\pi\pi} \left[ 1 + \frac{3m^2}{D_\rho(m^{\text{inv}}_{\pi\pi})} \right]^2 p_\gamma^3 p_\pi^3$$

(4.2)

where:

- $C$ constant  
- $\alpha_{\text{em}}$ fine structure constant  
- $m^{\text{inv}}_{\pi\pi}$ invariant mass of the $\pi\pi$ system  
- $p_\gamma$ photon momentum in $\eta'$ rest frame  
- $p_\pi$ pion momentum in the dipion rest frame  
- $f_\pi$ pion decay constant  
- $m$ mass of $\rho_0$

and,

$$D_\rho(m^{\text{inv}}_{\pi\pi}) = m^{\text{inv}}_{\pi\pi} - m^2 - \Pi(m^{\text{inv}}_{\pi\pi})$$

(4.3)

with:

$$\Pi(m^{\text{inv}}_{\pi\pi}) \approx C_2 \cdot m^{\text{inv}}_{\pi\pi} + C_3 \cdot (m^{\text{inv}}_{\pi\pi})^2$$

where $C_2$ and $C_3$ are constants. (4.4)

On figure 4.2, we computed the $sPlot$ for the invariant mass of the $\pi\pi$-system for the signal $B^+ \rightarrow \eta'(\pi^+\pi^-)K^+$. The distribution exhibits the same peculiarities as stated above: the mass of $\rho$ is a bit shifted to the left (750 MeV$/c^2$) and the damping is more dramatic than for a simple $e^+e^- \rightarrow \rho$ resonance peak. The fit is performed using a $\chi^2$ method computing $C_1$, $C_2$ and $C_3$ parameters of equations 4.3 and 4.4. Table 4.1 shows the convergence value of $C_1$, $C_2$ and $C_3$ and the $\chi^2$ per degree of freedom.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$(1.32 \pm 0.04) \cdot 10^5$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.9808 $\pm$ 0.0003</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$(-1.5156 \pm 0.0002) \cdot 10^3$</td>
</tr>
<tr>
<td>$\chi^2_{\text{ndof}}$</td>
<td>2.62</td>
</tr>
</tbody>
</table>

Table 4.1: Parameters of the $\pi\pi$-invariant mass $\chi^2$ minimizing fit.
The results are in agreement with other experiments ([14],[15]) since the dipion mass spectrum exhibits the expected shape. In particular, we observed the same peak shift of 20 MeV, specific to $B^+ \to \eta' K^+$ decays.
Chapter 5

Branching fraction

We compute the branching fraction of \( B^+ \to \eta' K^+ \) as described in section 1.2.

5.1 Efficiencies

5.1.1 Partial efficiencies

Geometrical acceptance

The geometrical acceptance is computed from simulated data counting the fraction of events that are in the detector acceptance. We obtain:

\[
\epsilon_{\text{geom}} = (16.66 \pm 0.16)\%
\]  \hspace{1cm} (5.1)

Efficiency for selection and stripping

The selection and stripping efficiencies are measured simultaneously from the Monte-Carlo sample by dividing the number of events remaining after selection and stripping \( N_f \) by the initial number of events in the acceptance \( N_i \):

\[
\epsilon_{\text{sel}} \cdot \epsilon_{\text{strip}} = \frac{N_f}{N_i} = (1.50 \pm 0.03)\%
\]  \hspace{1cm} (5.2)

Trigger efficiency

The trigger efficiency is computed from Monte-Carlo simulated data including the trigger line cuts in the set of selection cuts and counting the number of events passing the selection with \( (N_{\text{trig}}) \) and without \( (N_f) \) trigger requirements:

\[
\epsilon_{\text{trig}} = \frac{N_{\text{trig}}}{N_f} = (41.1 \pm 1.3)\%
\]  \hspace{1cm} (5.3)

5.1.2 Total efficiency

The the total efficiency determine from Monte-Carlo data is the product of each partial efficiency:

\[
\epsilon_{\text{tot}} = \epsilon_{\text{geom}} \cdot \epsilon_{\text{strip}} \cdot \epsilon_{\text{sel}} \cdot \epsilon_{\text{trig}} = (0.103 \pm 0.004)\%
\]  \hspace{1cm} (5.4)
CHAPTER 5. BRANCHING FRACTION

5.2 Results

Assuming a total efficiency retrieved from Monte-Carlo data, with the number of signal obtained from final selection method on real data: \( N_S = 2624 \pm 73 \), the measured branching ratio for the decay \( B^+ \rightarrow K^+\eta' \) is:

\[
B(B^+ \rightarrow K^+\eta') = \frac{N_S}{2 \cdot \mathcal{L} \cdot \sigma_{bb} \cdot f_u \cdot \epsilon_{tot} \cdot B(\eta' \rightarrow \pi\pi\gamma)} = [37.3 \pm 1.0 \text{ (stat.)} \pm 7.4 \text{ (syst.)}] \cdot 10^{-6}
\]

The result is more than 4\( \sigma \) away from HFAG mean value: \( (70.0 \pm 2.5) \cdot 10^{-6} \). In the next section, we measure the detection efficiency as a function of the longitudinal and transverse momenta of the photon in order to study this discrepancy.

5.3 Photon detection efficiency

The reconstruction of \( B^+ \rightarrow \eta'(\pi\pi\gamma)K^+ \) requires the photons to be detected with a high level of precision. The quality of this detection is likely to depend on the photon momentum itself. We mapped the detection efficiency as a function of the longitudinal and transverse momenta of the photon.

For this purpose, we compute the branching fraction associated to given transversal and longitudinal momenta with the efficiency retrieved from Monte-Carlo analysis and compared it to the expected PDG value. Fig. 5.1 shows the binned distributions of the ratio of the measured branching fraction \( B(\text{meas}) \) over the P.D.G. value \( B(\text{P.D.G.}) \) for five ranges on \( p_T(\gamma) \) (left) and five ranges of values on \( p_z(\gamma) \) (right). To preserve enough statistics in each cell, the ranges are chosen in order to have the same number of events in each bin of \( p_T(\gamma) \) and \( p_z(\gamma) \).

![Sliced distribution of the number of events](image)

(a) \( N_{\text{events}} \) as a function of the cuts on \( p_T(\gamma) \)

(b) \( N_{\text{events}} \) as a function of the cuts on \( p_z(\gamma) \)

Figure 5.1: Sliced distribution of the number of events

The left part of Fig. 5.2 shows the color-scaled value of the ratio \( B(\text{meas})/B(\text{P.D.G.}) \) for the same four ranges in \( p_T(\gamma) \) and \( p_z(\gamma) \) simultaneously. The cells containing not enough data to perform a reliable value of \( B(\text{meas})/B(\text{P.D.G.}) \) are painted in white. These cells correspond to photons with a high longitudinal momentum and low transverse momentum. Photons with these properties are propagating in the vicinity of the beam pipe and most of them are lost. It explains why we get lower amounts of well reconstructed \( B \) candidates, hence, less signal events, for photons with these particular momenta.
The largest discrepancy is observed for photons with low total momentum, which means that, when retrieving efficiencies from Monte-Carlo simulated data, the detection of real photons is much performant for high energy levels than for low ones.

In most region of phase space, we observe efficiencies lower than 1. That is the efficiency obtained from Monte-Carlo simulation is overestimated most of the time compared to the real efficiency.

This result is not in good agreement with LHCb other estimates of the photon detection efficiencies [17], but the dependance on transverse momentum follows a similar trend.
Chapter 6

CP violation and charge asymmetry

The large yield of the $B^+ \rightarrow \eta' K^+$ signal in real data makes of this decay an interesting candidate to highlight direct CP violation through charge asymmetry measurements.

If $N_{\text{meas}}^+$ and $N_{\text{meas}}^-$ are the number of events we obtain for each $B$ charge after the whole selection procedure, we have:

$$N_{\text{meas}}^+ = N_{B^+} \cdot B(B^+ \rightarrow \eta' K^+) \cdot \epsilon_+$$
$$N_{\text{meas}}^- = N_{B^-} \cdot B(B^- \rightarrow \eta' K^-) \cdot \epsilon_-$$

where $N_{B^+}$ and $N_{B^-}$ are the initial number of produced $B^+$ and $B^-$ and $\epsilon_+$ and $\epsilon_-$ characterize the detection efficiency of the $K^+$ with regard to the $K^-$. We define $A_P(B^\pm)$, the production asymmetry of $B^+$ with respect to $B^-$ by:

$$A_P(B^\pm) = \frac{N_{B^-} - N_{B^+}}{N_{B^-} + N_{B^+}}$$

(6.1)

Similarly, the detection asymmetry is defined as:

$$A_D(K^+ K^-) = \frac{\epsilon_- - \epsilon_+}{\epsilon_- + \epsilon_+}$$

(6.2)

So is the measured asymmetry:

$$A_{\text{RAW}}^{\text{sig}}(B^+ \rightarrow \eta' K^+) = \frac{N_{\text{meas}}^- - N_{\text{meas}}^+}{N_{\text{meas}}^- + N_{\text{meas}}^+}$$

(6.3)

Assuming small asymmetries:

$$A_{\text{RAW}}^{\text{sig}} \approx 0$$
$$A_D \approx 0$$
$$A_P \approx 0$$

one has for the physical asymmetry:

$$A_{CP}(B^+ \rightarrow \eta' K^+) = A_{\text{RAW}}^{\text{sig}}(B^+ \rightarrow \eta' K^+) - A_D(K^+ K^-) - A_P(B^+)$$

(6.4)
CHAPTER 6. CP VIOLATION AND CHARGE ASYMMETRY

6.1 Raw asymmetry

The raw asymmetry is the asymmetry that is measured directly by comparing the number of events associated to the channel \( B^+ \to \eta' K^+ \) with the events from the decay \( B^- \to \eta' K^- \). For this purpose, we distinguish positively charged from negatively charged \( B \) so that the total number of signal events is:

\[
N_{S}^{\text{tot}} = N_{S}^{+} + N_{S}^{-}
\]

(6.5)

We define the charge asymmetry of the signal \( A_{\text{sig}}^{\text{RAW}} \) in the following way:

\[
N_{S}^{+} = N_{S}^{\text{tot}} \left( 1 - \frac{A_{\text{bkg}}^{\text{RAW}}}{2} \right)
\]

(6.6)

\[
N_{S}^{-} = N_{S}^{\text{tot}} \left( 1 + \frac{A_{\text{bkg}}^{\text{RAW}}}{2} \right)
\]

(6.7)

The same equations can be written for the background as well.

![Figure 6.1: Extended likelihood fits of the distribution of mass of \( B^+ \) (left) and \( B^- \) (right).](image)

The fit is performed on the distribution of \( B^+ \) and \( B^- \) masses simultaneously with \( \eta' \) mass using \( A \) and \( N \) are the two parameters coded in RooFit as RooFormulaVar. The fits presented on Fig. 6.1 are both composed of two Gaussians to model the signal and of a Chebyshev polynomials of second order for the background. For each category of charge, every associated parameters are left free and varied to maximize the Likelihood logarithm. Table 6.1 shows the final value of the main parameters for both categories of charge.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B ) mass</td>
<td>( \bar{m}_B = 5282.42 \pm 0.59 \text{ MeV/c}^2 )</td>
</tr>
<tr>
<td>( B ) mass width (1st Gaussian)</td>
<td>( \sigma_B(G1) = 19.38 \pm 0.18 \text{ MeV/c}^2 )</td>
</tr>
<tr>
<td>( \eta' ) mass</td>
<td>( \bar{m}_{\eta'} = 959.61 \pm 0.44 \text{ MeV/c}^2 )</td>
</tr>
<tr>
<td>( \eta' ) mass width (1st Gaussian)</td>
<td>( \sigma_{\eta'}(G1) = 11.10 \pm 0.74 \text{ MeV/c}^2 )</td>
</tr>
<tr>
<td>Total number of signal</td>
<td>( N_S = N_S^+ + N_S^- = 2600 \pm 77 )</td>
</tr>
<tr>
<td>Total number of background</td>
<td>( N_B = N_B^+ + N_B^- = 6397 \pm 98 )</td>
</tr>
<tr>
<td>Background Asymmetry</td>
<td>( A_{\text{bkg}}^{\text{RAW}} = -0.052 \pm 0.013 )</td>
</tr>
<tr>
<td>Signal Asymmetry</td>
<td>( A_{\text{sig}}^{\text{RAW}} = 0.004 \pm 0.023 )</td>
</tr>
</tbody>
</table>

Table 6.1: Retrieved values of the main parameters of the fit
We note that background and signal asymmetries are weakly anti-correlated: $C_{A_{SB}} = -0.20$. This means we can consider our signal asymmetry to be only weakly sensitive to the background asymmetry.

The background asymmetry is clearly different from zero. The $pp$ initial state yields the positively charged particles to be more probably produced than the negative ones in $pp$ collisions.

### 6.2 Detection asymmetry

Due to their different composition in quarks, $K^+$ and $K^−$ interact differently with the detector material. This asymmetry is contributing in the total asymmetry we observe and is defined in terms of the detection efficiencies for kaons, $\epsilon_−$ and $\epsilon_+$:

$$A_D(K^+K^-) = \frac{\epsilon_− - \epsilon_+}{\epsilon_− + \epsilon_+}$$  \hspace{1cm} (6.8)

In the next parts, we report on a new method of mapping these efficiencies in terms of the momentum of the positively and negatively charged kaons from $D^0 \rightarrow K^+K^−$ decays.

#### 6.2.1 Cross-section of kaons in matter

The total and elastic cross-sections for interaction of $K^+$ and $K^-$ with proton as a function of their energy are given on Fig. 6.2 from Particle Data Group [6].

![Cross-section of kaon with proton](image)

(a) Total and elastic cross-sections for $K^+p$

(b) Total and elastic cross-section for $K^-p$

Figure 6.2: Cross-section of kaon with proton extracted from PDG [6]

Independently of the charge, the total cross-sections are converging to similar values for
large energies. For 200 GeV kaons, the respective cross-sections are:

\[ \sigma(K^+ p) = (19.910 \pm 0.11) \text{ mb} \]
\[ \sigma(K^- p) = (20.790 \pm 0.05) \text{ mb} \]

The property of the kaons to be indistinguishable in terms of cross-section at high energy levels, can be used as a condition for fitting the detection efficiency as explained in the following section. A systematic uncertainty of ±1% will be assumed on this hypothesis.

6.2.2 Computation of the detection asymmetry using \( D^0 \rightarrow K^+K^- \) decay

The particularity of \( D^0 \rightarrow K^+K^- \) is that it exhibits no time-independent CP violation. Since the production asymmetry has no reason to be considered here, all asymmetry measured between charges should come from the detection process, making of \( D^0 \rightarrow K^+K^- \) an appropriate channel to determine the effective detection asymmetry between \( K^+ \) and \( K^- \).

The detection efficiency of the kaon is directly related to its interaction with the matter of the detector and, hence, to the cross-section itself. The ratio between the efficiencies should then be the same as the ratio of the cross-sections:

\[ \frac{\sigma(K^+ p)}{\sigma(K^- p)} = \frac{\epsilon_+}{\epsilon_-} \quad (6.9) \]

As previously pointed out, we assume that this expression converge towards 1 for high energy kaons:

\[ \frac{\epsilon_+}{\epsilon_-} \bigg|_{\geq 135 \text{ GeV}} = 1 \quad (6.10) \]

Mapping process

We first performed a mapping of the number of events for the different ranges of \( p_{K^+} \) and \( p_{K^-} \) as drawn on Fig. 6.3. \( N_{ij} \) corresponds to the number of measured events passing the \( i^{\text{th}} \) cut on \( p_{K^+} \) and the \( j^{\text{th}} \) cut on \( p_{K^-} \).

![Figure 6.3: Schematic representation of the matrix of the number of \( D^0 \rightarrow K^+K^- \) events for different combination of ranges cuts on \( p_{K^+} \) and \( p_{K^-} \).]
The widths between each cut value are the same for $p_{K^+}$ and $p_{K^-}$ so that the representation is completely symmetric with respect to the diagonal. The ranges of cut are determined in order to keep enough events in the bins of interest.

$N_{ij}$ (resp. $N_{ji}$) can be expressed as the number of events initially produced $M_{ij}$ (resp. $M_{ji}$) which passed the $ij$-momentum selection [ $i^{th}$ cut (resp. $j^{th}$) on $p_{K^+}$ and $j^{th}$ cut (resp. $i^{th}$) on $p_{K^-}$] with a given charge-depending efficiency $\epsilon^\pm$:

\[
\begin{cases} 
N_{ij} = M_{ij} \cdot \epsilon_i^+ \cdot \epsilon_j^- \\
N_{ji} = M_{ji} \cdot \epsilon_j^+ \cdot \epsilon_i^-
\end{cases}
\]

Since, there is no production asymmetry in $D^0 \to K^+K^-$, one can assume: $M_{ij}/M_{ji} = 1$. Hence, we can define the ratio between the two populations:

\[
F_{ij} = \frac{N_{ij}}{N_{ji}} = \left( \frac{M_{ij}}{M_{ji}} \right) \frac{\epsilon_i^+ \cdot \epsilon_j^-}{\epsilon_j^+ \cdot \epsilon_i^-}
\] (6.11)

Defining $\rho_i = \epsilon_i^+ / \epsilon_i^-$ and $\rho_j = \epsilon_j^+ / \epsilon_j^-$, one has:

\[
F_{ij} = \frac{\rho_i}{\rho_j}
\] (6.12)

Using eq. 6.10, one imposes the following conditions:

\[
\begin{cases} 
\rho_i = 1 & \text{if } i \text{ is such that } p_i \in [135 - 200] \text{ GeV, } i \neq j \\
\rho_j = 1 & \text{if } j \text{ is such that } p_j \in [135 - 200] \text{ GeV, } j \neq i
\end{cases}
\] (6.13)

$\chi^2$ Fitting

The $p_{K^+}$ and $p_{K^-}$ axes are both sliced into 20 slices whose widths are established in order to keep a sufficient number of events in each bin. The binning is the same for $p_{K^+}$ and $p_{K^-}$. Since the statistics is mainly distributed around 20-30 GeV, we used 17 same-sized bins between 0 and 102 GeV. We counted the remaining events between 102 and 200 GeV and establish the widths of the 3 bins left such that they contain the same number of events. The whole binning is reported on table D.1.

The resulting canvas has been fitted using a Minuit $\chi^2$ minimization method whose $\chi^2$ function is:

\[
\chi^2 = \sum_{i=1}^{n_{bins}-1} \sum_{j=i+1}^{n_{bins}} \frac{\left( N_{ij} - \rho_i \rho_j \right)^2}{N_{ij} \left( \frac{1}{N_{ij}} + \frac{1}{N_{ji}} \right)} \quad \text{if } N_{ij}, N_{ji} > 500,
\] (6.14)

To avoid uncertainties due to low statistics, the cells containing less than 500 events were ignored in the computation.

For a total canvas containing $20 \cdot 20 = 400$ cells, one has 190 terms: $\frac{20! 19!}{2!}$ (to whom we subtract the terms we do not take into account because of the low statistics) involved in the $\chi^2$ computation, 19 free parameters and the last one fixed to 1 in order to satisfy condition 6.13. The $\chi^2$ is minimized for the parameters $\rho_i$ reported in Appendix D in Table D.1. Fig. 6.4 shows the value of $\rho_i$ for each momentum bin.
CHAPTER 6. CP VIOLATION AND CHARGE ASYMMETRY

Figure 6.4: Values of the ratio of the efficiencies of detection for the $K^+$ relative to $K^-$ $\rho = \frac{\epsilon_+}{\epsilon_-}$ as a function of the momentum of the kaon in GeV. The last bin is fixed at $\rho = 1$.

**Detection asymmetry**

The parameters from the fit describe the ability to detect positive kaons compared to negative ones. For a given range in momentum, when multiplying the number of $K^-$ by the corresponding parameter, one should then obtain the number of detected $K^+$. Hence, we define the distribution of the positively charged kaons momenta $D(K^+)$ as a corrected distribution of the negatively charged kaons momenta, $D_{corr}(p_{K^-})$ as follows:

$$D(K^+) = D_{corr}(p_{K^-}) = \rho \cdot D(p_{K^-})$$ (6.15)

where $D(p_{K^-})$ is the distribution of the events as a function of the momentum of the negatively charged kaon for the set of selection stated in Tables 2.1 and 2.2 and $\rho$ corresponds to the ratio between the detection efficiency for $K^+$ and the detection efficiency for $K^-$. 

The detection asymmetry is computed by comparing the number of events in the distribution of $p_{K^-}$ to the number of events in the distribution of $p_{K^+}$ obtained from the correction.

As each parameter $\rho_i = \frac{\epsilon_i^+}{\epsilon_i^-}$ is computed for a specific range of momentum $p_{K^-}^i$, the distribution of the kaons momenta needs to be binned in the same way in order to multiply it with the appropriate $\rho_i$. Therefore, we have:

$$p_{K^+}^i = \rho_i \cdot p_{K^-}^i \quad \forall i = 1, \ldots, 20$$

Fig. 6.5 shows the binned distribution of $p_{K^-}$ (blue points) and the corrected distribution (red stars). The difference between the number of events of each distribution gives us the following detection asymmetry:

$$A_D = -0.004 \pm 0.003 \text{ (stat.)} \pm 0.010 \text{ (syst.)}$$ (6.16)

The statistical error as been computed by taking into account the fact that the $\rho_i$’s are not correlated. The covariance matrix (Tab.D.2) has been extracted from Minuit output and used to determine the cross-terms in the total uncertainty computation.
CHAPTER 6. CP VIOLATION AND CHARGE ASYMMETRY

6.3 Production asymmetry

6.3.1 Production asymmetry computed from LHCb further analysis

As previously pointed out, since $B^+$ and $B^-$ don’t have the same composition in quarks ($\bar{u}b$ and $\bar{u}b$), they are not produced exactly at the same rate in $pp$ collisions. The proton composition ($uud$) makes the $B^+$ production more probable than the $B^-$ one. This effect yields a non-zero production asymmetry (eq. 6.1).

This asymmetry has already been measured at LHCb [18]. From $D^{*+} \rightarrow D^0(K^+\pi^-)\pi^+$ decays, one established that: $A_D = -0.010 \pm 0.002$ under the assumption that the pion detection asymmetry is zero. From $B^+ \rightarrow J/\psi K^+$ decays, using the measured raw asymmetry and knowing its established $CP$ asymmetry, one can deduce the production asymmetry. It has been found to be compatible with zero: $A_P = -0.003 \pm 0.009$ [19].

In the following, we take the production asymmetry fixed to zero with one percent uncertainty: $A_P = 0.000 \pm 0.010$. This effectively decouples $A_{\text{sig}}(B^+ \rightarrow \eta'K^+)$ from $A(J/\psi K^+)$. 

6.4 Physical asymmetry

The effective asymmetry of the signal is computed by using eq. 6.4:

$$A_{\text{CP}}(B^+ \rightarrow \eta'K^+) = A_{\text{sig}}^{\text{RAW}}(B^+ \rightarrow \eta'K^+) - A_D(K^+K^-) - A_P(B^+)$$

$$= [0.004 \pm 0.023 \text{ (stat.)}] - [-0.004 \pm 0.003 \text{ (stat.)} \pm 0.010 \text{ (syst.)}] - [0.000 \pm 0.010 \text{ (syst.)}]$$

$$= 0.008 \pm 0.023 \text{ (stat.)} \pm 0.014 \text{ (syst.)}$$

The physical $CP$ asymmetry is compatible with zero. Since the statistical uncertainty remains larger than the systematic uncertainty, the precision of the result can still be improved by increasing the size of the statistical sample.
Chapter 7

Conclusion

We have measured the branching fraction for $B^+ \to \eta' K^+$ decay using 2011 LHCb data:

$$\text{BR}(B^+ \to \eta' K^+) = [37.3 \pm 1.0 \text{ (stat.)} \pm 7.4 \text{ (syst.)}] \cdot 10^{-6}$$

The integrated luminosity $L = (1017 \pm 36) \text{ pb}^{-1}$ allows to get a significant signal yield and to reduce the statistical uncertainty to 3%. The discrepancy with the HFAG value is partially understood when considering the influence of the photon detection efficiency. The photon detection efficiency exhibits a clear dependence to the photon momentum. Although this dependence follows a trend which is similar to our expectation, the efficiency values are far from other estimates obtained at LHCb. This could be due to a bad modelization of the trigger in the Monte-Carlo, yielding a misestimation of the trigger efficiency.

The charge asymmetry has been obtained from the measured raw asymmetry, corrected from the detection asymmetry while the production asymmetry is assumed to be zero. We developed a new method based on $D^0 \to K^+ K^-$ to evaluate the detection asymmetry for kaons relative to their respective momenta. The detection efficiency mapping is performed by using the fact that the cross-sections for $K^+ p$ and $K^- p$ are equal for high energy kaons. We obtained the following physical asymmetry for a signal yield of $2600 \pm 77$:

$$A_{\text{CP}} = 0.008 \pm 0.023 \text{ (stat.)} \pm 0.014 \text{ (syst.)}$$

This result is compatible with Babar experiment [20] (signal yield: $Y_S = 4592 \pm 119$), and Belle experiment [21] (signal yield: $Y_S = 1895.7 \pm 59.5$).

2012 data will allow to multiply by 2.5 the luminosity and decrease the statistical uncertainty on the charge asymmetry by a factor of 1.6.
Appendix A

Partial reconstruction

A.1 $\eta'$ mass reconstruction from kinematics

Particles energies and momenta are both measured with LHCb calorimeters. The latter have a finite precision contributing to the uncertainties on the measurement. The photon energy and momentum are poorly described by LHCb calorimeters. Our idea was to derived them from kinematics to avoid the measurement uncertainties. They are then computed from the $B^+ \rightarrow \eta' K^+$ decay quantities listed on figure A.1.

![Diagram](image)

Figure A.1: Notations used for kinematics

The unitary vector $\vec{d}$ describing the propagation of the $B^+$ is known such that we can write :

$$\vec{p}_B = p_B \vec{d}$$

The conservation of energy and momentum yields :

$$P_{B^+} = P_{K^+} + P_{2\pi} + P_\gamma$$

such that

$$P_{B^+} - P_{K^+} = P_{2\pi} + P_\gamma$$  \hspace{1cm} (A.1)

From conservation of 4-vector norm, we get :

$$(P_{2\pi} + P_\gamma)^2 = m_{\eta'}^2$$

and so,

$$(P_{B^+} - P_{K^+})^2 = m_{\eta'}^2$$  \hspace{1cm} (A.2)

We compute then :

$$P_{B^+} \cdot P_{2\pi} = (P_{K^+} + P_{2\pi}) \cdot (P_{2\pi}) = P_{K^+} \cdot P_{2\pi} + m_{2\pi}^2$$
Using eq. A.2, we get the following system:

\[(P_{B+} - P_{K^+}) \cdot P_{2\pi} = m_{2\pi}^2\]  \hspace{1cm} (A.3)
\[(P_{B+} - P_{K^+})^2 = m_{\eta'}^2\]  \hspace{1cm} (A.4)

Developing A.3 and A.4,

\[(E_{B+} - E_{K^+})E_{2\pi} = m_{2\pi}^2 + \vec{p}_{B+} \cdot \vec{p}_{2\pi} - \vec{p}_{K^+} \cdot \vec{p}_{2\pi}\]  \hspace{1cm} (A.5)
\[(E_{B+} - E_{K^+})^2 = m_{\eta'}^2 + p_{B+}^2 + p_{K^+}^2 - 2\vec{p}_{B+} \cdot \vec{p}_{K^+}\]  \hspace{1cm} (A.6)

Replacing \(p_{K^+}^d = \vec{p}_{K^+} \cdot \vec{d}\) and \(p_{2\pi}^d = \vec{p}_{2\pi} \cdot \vec{d}\) in A.5 and A.6:

\[(E_{B+} - E_{K^+})E_{2\pi} = m_{2\pi}^2 + p_B^d + p_{2\pi}^d - \vec{p}_{K^+} \cdot \vec{p}_{2\pi}\]  \hspace{1cm} (A.7)
\[(E_{B+} - E_{K^+})^2 = m_{\eta'}^2 + p_B^d + p_{K^+}^2 - 2p_B^d p_{K^+}^d\]  \hspace{1cm} (A.8)

We take the square of A.7 and multiply A.8 by \(E_{2\pi}^2\) so that we can equalize them and find a second order equation for \(p_{B+}\):

\[
\left[ E_{2\pi}^2 - (p_{2\pi}^d)^2 \right]_A p_{B+}^2 - 2 \left[ E_{K^+} E_{2\pi} + p_{2\pi}^d (m_{2\pi}^2 - \vec{p}_{K^+} \cdot \vec{p}_{2\pi}) \right]_B p_{B+} + \left[ E_{2\pi}^2 (m_{\eta'}^2 + p_{K^+}^2) - (m_{2\pi}^2 - \vec{p}_{K^+} \cdot \vec{p}_{2\pi})^2 \right]_C = 0
\]

(A.9)

\(A, B, C\) are totally determined by the experiment such that we can easily calculate the two solutions for \(p_{B+}\).

Inserting these solutions for \(p_{B+}\) in A.7, we can deduce \(E_{B+}\):

\[E_{B+} = \frac{1}{E_{2\pi}} \left[ E_{K^+} E_{2\pi} + m_{2\pi}^2 + p_B^d p_{2\pi}^d + \vec{p}_{K^+} \cdot \vec{p}_{2\pi} \right]\]  \hspace{1cm} (A.10)

whose solution can easily be found for the two values of \(p_{B+}\).

The \(B^+\) mass is then deduced from the invariant mass formula: \(m_{B^+}^2 = E_{B^+}^2 - p_{B+}^2\). We get two solutions which are close together and which depend on the two values of \(p_{B+}\). Knowing, \(\vec{p}_{B+}\) and \(E_{B+}\), we compute \(k\) and \(k\) from energy and momentum conservation laws (eq. A.1).

### A.2 Significance optimization

We optimize the event selection using a Monte-Carlo simulation of the \(B\) production and decay sequences and of the detector response. The well chosen set of cuts listed above (Tab A.1) allows to keep enough events associated to a real signal.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance of (B^+)’s flight</td>
<td>(D_f(B^+) &gt; 20) mm</td>
</tr>
<tr>
<td>PID(K(^+))</td>
<td>(DDL_K(K^+) &gt; 5)</td>
</tr>
<tr>
<td>Photon’s energy</td>
<td>(E_\gamma &lt; 30000) MeV</td>
</tr>
<tr>
<td>Transverse momentum of (B^+)</td>
<td>(p_T(B^+) &gt; 4000) MeV/c</td>
</tr>
</tbody>
</table>

Table A.1: Set of cuts chosen to maximize the significance
The $B^+$ resonance resolution depends on the resolution on the direction of propagation. Given uncertainties on the position of the primary and secondary vertices (PV and SV on Fig. A.2), the best angular resolution will be achieved when $B^+$ is flying a great distance before decaying. This motivates a cut on the distance of flight.

![Diagram](image)

Figure A.2: Influence of the $B^+$ distance of flight ($D_f$) on the direction uncertainty

In addition to this constraint, a particle identification (PID) cut for $K^+$ provides a way to distinguish kaons from pions, a constraint on the photon energy suppresses very energetic photons coming from other interactions and a lower bound to the transverse momentum of $B^+$ favours signal events.
Appendix B

Probability distribution functions

B.1 Gaussian distribution

The gaussian p.d.f. is defined as:

\[ G(m_B) = \frac{1}{\sqrt{2\pi}\sigma_G} \exp\left\{-\frac{1}{2}\left(\frac{m_B - \bar{m}_B}{\sigma_G}\right)^2\right\} \]  

(B.1)

where:

- \( \sigma_G \): width of the Gaussian
- \( \bar{m}_B \): mean of the Gaussian

B.2 Crystal Ball Shape

The Crystal Ball shape is a Gaussian distribution with a radiative power-law tail. The equation of this distribution is:

\[ CBS(m_B) = \begin{cases} 
\left(\frac{s_{CB}}{|\alpha_{CB}|}\right)^{s_{CB}} e^{-\frac{1}{2}s_{CB}^2} & \text{if } m_B < \alpha_{CB} \\
\left(\frac{s_{CB}}{|\alpha_{CB}|} - |\alpha_{CB}| - m_B\right)^{s_{CB}} & \text{if } m_B > \alpha_{CB} 
\end{cases} \]

(B.2)

\[ \left(\frac{s_{CB}}{|\alpha_{CB}|} - |\alpha_{CB}| - m_B\right)^{s_{CB}} \]

where:

- \( \sigma_{CB} \): width of the Crystal Ball gaussian component
- \( \bar{m}_B \): mean of the gaussian part of the Crystal Ball

B.3 Chebyshev Polynomials

The Chebyshev polynomials of the first kind, can be defined by the following integral [22]:

\[ C_n(z) = \frac{1}{4\pi i} \oint ((1 - z^2)(z')^{-n-1} (1 - 2zz' + z'^2) dz' \]

(B.4)

A well chosen reorganization of power terms in Chebyshev polynomials results in much lower correlations between the coefficients in the fit making it more stable than using a simple polynomial \( \chi^2 \) minimization.
APPENDIX B. PROBABILITY DISTRIBUTION FUNCTIONS

B.4 Phase-space background p.d.f.

The Argus function [23] is an empirical formula to model the phase space of multi-body decays. It is defined as:

\[
A(m_B) = m_B \sqrt{1 - \left( \frac{m_B}{m_f} \right)^2} \cdot \exp \left( c \left( 1 - \left( \frac{m_B}{m_f} \right)^2 \right) \right) \tag{B.5}
\]

where:

- \( m_f \) | end value of the Argus function
- \( c \) | describes the shape of the Argus function
Appendix C

Data analysis from Stripping 17

C.1 Selection requirements

The $B$ candidates from Stripping 17 are all $K\pi\pi\gamma$ combinations fulfilling maximum significance selection conditions given in Table C.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of $B$ meson</td>
<td>$4500 &lt; m_B &lt; 5700 \text{ MeV}/c^2$</td>
</tr>
<tr>
<td>Transverse momentum of $B$</td>
<td>$p_{T}(B) &gt; 4000 \text{ MeV}/c$</td>
</tr>
<tr>
<td>Angle btw $p_B$ and primary vertex direction</td>
<td>$\alpha(p_B) &gt; 0.999994$</td>
</tr>
</tbody>
</table>

Table C.1: Constraints on mother particle for Stripping 17. The condition in grey is required beforehand in the stripping before building the $n$Tuple.

The daughter particles are also required to satisfy a set of conditions (given on Table C.2):

<table>
<thead>
<tr>
<th>Particle</th>
<th>Variable</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta'$</td>
<td>$\eta'$ mass</td>
<td>$</td>
</tr>
<tr>
<td>$K$</td>
<td>Total momentum of the kaon</td>
<td>$p(K^\pm) &gt; 3000 \text{ MeV}/c$</td>
</tr>
<tr>
<td></td>
<td>Transverse momentum of the kaon</td>
<td>$p_{T}(K^\pm) &gt; 2500 \text{ MeV}/c$</td>
</tr>
<tr>
<td></td>
<td>Particle identification (PID)</td>
<td>$\text{Prob}(K) &gt; 0.1$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Total momentum of the pion</td>
<td>$p(\pi^\pm) &gt; 2000 \text{ MeV}/c$</td>
</tr>
<tr>
<td></td>
<td>Transverse momentum of the pions</td>
<td>$p_{T}(\pi^\pm) &gt; 2500 \text{ MeV}/c$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Energy of the photon</td>
<td>$E_\gamma &gt; 1400 \text{ MeV}$</td>
</tr>
<tr>
<td></td>
<td>Calorimeter Confidence Level</td>
<td>$CL(\gamma) &gt; 0.1$</td>
</tr>
</tbody>
</table>

Table C.2: Constraints on daughter particles for Stripping 17. The conditions in grey are required beforehand in the stripping before building the $n$Tuple.
C.2 Results of the Fit

We present the result of the extended maximum likelihood fit for a sample of Stripping 17 real data whose $\eta'$ are constrained to their nominal mass and which satisfies the requirements for a maximum significance previously stated (Tab. C.1 and C.2). The probability distribution function composing the fit are the ones listed in section 3.2.

From the results of the 2D-fit, we compute the significance of the signal from stripping 17:

$$S = 29.1$$  \hspace{1cm} (C.1)

The fit corroborate our assumptions by showing a huge domination of combinatorial background, a weak contribution of non-$\eta'$ feed-down and a very tiny contribution to feed-down due to misreconstructed $B$ with a rather well reconstructed $\eta'$. 

Figure C.1: 2D-Fit of the distributions of $m_{\eta'}$ and $m_B$ for a sample of real data. Signal shape is red dashed. Combinatorial background is green dashed. Resultant fit is blue. The final values of the parameters varied in the Log Likelihood maximization are reported.
Appendix D

Detection asymmetry

D.1 Detection efficiency $\chi^2$ fit from $D^0 \rightarrow K^+K^-$

<table>
<thead>
<tr>
<th>Bin</th>
<th>Range $p_K$</th>
<th>$\rho_i$</th>
<th>Bin</th>
<th>Range $p_K$</th>
<th>$\rho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[0,6000]</td>
<td>0.831 ± 0.045</td>
<td>11</td>
<td>[60000,66000]</td>
<td>0.999 ± 0.004</td>
</tr>
<tr>
<td>2</td>
<td>[6000,12000]</td>
<td>0.987 ± 0.004</td>
<td>12</td>
<td>[66000,72000]</td>
<td>0.996 ± 0.005</td>
</tr>
<tr>
<td>3</td>
<td>[12000,18000]</td>
<td>1.019 ± 0.004</td>
<td>13</td>
<td>[72000,78000]</td>
<td>0.996 ± 0.006</td>
</tr>
<tr>
<td>4</td>
<td>[18000,24000]</td>
<td>1.017 ± 0.004</td>
<td>14</td>
<td>[78000,84000]</td>
<td>1.002 ± 0.005</td>
</tr>
<tr>
<td>5</td>
<td>[24000,30000]</td>
<td>1.013 ± 0.004</td>
<td>15</td>
<td>[84000,90000]</td>
<td>1.010 ± 0.005</td>
</tr>
<tr>
<td>6</td>
<td>[30000,36000]</td>
<td>1.013 ± 0.004</td>
<td>16</td>
<td>[90000,96000]</td>
<td>1.006 ± 0.006</td>
</tr>
<tr>
<td>7</td>
<td>[36000,42000]</td>
<td>1.009 ± 0.004</td>
<td>17</td>
<td>[96000,102000]</td>
<td>1.010 ± 0.006</td>
</tr>
<tr>
<td>8</td>
<td>[42000,48000]</td>
<td>1.003 ± 0.004</td>
<td>18</td>
<td>[102000,114000]</td>
<td>1.013 ± 0.006</td>
</tr>
<tr>
<td>9</td>
<td>[48000,54000]</td>
<td>0.998 ± 0.004</td>
<td>19</td>
<td>[114000,135000]</td>
<td>1.012 ± 0.006</td>
</tr>
<tr>
<td>10</td>
<td>[54000,60000]</td>
<td>1.002 ± 0.004</td>
<td>20</td>
<td>[135000,250000]</td>
<td>1.0 (fixed)</td>
</tr>
</tbody>
</table>

Table D.1: Value of the cut ranges for each bin on $p_{K^+}$ and $p_{K^-}$ and associated $\chi^2$ fit parameters
### APPENDIX D. DETECTION ASYMMETRY

Table D.2: Covariance matrix of the 19 free parameters $\rho_i$ extracted from the Minuit output

<table>
<thead>
<tr>
<th>$\rho_i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.070</td>
<td>0.075</td>
<td>0.075</td>
<td>0.073</td>
<td>0.073</td>
<td>0.072</td>
<td>0.071</td>
<td>0.070</td>
<td>0.073</td>
<td>0.064</td>
<td>0.062</td>
<td>0.059</td>
<td>0.057</td>
<td>0.054</td>
<td>0.051</td>
<td>0.048</td>
<td>0.053</td>
<td>0.053</td>
</tr>
<tr>
<td>2</td>
<td>0.070</td>
<td>1.000</td>
<td>0.897</td>
<td>0.903</td>
<td>0.894</td>
<td>0.883</td>
<td>0.880</td>
<td>0.866</td>
<td>0.856</td>
<td>0.885</td>
<td>0.777</td>
<td>0.750</td>
<td>0.723</td>
<td>0.691</td>
<td>0.656</td>
<td>0.622</td>
<td>0.586</td>
<td>0.644</td>
<td>0.648</td>
</tr>
<tr>
<td>3</td>
<td>0.075</td>
<td>0.897</td>
<td>1.000</td>
<td>0.958</td>
<td>0.953</td>
<td>0.946</td>
<td>0.941</td>
<td>0.927</td>
<td>0.916</td>
<td>0.943</td>
<td>0.831</td>
<td>0.802</td>
<td>0.772</td>
<td>0.739</td>
<td>0.702</td>
<td>0.665</td>
<td>0.626</td>
<td>0.688</td>
<td>0.692</td>
</tr>
<tr>
<td>4</td>
<td>0.075</td>
<td>0.903</td>
<td>0.958</td>
<td>1.000</td>
<td>0.960</td>
<td>0.954</td>
<td>0.956</td>
<td>0.937</td>
<td>0.928</td>
<td>0.970</td>
<td>0.842</td>
<td>0.814</td>
<td>0.784</td>
<td>0.750</td>
<td>0.712</td>
<td>0.675</td>
<td>0.636</td>
<td>0.699</td>
<td>0.703</td>
</tr>
<tr>
<td>5</td>
<td>0.073</td>
<td>0.894</td>
<td>0.953</td>
<td>0.960</td>
<td>1.000</td>
<td>0.948</td>
<td>0.947</td>
<td>0.933</td>
<td>0.924</td>
<td>0.949</td>
<td>0.839</td>
<td>0.810</td>
<td>0.780</td>
<td>0.746</td>
<td>0.709</td>
<td>0.671</td>
<td>0.632</td>
<td>0.695</td>
<td>0.699</td>
</tr>
<tr>
<td>6</td>
<td>0.073</td>
<td>0.883</td>
<td>0.946</td>
<td>0.954</td>
<td>0.948</td>
<td>1.000</td>
<td>0.941</td>
<td>0.927</td>
<td>0.920</td>
<td>0.945</td>
<td>0.836</td>
<td>0.807</td>
<td>0.778</td>
<td>0.745</td>
<td>0.707</td>
<td>0.669</td>
<td>0.631</td>
<td>0.694</td>
<td>0.698</td>
</tr>
<tr>
<td>7</td>
<td>0.072</td>
<td>0.880</td>
<td>0.941</td>
<td>0.956</td>
<td>0.947</td>
<td>0.941</td>
<td>1.000</td>
<td>0.928</td>
<td>0.922</td>
<td>0.954</td>
<td>0.836</td>
<td>0.808</td>
<td>0.779</td>
<td>0.746</td>
<td>0.708</td>
<td>0.671</td>
<td>0.633</td>
<td>0.696</td>
<td>0.700</td>
</tr>
<tr>
<td>8</td>
<td>0.071</td>
<td>0.866</td>
<td>0.927</td>
<td>0.937</td>
<td>0.933</td>
<td>0.927</td>
<td>0.928</td>
<td>1.000</td>
<td>0.925</td>
<td>0.927</td>
<td>0.827</td>
<td>0.799</td>
<td>0.771</td>
<td>0.738</td>
<td>0.702</td>
<td>0.665</td>
<td>0.627</td>
<td>0.689</td>
<td>0.694</td>
</tr>
<tr>
<td>9</td>
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Bibliography


[17] I. Belyaev et al. Study of $\pi^0/\gamma$ reconstruction efficiency with 2011 data.


