Time Resolution Studies of $B^0 \to D\pi$

Abstract

This analysis presents a measurement of resolutions of real and simulated $B^0 \to D\pi$ at LHCb. The effective resolution obtained in a lifetime biased MC sample is extracted from a triple gaussian fit to the distribution of $\Delta t = t_{\text{true}} - t_{\text{reco}}$. The corresponding value is $\sigma_{\text{eff}} = (0.045188 \pm 0.000170)$ ps. The per-event effective resolution as a function of the per-event decay time error with 22 slices is best fitted with a quadratic $\chi^2$ fit and shows values which could be taken for the calibration. The $t_{\text{reco}}$ distribution in the lifetime unbiased MC was weighted and the corresponding effective resolution extracted from a RooDecay fit is $\sigma_{\text{eff}} = (0.042845 \pm 0.003047)$ ps which is compatible with the previously obtained $\sigma_{\text{eff}}$. A better background reduction would be needed to get a satisfactory fit to the data in order to get significant values of the resolutions of the signal and the background.
## Contents

1 Introduction .......................................................................................................................... 3

2 The LHCb experiment .......................................................................................................... 3

3 Results using Monte Carlo data ......................................................................................... 4
   3.1 Δt distribution .................................................................................................................. 4
   3.2 Per-event decay time error .............................................................................................. 6
   3.3 Lifetime unbiased Monte Carlo ...................................................................................... 7
      3.3.1 Weight calculation ................................................................................................... 9
      3.3.2 Computation of the resolution in the lifetime unbiased MC sample ....................... 10

4 Results in Data ..................................................................................................................... 11
   4.1 Computation of the weights ............................................................................................ 11
   4.2 Background reduction .................................................................................................... 11
   4.3 Data Fit .......................................................................................................................... 13

5 Conclusion ............................................................................................................................ 16

6 Appendix ................................................................................................................................ 17
   6.1 Computation of the asymmetric errors in figures 7a and 7b ........................................... 17
1 Introduction

The weak force causes decays in which particles made of heavy quarks decay into particles made of their lighter cousins. The CKM (Cabibbo-Kobayashi-Maskawa) matrix describes the rates of these decays and gives rise to CP violation. The latter can be described by the unitary triangle whose area measures to amount of CP violation caused by the weak force. $\gamma = \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$ is the least-well known unitary angle of the triangle. It can be measured in interference between $b \rightarrow c$ and $b \rightarrow u$ transitions. $B^0 \rightarrow D\pi$ decays whose feynman graphs are shown in figure 1 will be considered in this TPIV project.

![Figure 1: Feynman graphs of the $B^0 \rightarrow D\pi$ decays.](image)

The $B^0$ meson oscillates in its antimatter counterpart with a frequency dependent on $\Delta m$. The amplitude of the time-dependent asymmetry shown in figure 2 encodes sensitivity to CP violation. The experimental resolution smears this oscillation, reducing the sensitivity. We will first compute the resolutions in lifetime biased and lifetime unbiased Monte Carlo samples. We will then attempt to compute this resolution in data. This could give us a method to compare MC and data resolutions which could be then used for future analysis.

![Figure 2: Asymmetry as a function of the $B^0$ decay time [6].](image)

2 The LHCb experiment

The LHCb experiment is one of the 4 experiments of the Large Hadron Collider (LHC) at CERN. It is a specialized b-physics experiment. One of its primary goals is to measure the characteristics of CP-violation in the decays of b-hadrons.

The LHCb detector is a single-arm forward spectrometer whose layout is shown in figure 3. The geometry of the detector is motivated by the fact that the two b-hadrons are mainly produced in the same forward (or backward) cone. The following list describes the basic aspects of each part of the LHCb detector:

**The tracking system** consists of the VErtex LOcator (VELO), the Tracker Turicensis (TT) and the tracking stations T1 T2 T3. The $B \rightarrow D\pi$ decay occurs in the VELO. It estimates with a precision of up to $10 \mu m$ the distance between the proton-proton collision and the B-mesons decay. This measurement is indirect since only charged decay products are directly measured.
The Cherenkov detectors RICH1 and RICH2 measure the momenta of charged particles. RICH1 covers the low momentum charged particle range from about 2 to 60 GeV/c and uses Aerogel and C$_4$F$_{10}$ as radiators, while RICH2 covers the high momentum range from about 15 GeV/c to 100 GeV/c, using a CF$_4$ radiator.

The spectrometer magnet deflects charged particles in the horizontal plane with a B-field of about 4Tm. This is required for the measurement of the particles’ momenta.

The calorimeter system is composed of a hadronic calorimeter (HCAL) and an electromagnetic calorimeter (ECAL). It allows the identification of electrons, photons and hadrons as well as the measurement of their energies and positions.

The muon detection system consists of 5 muon chambers (M1-M5). They are placed at the far-end of the LHCb detector due to the high lifetime of the muon ($\approx 2.2\ \mu s$) and its low interaction with matter.

![Figure 3: Side view of the LHCb detector](image)

3 Results using Monte Carlo data

3.1 $\Delta t$ distribution

The first MC sample that is studied is lifetime biased. This sample is constructed with lifetime biased variables. One of the major aspects of this sample is a cut on the reconstructed decay time of the $B^0$ such that $t_{ reco} > 0.2\ \text{ps}$. Every $B_0$ of the sample has therefore travelled a certain distance before decaying into a $D\pi$ pair (figure 4). This cut is made to remove all the prompt $D\pi$ candidates from the sample ($D\pi$ candidates created at the primary vertex).
Figure 4: Sketch of a $B \to D\pi$ decay generated by a lifetime biased MC sample.

First, we subtract the generated decay time (called the true decay time) from the reconstructed one: $\Delta t = (t_{\text{true}} - t_{\text{reco}})$. The corresponding distribution is then fitted with a triple gaussian model of the form:

$$F(\Delta t) \propto \frac{f_{\text{core}}}{\sigma_{\text{core}}} e^{-\frac{1}{2} \frac{(\Delta t - \mu_{\text{core}})^2}{\sigma_{\text{core}}^2}} + \frac{f_{\text{mid}}}{\sigma_{\text{mid}}} e^{-\frac{1}{2} \frac{(\Delta t - \mu_{\text{mid}})^2}{\sigma_{\text{mid}}^2}} + \frac{f_{\text{tail}}}{\sigma_{\text{tail}}} e^{-\frac{1}{2} \frac{(\Delta t - \mu_{\text{tail}})^2}{\sigma_{\text{tail}}^2}}$$  \hspace{1cm} (1)

Where $f_{\text{core}}$, $f_{\text{mid}}$ and $f_{\text{tail}}$ are weighted so that:

$$f_{\text{core}} + f_{\text{mid}} + f_{\text{tail}} = 1$$  \hspace{1cm} (2)

This is done automatically by the fit by making $f_{\text{core}}$ and $f_{\text{mid}}$ as free parameters. $f_{\text{tail}}$ will then be fixed using expression 2.

$\sigma_{\text{core}}$, $\sigma_{\text{mid}}$ and $\sigma_{\text{tail}}$ are the three gaussian widths. The hierarchy of the three gaussians ($\sigma_{\text{core}} < \sigma_{\text{mid}} < \sigma_{\text{tail}}$) is respected by forcing:

$$\sigma_{\text{mid}} = \sigma_{\text{core}} + \delta \sigma_{\text{mid}} \quad \delta \sigma_{\text{mid}} > 0$$  \hspace{1cm} (3)

$$\sigma_{\text{tail}} = \sigma_{\text{mid}} + \delta \sigma_{\text{tail}} \quad \delta \sigma_{\text{tail}} > 0$$  \hspace{1cm} (4)

The final gaussian described by expression 1 has the effective resolution:

$$\sigma_{\text{eff}} = \sqrt{f_{\text{core}} \cdot \sigma_{\text{core}}^2 + f_{\text{mid}} \cdot \sigma_{\text{mid}}^2 + f_{\text{tail}} \cdot \sigma_{\text{tail}}^2}$$  \hspace{1cm} (5)

The $\Delta t$ distribution is displayed in figure 5 with its corresponding maximum likelihood fit in blue. The three gaussians are also shown in dashed lines.

The tail-gaussian (in green) seems to have a very small impact compared to the other two gaussians. However, removing it leads to a poorer fit to the MC data points.

The pull plot is shown at the bottom of the $\Delta t$ distribution where most of the data points are concentrated between -3 and 3$\sigma$.

Finally, the extracted effective resolution is:

$$\sigma_{\text{eff}} = (0.045188 \pm 0.000170) \, \text{ps}$$  \hspace{1cm} (6)
3.2 Per-event decay time error

The error on $\Delta t$ corresponds to the error on $t_{\text{reco}}$ since $t_{\text{true}}$ has no error. This means that $\delta|\Delta t| = \delta|t_{\text{true}} - t_{\text{reco}}| = \delta t_{\text{reco}}$.

The distribution of $\delta t_{\text{reco}}$ is shown in figure 6.

On the x-axis, this distribution is chopped in slices of equal number of events. Each slice $i$ has its average $\delta t_{\text{reco}}$ noted $<\delta_i>$ and called the per-event decay time error.
For each slice $i$, the difference between the true and the reconstructed decay times $\Delta t$ is fitted with a double-gaussian model (the number of events for each slice is too small to require a triple-gaussian model). A per-event effective resolution $\sigma(<\delta_i>)$ is then extracted from the fit. $\sigma(<\delta_i>)$ can then be compared to $<\delta_i>$. In figures 7a and 7b respectively, a linear and a quadratic $\chi^2$ fit to the effective resolution distribution were produced in the form $\sigma(<\delta_i>) = q_1 <\delta_i> + q_0$ and $\sigma(<\delta_i>) = q_2 <\delta_i>^2 + q_1 <\delta_i> + q_0$.

Since the distribution of $\delta t_{\text{reco}}$ shown in figure 6 is asymmetric, the errors on $<\delta>$ should be asymmetric too. These errors are expressed in the appendix in expressions 8 and 9.

![Figure 7: Per-event effective resolution as a function of the per-event decay time error for simulated B → Dπ. $q_0$, $q_1$ and $q_2$ are obtained from a $\chi^2$ fit to the effective resolution (shown in red). The number of slices was fixed to 22. Pull plots are displayed at the bottom of each figure.](image)

Both plots display fits with $\chi^2$/NDOF close to 1. This underlines the good quality of the fits. The pull plot of the linear fit has a U-shape, while the pull plot of the quadratic fit presents no particular shape. This aspect indicates that the parameters obtained from the quadratic fit could be the ones chosen for the calibration.

While being small, the value of the offset ($q_0 = 0.0084 \pm 0.0024$) has to be taken into account to get a satisfying fit. The calibration values extracted from the quadratic $\chi^2$ fit are $q_1 = 0.8154 \pm 0.1503$ and $q_2 = 5.0470 \pm 2.3480$.

### 3.3 Lifetime unbiased Monte Carlo

The second MC sample that is studied is a lifetime unbiased sample. The cut $t_{\text{reco}} > 0.2$ ps was not applied here. Therefore, for very low decay times, prompt $D\pi$ can be falsely associated as decay products of a prompt $B^0$ (a sketch of this situation is shown in figure 8). This aspect is observed as a resolution measured for decay times smaller than 0.

The true decay time $t_{\text{true}}$ of the $B^0$ must follow a decreasing exponential distribution. Figure 9 shows that this is not the case. Near 1 ps, the $t_{\text{true}}$ distribution loses its decreasing exponential shape. This is linked to the experimental acceptance which rejects some of the low decay time events.

As for the reconstructed decay time $t_{\text{reco}}$, its distribution should be peaked at 0 with negative values linked to the resolution. Figure 10 shows that the distribution is not peaked at 0. These problems can be solved by computing weights which are values added to the MC ntuple.
Figure 8: Sketch of a $B \to D\pi$ decay generated by a lifetime unbiased MC sample. The decay time of the $B^0$ is chosen to be very small. Therefore, the two prompt $D\pi$ shown in green are falsely associated as decay products of the $B^0$.

Figure 9: Distribution of $t_{\text{true}}$ in the lifetime unbiased MC sample.

Figure 10: Distribution of $t_{\text{treco}}$ in the lifetime unbiased MC sample.
3.3.1 Weight calculation

A $B^0$ decay model is made with the PDG value of the lifetime of the $B^0$: $\tau = (1.519 \pm 0.007)$ ps. It is shown in figure 11.

This $B^0$ decay model is divided by the histogram version of the $t_{\text{true}}$ distribution shown in figure 9. It is then normalized so that the number of events in the weighted sample is the same as the one in the unweighted sample. The resulting plot shown in figure 12 displays the weight values as a function of $t_{\text{true}}$.

For each event with a certain value of $t_{\text{true}}$, the corresponding weight displayed in figure 12 is extracted and applied as a multiplicative factor to the value of $t_{\text{reco}}$ (shown in figure 10) of the same event.

The resulting plot of $t_{\text{reco}}$ is shown in figure 13. It is possible to see that the distribution is now correctly peaked at 0. The weighting method has worked successfully.

Figure 11: $B_0$ decay model. The lifetime is fixed to PDG value: $\tau = \tau_{B^0} = 1.52$ ps

Figure 12: Weights as a function of the value of $t_{\text{true}}$. 
3.3.2 Computation of the resolution in the lifetime unbiased MC sample

The maximum likelihood fit of the weighted distribution of $t_{\text{reco}}$ shown in figure 13 was made with a RooDecay model. It is the convolution of a gaussian model with a decreasing exponential model. The gaussian model used here consists of two gaussians with widths and yields called respectively $\sigma_{\text{core}}, \sigma_{\text{tail}}$ and $f_{\text{core}}, f_{\text{tail}}$. The effective resolution $\sigma_{\text{eff}}$ is then computed using expression 5 (without $\sigma_{\text{mid}}$). All of the parameters, including the decay time $\tau$ and the mean $\mu$ are set free. The $t_{\text{reco}}$ distribution with its fit in blue are shown in figure 14.

![Figure 13: Weighted distribution of $t_{\text{reco}}$.](image1)

![Figure 14: Weighted $t_{\text{reco}}$ distribution with its maximum likelihood fit in blue for Prompt MC $D\pi$ candidates.](image2)
The fit is satisfactory with a $\chi^2$/NDOF very close to 1. Furthermore, the pull plot shown at the bottom indicates that every event is inside the interval $[-3\sigma, 3\sigma]$. The fit is well centred at 0 and the value of $\tau$ obtained is compatible with the PDG value ($\tau_{PDG} = (1.520 \pm 0.004) \text{ ps}$). The extracted effective resolution is:

$$\sigma_{\text{eff}} = (0.042845 \pm 0.003047) \text{ ps}$$

(7)

This value is compatible with the one obtained from the $\Delta t$ distribution and shown in expression 6. Indeed, their relative pull is equal to 0.77$\sigma$.

4 Results in Data

The analysed data is a lifetime unbiased sample with magnetic fields up and down of 2011 and 2012. The aim is to compute the resolutions of the background and the signal of the reconstructed decay time $t_{\text{reco}}$ and to compare them to what was obtained in MC (shown in figure 14). This would then give us a ratio between MC and data which could then be used for future analysis.

4.1 Computation of the weights

The data has to be weighted with the same weights than the ones used in MC. In order to do that, the distribution of $t_{\text{reco}}$ was chopped in bins. For each bin, the average value of the corresponding weights was computed. This computation was only done for values of $t_{\text{reco}}$ bigger than $-0.4 \text{ ps}$ because the number of values below is too small to get some significant weights. These weights can then be used in data. However, we will see that data has got lots of values below $-0.4 \text{ ps}$. To solve this problem, we simply applied the average weight that was obtained for the bin at $-0.4 \text{ ps}$ to the values below $-0.4 \text{ ps}$.

4.2 Background reduction

Figures 15a and 15b show the mass distributions of the B and D mesons in data. No clear peak can be seen in the B plot and big one is noticed in the D plot. This tells us that the B-meson distribution is submerged by background and that this background consists partly of prompt $D\pi$ candidates.

Figure 15: Mass distributions of the B and D mesons without any attempt to reduce the background.
The plot of the weighted $t_{\text{reco}}$ distribution obtained without any background reduction is shown in figure 16. An asymmetry between the positive and the negative values would indicate the presence of a clear signal component. However, this signal part seems to be too low to get a satisfying fit, meaning that signal selection cuts have to be made to reduce the background.

Figure 16: Distribution of $t_{\text{reco}}$ obtained without any background reduction.

The following list enumerates the cuts that were made to reduce the background:

**Trigger cuts:** The list of trigger cuts was directly taken from [2]. They are: Hlt1 All Tracks L0 TOS; Hlt2 Topo2,3,4BodyBBDT TOS; and Hlt2 Inclusive Phi TOS.

**DLL cuts:** They include a DLL cut on the bachelor particles: $\text{DLL}_{K\pi} < 4$ for the pions (this is the part that is kept and not the part that is suppressed, this applies for all of the cuts shown below); and DLL cuts on the daughter particles: $\text{DLL}_{K\pi} < 8$ for the pions and $\text{DLL}_{K\pi} > -2$ for the kaons.

**Momentum Transverse (PT) cuts:** The momentum transverse distributions of the background and the signal have to be distinguished in order to make cuts which would reduce the amount of background in the data sample. A ”background-heavy” sample is obtained by taking only events with a B mass greater than $5600\text{MeV}/c^2$ in the data sample. MC is known to simulate very well the signal. Therefore, by setting the background category of the B to 0 in MC, a good ”signal-heavy” sample can be obtained. The momentum transverse distributions of the $\pi$ and the D mesons are displayed in figures 17a and 17b respectively with MC (the signal component) in red and data (the background component) in blue. Both distributions have been normalized to an area of 1. The corresponding cuts that were applied are $\text{PT}(\pi) > 2000\text{MeV}/c$ and $\text{PT}(D) > 15000\text{MeV}/c$. Unfortunately these ”vertical” cuts are not ideal because they still keep some background while reducing some signal. A refined background reduction would consist on constructing BDTs which would end up cutting most of the background areas while keeping most of the signal. However, this technique probably goes beyond the scope of a TPIV.

**Mass cuts:** After applying the cuts described above. The mass distributions of the B and D mesons shown in figures 18a and 18b are obtained. By comparing these distributions to the ones displayed in figures 15a and 15b we can clearly see that a lot of background has been suppressed. However, figure 15b shows that there still remains some flat background not related to the prompt $D\pi$. The final cuts that are made consist on cutting...
around the mass peaks. Therefore, the following cuts are applied: $5229 \text{MeV}/c^2 < m(B^0) < 5329 \text{MeV}/c^2$ and $1840\text{MeV}/c^2 < m(D^\pm) < 1900\text{MeV}/c^2$.

(a) Transverse momentum distributions of the bachelor pion

(b) Transverse momentum distributions of the bachelor D-meson

Figure 17: Transverse momentum distributions of the bachelor pions and D mesons. Cuts on the data were made to reproduce the background. The background category was set to 0 in MC to simulate the signal in data.

(a) Mass distribution of the B-meson.

(b) Mass distribution of the D-meson.

Figure 18: Mass distributions of the B and D mesons after applying trigger, DLL and PT cuts.

4.3 Data Fit

The distribution of $t_{\text{reco}}$ in data is fitted after applying the weights and reducing a part of the background (shown in sections 4.1 and 4.2). In the same way that what was done with MC, the signal is fitted with a RooDecay model (the convolution of a gaussian model and a decreasing exponential model). Except that the gaussian model is simply a single gaussian instead of a double gaussian. As for the background, it is fitted with a double gaussian model. Both distributions are centred at fixed $\mu = 0.0\text{ps}$. Figure 19 shows the corresponding plot. The double gaussian model is displayed in red with its core and tail gaussians in cyan and magenta. The RooDecay model is drawn in green. The overall fit which is the sum of the two fits is shown in blue. The latter seems to fit the data quite well. However, the resolution obtained for the signal is very different from the effective resolution obtained for the background. We would like both resolutions to be compatible with each other since they both correspond to the same prompt $D\pi$ candidates. Furthermore, the decay time obtained from the RooDecay fit is not compatible
with the PDG value \( \tau = (1.519 \pm 0.007) \) ps. These two aspects imply that the results of this fit cannot be considered as final results.

As an attempt to resolve these problems, the first idea is to fix the \( \tau \) value to the PDG value \( (\tau = 1.52 \) ps). The plot is shown in figure 20. The fit is not very satisfactory. Both resolutions (of background and signal) are still very different. Around 0.4 ps, where the decreasing exponential component is already dominant, the fit seems to differ very much from the data points which seems to experience a small bump. This bump may be linked to background which was not effectively taken away from the various cuts which were applied. Furthermore, at high \( t_{\text{reco}} \) values the data points are below the fit (an aspect which can be seen in the pull plot). This shows that in this region where no prompt \( D_1 \pi \) candidates are present the \( t_{\text{reco}} \) distribution does not follow a satisfying \( B^0 \) decay distribution.

Then, the \( \tau \) parameter is set free and the RooDecay resolution is fixed to the resolution obtained with the double gaussian model. The corresponding plot is shown in figure 21. The fit is quite satisfactory but the \( \tau \) parameter is unfortunately not compatible with the PDG value.

Finally, in figure 22 the fit consists of the RooDecay resolution fixed to the one extracted from the double gaussian model and the \( \tau \) parameter fixed to the PDG value. Here, the fit is quite bad. The situation is quite similar the one seen in figure 20.

None of the resolutions extracted from 4 fits can be used as final results for future analysis. Some better background reduction should be performed in order to compute correct resolution parameters.

![Figure 19: The RooDecay resolution and \( \tau \) are free parameters.](image-url)
Figure 20: $\tau$ is fixed to the PDG value. The RooDecay resolution is a free parameter.

Figure 21: The RooDecay resolution is fixed to the effective resolution extracted from the double gaussian model. $\tau$ is a free parameter.
Figure 22: The RooDecay resolution is fixed to the effective resolution extracted from the double
 gaussian model. \( \tau \) is fixed to the PDG value.

5 Conclusion

The fit of \( \Delta t = t_{\text{true}} - t_{\text{reco}} \) in the lifetime biased MC sample is very satisfactory. The per-event
effective resolution as a function of the per-event decay time error is best fitted with a quadratic \( \chi^2 \) fit. It was also noticed that an offset \( q_0 \) is needed. In the lifetime unbiased MC sample,
the wanted \( t_{\text{reco}} \) distribution was obtained using weights. The corresponding fit was made
with the convolution of a double gaussian model and a decreasing exponential function. The
decay time obtained is in accordance with the PDG value and the extracted effective resolution
is compatible with the one previously obtained from the fit of \( \Delta t \) in the lifetime biased MC
sample.

In data, the weights computed in MC were used to obtain the wanted distribution of \( t_{\text{reco}} \). The
cuts applied to remove some background were unfortunately not effective enough to obtain
a satisfying fit to the distribution of \( t_{\text{reco}} \). Hence, no ratio between the effective resolutions in
data and MC could be calculated. An idea would be to construct BDTs which would suppress
background without taking away too much signal. This could be done in a future analysis.
6 Appendix

6.1 Computation of the asymmetric errors in figures 7a and 7b

In a asymmetric normal distribution as the one shown in figure 6, the asymmetric errors for each slice $i$ (with average $\mu$) are expressed as:

\[
\sigma(i)_{<\mu} = \sqrt{\frac{1}{N_{<\mu}} \sum_{j<\mu} (x_j - \mu)^2}
\]

\[
\sigma(i)_{>\mu} = \sqrt{\frac{1}{N_{>\mu}} \sum_{j>\mu} (x_j - \mu)^2}
\]

References

[1] S. Blusk and C. Fitzpatrick : *A measurement of the CP-violating phase $\phi_s$ in the decays $B^0_s \rightarrow D^+_s D^-_s$* LHCb-ANA-2014-068 (September 16, 2014)


