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Studies of momentum scale calibration using $J/\psi \rightarrow \mu^+\mu^-$

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Abstract

The use of $J/\psi \rightarrow \mu^+\mu^-$ to calibrate the momentum scale is discussed. A procedure is developed and tested using simulated data from DC' 06. It is shown that there is a ~ 1 MeV bias on the J/ψ mass present in the simulation. The majority of this bias is shown to be due to the energy loss correction.

1 Introduction

In order for the track fit to give an unbiased momentum estimate the effect of the magnetic field seen by the particle together with any energy loss undergone in the detector must be accounted for. In studies with simulated data it is easy to check that these requirements are met by comparing reconstructed quantities to their true values. With real data other tests have to be made. One check is compare the reconstructed masses of resonances to their known values. Studies of the decay $J/\psi \rightarrow \mu^+\mu^-$ provide a powerful check due to its high yield and clean signature. In 2 fb^{-1} of data 2×10^9 events are expected [1] with a S/B of ~ 3 (Fig. 1). The large statistics available will allow to study the mass distribution as a function of the track momentum and pseudorapidity.

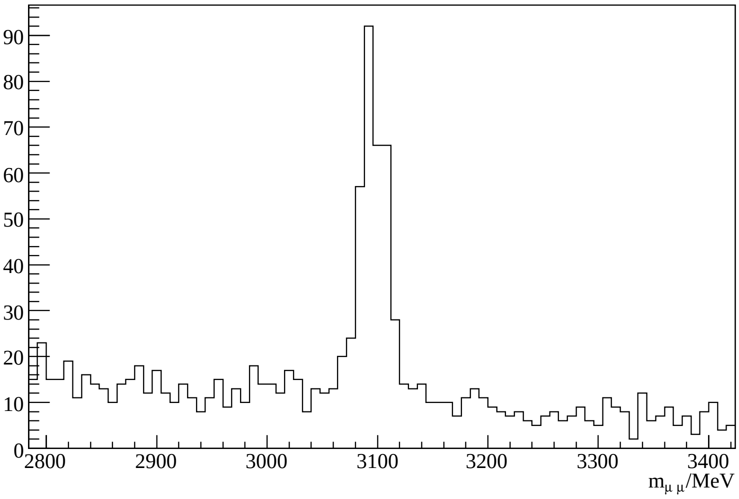

Figure 1: Selected $J/\psi \rightarrow \mu^+\mu^-$ candidates in 1.1 million L0 stripped minimum bias events using the cuts described in Section 3.

In this note the effect of an imperfect knowledge of the magnetic field and energy loss correction on the J/ψ mass are discussed. The note is structured as follows. First, the kinematics of two body decays is discussed and a strategy for calibrating the momentum scale developed. This strategy is then applied to simulated data. In doing this a small bias is observed in the J/ψ mass. This will be shown to be due to the energy loss correction. Finally, the mass resolution is investigated.

The observation of a bias is not unexpected since the situation regarding the modeling of energy loss in both the simulation and reconstruction is known to be confused energy loss is well understood and given by the Bethe-Bloch equation [2]:

$$\frac{dE}{dx} \sim \frac{Z\rho}{A\beta^2}\left[ln\left(\frac{2m_e\beta^2\gamma^2}{I}\right) - \beta^2 - \frac{\delta}{2}\right] \tag{1}$$

where I is the mean excitation energy of the material and δ is a correction for the density effect [3]. If the dependence on $\beta\gamma$ and the variation of the excitation energy with material type are assumed to be small this reduces to:

$$\frac{dE}{dx} = \frac{c_{ion}\rho Z}{A}. \tag{2}$$

This formula is used in the track fit to correct for energy loss. The value of c_{ion} is tuned to minimize the bias on the momentum. Using a sample of tracks from the DC 04 production that have an average momentum of 10 GeV a value of 354 MeV mm^2 mol^{-1} is found [4]. Studies indicate that this value is also appropriate for the DC 06 production. However, the approximations made in Equation 2 are estimated to lead to systematic errors at the level of 10 - 20 % per track. In addition, it has been shown that with Geant4 version 7.1 which was used for the DC 04 and DC 06 [5] Monte Carlo productions the simulated energy loss does not have the dependence on the $\beta\gamma$ that is expected from Equation 1 [6, 7]. Instead of reaching the Fermi plateau at high $\beta\gamma$ the energy loss rises almost linearly. This means that even if the assumptions above are valid they may not be for data generated with the DC 06 version of Gauss. Studies have shown that this discrepancy is reduced in Geant4 version 8.3 and also if δ-rays are simulated [8].

Two other problems should be noted. First, the straggling function for energy loss is highly asymmetric: the mean of the distribution is $\sim$ 20 % higher than the most probable value (Fig. 2). Therefore, if the energy loss correction is tuned to the most probable value it will not be correct on average. In addition, the tuning procedure above assumes there are no other effects present that bias the momentum.

2 Method

For a two-body decay to muons relativistic kinematics gives:

$$m_{\mu\mu}^2 = (E_1 + E_2)^2 - (\vec{p_1} + \vec{p_2}) \cdot (\vec{p_1} + \vec{p_2}) \tag{3}$$

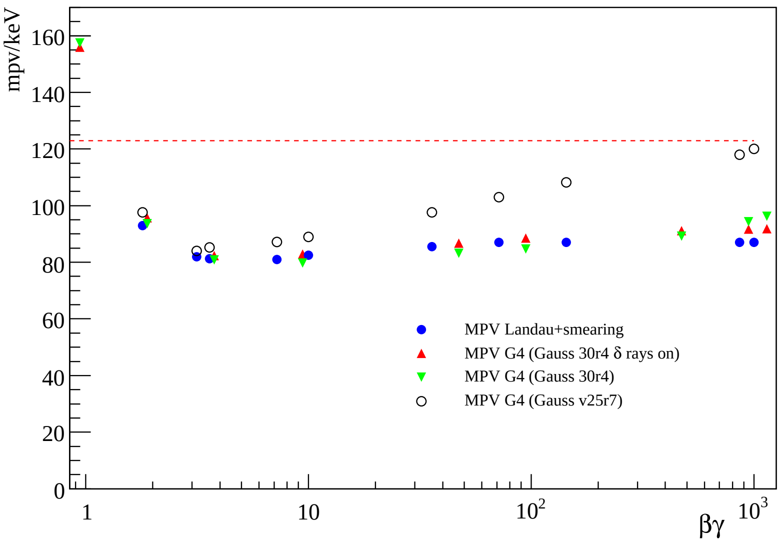

Figure 2: Energy loss in 300 μm silicon versus $\beta\gamma$ as given by various versions/settings of Geant4 and the model discussed in [7]. Note the suppressed scale. The dashed line is the value found for this thickness of silicon using Equation 2. The average momentum of muons from J/ψ decays is $\sim$ 30 GeV which corresponds to $\beta\gamma = 286$.

Collecting terms and making a first order Taylor expansion this reduces to:

$$m_{\mu\mu}^2 = m_\mu^2 R + 2 \cdot (p_1 p_2 - \vec{p_1} \cdot \vec{p_2}) \tag{4}$$

where:

$$R = 2 + \frac{p_1}{p_2} + \frac{p_2}{p_1}. \tag{5}$$

If the scale of the magnetic field is wrong by a factor α then the momentum of each particle needs to be scaled by $1+\alpha$. Assuming the particles originated in the decay $J/\psi \rightarrow \mu^+\mu^-$ then:

$$m_{J/\psi}^2 = m_\mu^2 R + 2 \cdot (1+\alpha)^2 \cdot (p_1 p_2 - \vec{p_1} \cdot \vec{p_2}). \tag{6}$$

Subtracting Equation 4 and 6 gives:

$$m_{\mu\mu}^2 = \frac{m_{J/\psi}^2 - m_\mu^2 R}{(1+\alpha)^2} + m_\mu^2 R. \tag{7}$$

For $\alpha \ll 1$ and $m_\mu^2 R \ll m_{J/\psi}^2$ this simplifies to:

$$\Delta m = \alpha \cdot m_{J/\psi}. \tag{8}$$

Thus, the scale factor is easily extracted from the difference between the measured and true mass and is independent of the J/ψ momentum.

The other possibility is that the momentum is biased due to a poor tuning of the energy loss correction. In this case $p \rightarrow p' - \beta_{ion}$. Assuming, the opening angle between the particles is small, the decay symmetric such that $p_1 \approx p_2$ and the correction β_{ion} is the same for both particles it follows from Equation 4 that:

$$m_{\mu\mu}^2 - m_{J/\psi}^2 = \beta_{ion} \cdot (p_1 + p_2) \cdot \theta^2 \tag{9}$$

where θ is the opening angle. In Fig. 3 the variable θ and R are plotted for selected J/ψ candidates. It can be seen that the approximations made are reasonable. From Equation 9 it follows that a wrong tuning of the energy loss correction results in a bias on the mass that depends on the square root of the J/ψ momentum. Furthermore, the bias scales as $\sqrt{\beta_{ion}}$ and is proportional θ. Therefore, for the case of LHCb the effect of energy loss on the J/ψ mass is small.

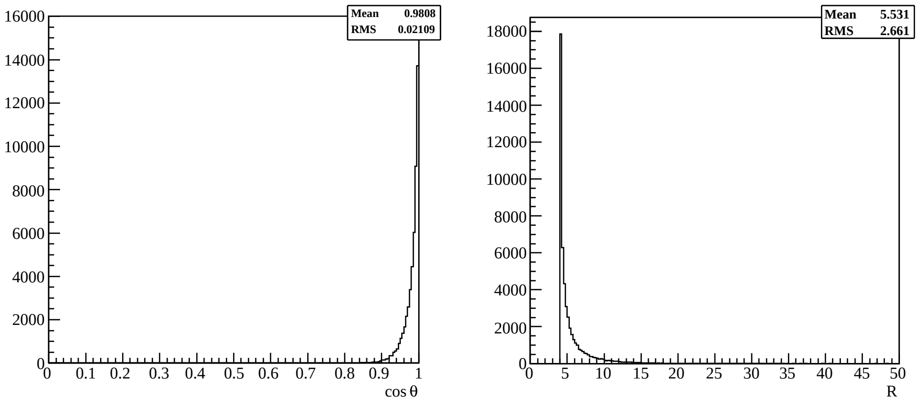

Figure 3: Distribution of cosθ and R in reconstructed $J/\psi \rightarrow \mu^+\mu^-$ candidates.

From the above discussion a strategy for calibrating the momentum scale using J/ψ decays is to scan several values of the constant c_{ion} in Equation 2. For each value of c_{ion} fits to the J/ψ mass distribution are made in bins of momentum. The resulting values are then fitted to a constant and the χ^2

of the fit extracted. The optimal value for the energy loss correction is the value for which the χ^2 is minimized. Any residual discrepancy between the observed and true J/ψ mass is attributed to the magnetic field scale and corrected by the scaling the field map.

3 Results

The J/ψ mass distribution has been studied with a sample of 110,000 inclusive J/ψ events from the DC 06 production [5]. A J/$\psi \rightarrow \mu^+\mu^-$ selection was made using the LoKi [9] analysis toolkit. First, muon candidates were selected by requiring that $\Delta L_{\mu\pi} < -8$ and pt > 500 MeV. Muon pairs with opposite charge were then fitted to a common vertex and the χ^2 required to be less than 10. Finally, in order to benefit from the improvements in the track fit [10] that have occurred since the DSTs were produced a re-fit of the tracks in selected candidates was made and the vertex refitted. This also allowed the effect of varying the energy loss correction and changing the field scale to be studied. Around 50,000 J/ψ candidates were reconstructed. The momentum spectra for the candidates that are associated to a true J/ψ using the Monte Carlo truth is shown in Fig. 4.

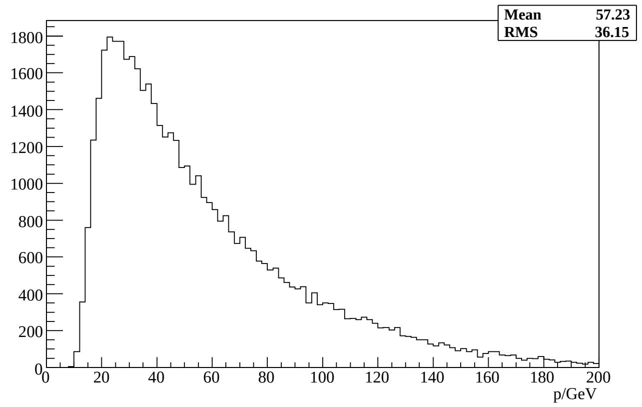

Figure 4: Momentum spectrum of reconstructed J/ψ candidates versus p/GeV.

Fig. 5 shows the resulting invariant mass distribution. Due to QED radiative corrections the distribution has a non-Gaussian tail towards low invariant mass. One way to account for this is to fit a Crystal Ball [11] which describes the radiative tail using a power law:

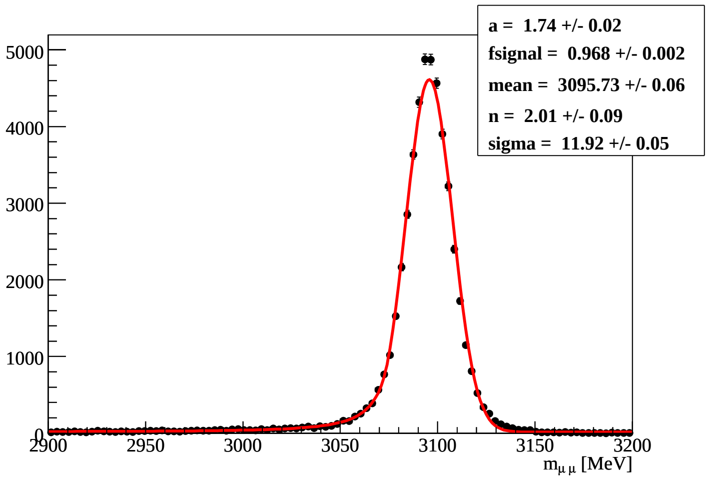

Figure 5: J/ψ mass distribution. The result of a fit to a Crystal Ball plus a flat background component is superimposed.

This procedure has some disadvantages. First, with small statistics the fit was found to be unstable due to correlations between the parameters. In addition, it is hard to judge how 'Gaussian' the distribution would be in the absence of radiative corrections. Finally, as will be seen a bias towards lower mass is seen with the Crystal Ball fit. These difficulties can be avoided in Monte Carlo studies by fitting the difference between the true and reconstructed invariant mass of the di-muon pair. Fig. 6 shows the distribution of this variable together with the result of a Gaussian fit. It can be seen that the distribution is well described by a single Gaussian [1].

Fig. 7 shows the bias on the J/ψ mass as a function of momentum in four cases:

- A Crystal Ball fit to the J/ψ mass distribution with the standard track fit.
- A Gaussian fit to the difference between the true and reconstructed invariant mass with the standard track fit.

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[1] A double Gaussian would fit better. However, for the studies in this note there is no need to introduce this additional complication.

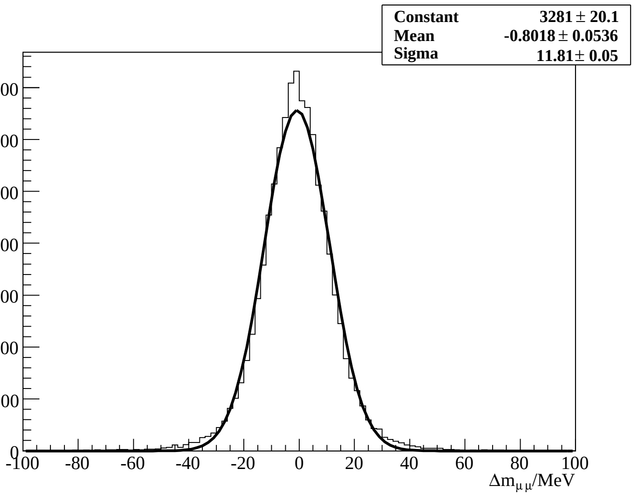

Figure 6: Di-muon invariant mass distribution. The result of a Gaussian fit is superimposed.

- A Gaussian fit to the difference between the true and reconstructed invariant mass with the energy loss correction turned off in the fit.
- A Gaussian fit to the difference between the true and reconstructed invariant mass with the magnetic field scaled downwards by 0.5 % in the track fit.

A bias of -1.2 MeV is seen with the Crystal Ball fit. Fitting a Gaussian to the difference between the true and reconstructed di-muon mass the bias is reduced to -0.8 MeV. This shows that 0.4 MeV of the bias is due to the radiative corrections [2].

The results for runs with the energy loss correction turned off in the fit and with the field scaled behave as expected from Section 2. In the former case the bias depends on the square root of the momentum as expected from Equation 9. For the latter case a 0.5 % bias independent of the J/ψ momentum is seen. Fig. 8 shows the results of three runs with different scalings of the field together with Equations 7 and 8. In the first formula the average value of R found for true J/ψ candidates of 5.5 (see Fig. 3) is used. It can be seen that for the J/$\psi \to \mu^+\mu^-$ case the approximations made in Equation 8 are valid and that the simulation behaves as expected.

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[2] This effect was seen in all the samples fitted in this note.

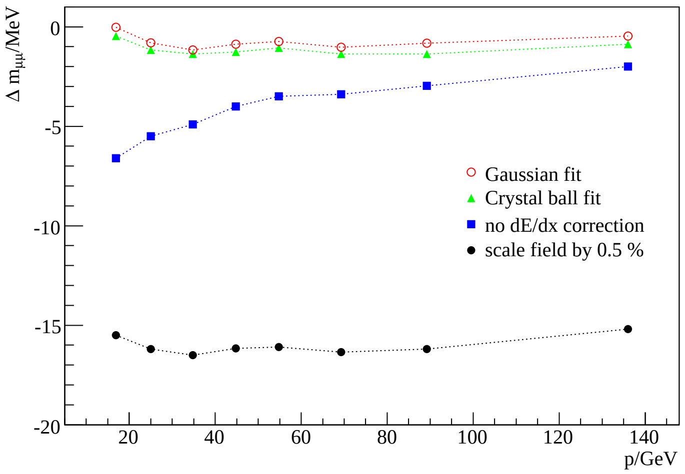

Figure 7: Bias on the J/ψ mass for the four cases discussed in the text.

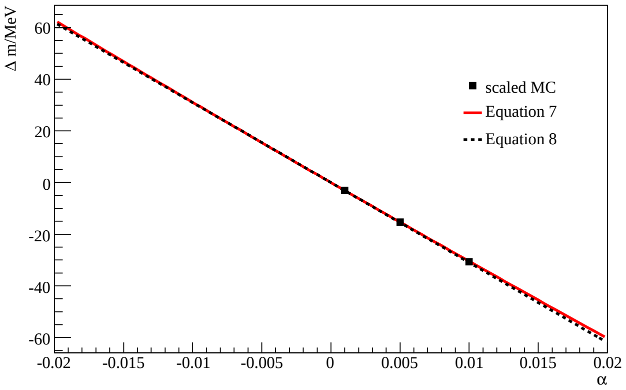

Figure 8: Effect of varying the magnetic field scale on the fitted bias. The points have been corrected for the -0.8 MeV bias seen in the run with nominal field. Equations 7 and 8 are superimposed.

3.1 Origin of the bias

The bias on the J/ψ mass seen with the standard track fit settings has been investigated in detail. The first question to answer is whether the value of c_{ion} used in the energy loss correction is optimal. Therefore, a scan of values of c_{ion} was made as described in Section 2. Fig 9 shows example plots of the bias versus momentum for three values of c_{ion} whilst Fig 10 shows the χ^2 of the constant fit versus c_{ion}. Fitting a parabola to the latter plot gives the minimum of the χ^2 to be at $c_{ion} = 317 \pm 13$ MeV mm^{2}mol^{-1} for which the bias is -1.18 ± 0.05 MeV mm^2 mol^{-1}. The value found for c_{ion} is lower than default value used in the reconstruction. However, it can also be seen (Fig. 9) that with $c_{ion} = 354$ MeV mm^{2}mol^{-1} the bias in the lowest bin is zero. This is consistent with the value of c_{ion} being tuned for tracks with p= 10 GeV as discussed in Section 1. There is no tuning of the energy loss correction that will remove the bias and give a linear momentum scale.

After the energy loss correction has been tuned the average bias can be removed by scaling the field by a factor of 0.00039 [3]. Fig. 11 shows the results of a run with $c_{ion} = 354$/MeV mm^2 mol^{-1} and a magnetic field scale factor $\alpha = 0.00039$. It can be seen that a residual bias remains that varies between -0.3 MeV to 0.5 MeV depending on the J/ψ momentum.

To study the influence of the Geant4 version and settings two samples of similar size to that used above were generated with Gauss v30r5, digitized with Boole v14r7 and reconstructed with Brunel v32r1. In the first sample the standard LHCb settings of Geant4 were used whilst in the second δ-rays were simulated [4]. Using the procedure discussed above the optimal tuning of the energy loss correction was extracted together with the bias. The results are summarized in Table 1. No significant difference is seen between the two versions of Geant. In both cases a bias of -1.2 MeV is seen. Turning on δ-rays the bias is reduced to ~ 1 MeV though higher statistics are needed to confirm this result. It is concluded that the bias is not strongly effected by the version and settings of Geant4.

Run	c_{ion}**/MeV mm^2 mol^{-1} at the minimum of the χ^2**	**Bias/MeV**
DC 06	317 ± 13	-1.18 ± 0.05
New Geant	300 ± 13	-1.22 ± 0.04
New Geant (δ-rays on)	330 ± 11	-1.05 ± 0.05

Table 1: Optimal dE/dx correction and bias for different settings of Geant4.

Finally, a sample of events were generated in which energy loss was turned

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[3] The option **MagneticFieldSvc.ScaleFactor** = 1.00039

[4] This was done by setting the Gauss option **Giga.ModularPL.CutForElectron** to 1.0 mm.

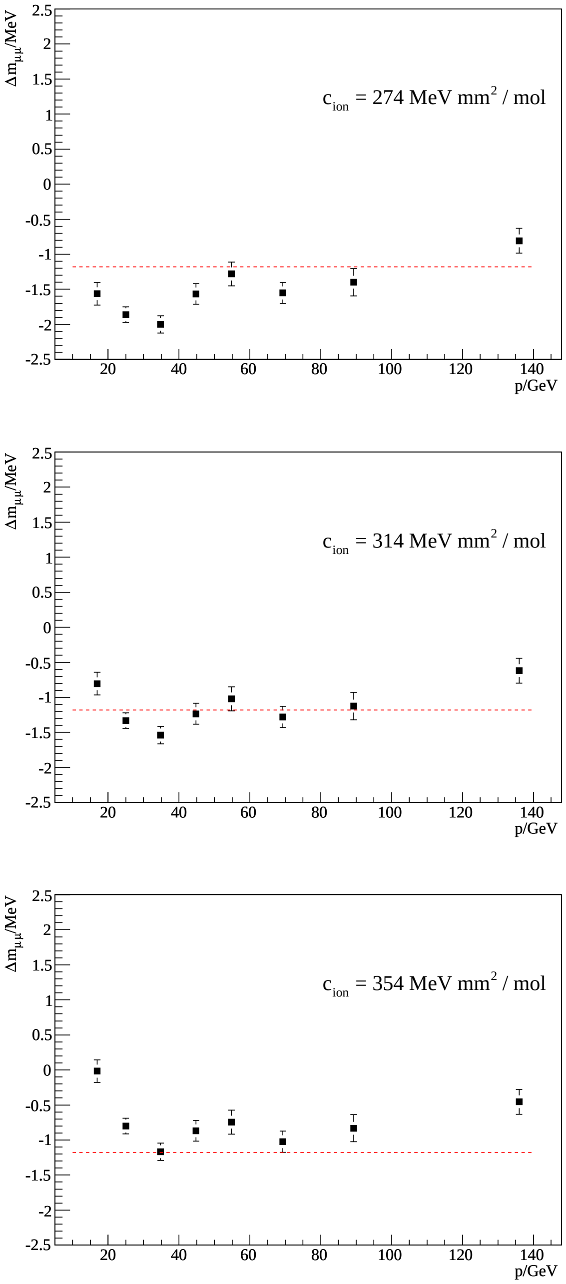

Figure 9: Examples of the $\Delta m_{\mu\mu}$/MeV versus p/GeV from which the χ^2 was extracted. The dashed line in each plot is the best fit to the run with $c_{ion} = 314$ MeV mm^{2}mol^{-1}.

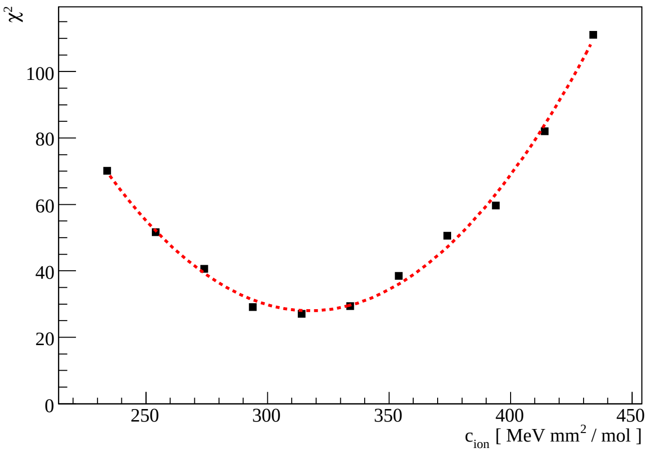

Figure 10: χ^2 versus c_{ion}/MeV mm^2 mol^{-1}. A fit to a parabola is superimposed.

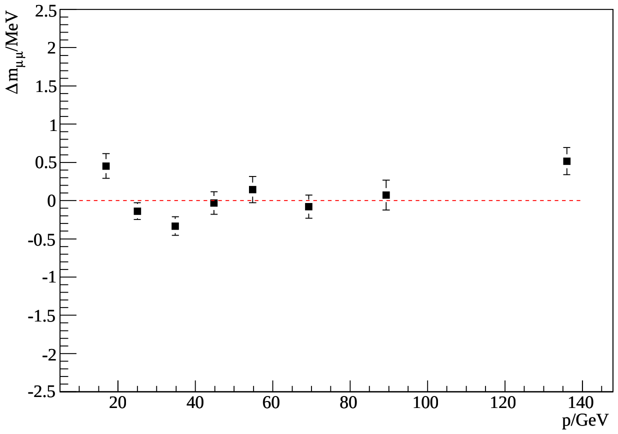

Figure 11: Bias on the $\Delta\text{m}_\mu\mu$ versus momentum with χ^2 versus c_{ion}/MeV mm^2 mol^{-1} and a magnetic field scale factor $\alpha = 0.00039$.

off in Geant4. Fig. 12 shows the bias on the J/ψ mass as a function of momentum for the sample where energy loss is turned off in Geant4. It can be seen that the bias is reduced to -0.18 ± 0.06 MeV. This demonstrates that the majority of the bias is related to the energy loss correction. Clearly, there is some tension between this statement and the conclusions drawn in the previous paragraph. This paradox would be resolved if the effects of the approximations made in modeling energy loss in the reconstruction (Fig. 2) cause the observed bias. First studies using the full Bethe Bloch formula will be presented in the next section.

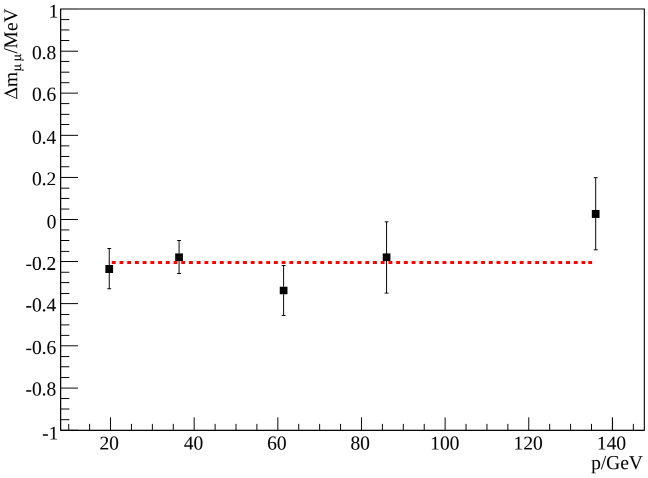

Figure 12: $\Delta m_{\mu\mu}$/MeV versus p/GeV for the Monte Carlo sample with energy loss switched off in Geant (and the reconstruction). The dashed line is the best fit to a constant.

The origin of the remaining -0.18 ± 0.06 MeV bias has also been investigated. Changing the settings of the Runga-Kutta algorithm used for extrapolation in the magnetic field was found to have no effect. Using an implementation of the analytic extrapolation technique described in [12] the bias is reduced to -0.14. This indicates that part of the bias is due to systematic effects in the Runga-Kutta algorithm. However, the origin of the remainder is unclear. One possibility is that it is a systematic effect originating in the extrapolation through the magnetic field in the Geant step.

4 Studies with the Bethe-Bloch formula

An implementation of the Bethe-Bloch formula (Equation 1) has recently become available in the LHCb reconstruction software [13]. This formula allows a more accurate estimate of the energy loss to be made in the reconstruction.

It also has the advantage that there are no free parameters to tune. Fig. 13 shows the bias on the J/ψ mass found using this formula for the DC 06 data sample. It can be seen that for J/ψ's with p > 40 GeV there is no bias.

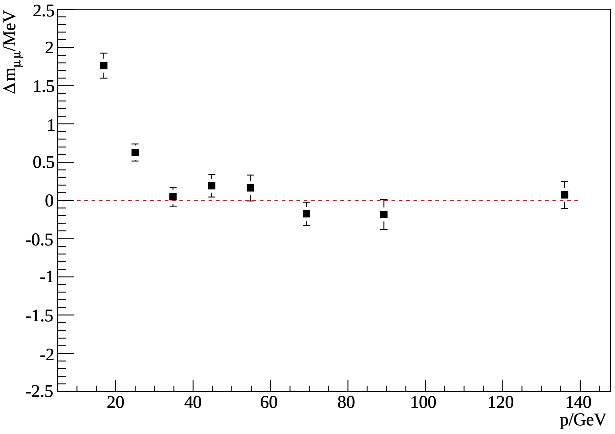

Figure 13: $\Delta m_{\mu\mu}$/MeV versus p/GeV for the DC 06 Monte Carlo sample reconstructed using the Bethe-Bloch formula (Equation 1) to correct for the effect of energy loss. The dotted line corresponds to no bias.

However, at lower momentum a bias of up to 1.8 MeV is seen.

Though the Bethe-Bloch formula has no free parameters a scale factor can be introduced and the tuning procedure described in the previous section used to find its optimal value. This procedure allows the deviation from the Bethe-Bloch formula seen above to be quantified. Table 2 shows the result of performing this procedure for the DC 06 Monte Carlo sample together with the samples generated with the new version of Geant4. It can be seen that

Run	**Scale factor at the minimum of the χ^2**	**Bias/MeV**
DC 06	0.72 ± 0.03	-1.06 ± 0.05
New Geant	0.63 ± 0.4	-1.15 ± 0.04
New Geant (δ-rays on)	0.75 ± 0.03	-0.89 ± 0.05

Table 2: Optimal dE/dx correction and bias for different settings of Geant4 using the Bethe-Bloch formula for to correct for energy loss in the reconstruction.

optimal agreement with the shape expected from the Bethe-Bloch formula is found with a scale factor of around ~ 70 %. As observed in the studies in the previous section the smallest bias is found for the data generated with δ-ray simulation turned on in Geant4. In this case the optimal settings to remove the bias are $f_{scale} = 0.75$ and a magnetic field scale factor $\alpha = 0.00029$. Fig 14 shows the results of a run with these settings. It can be seen that the bias varies between -0.4 MeV and 0.4 MeV depending on the momentum.

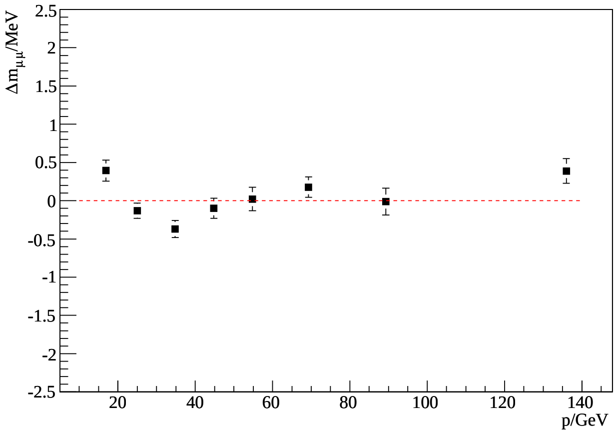

Figure 14: $\Delta m_{\mu\mu}$/MeV versus p/GeV for the Monte Carlo sample with δ-ray simulation reconstructed using the Bethe-Bloch formula to correct for the effect of energy loss with a scale factor of 0.75. The magnetic field was scaled by a factor $\alpha = 0.00029$ The dotted line corresponds to no bias.

From these studies it is concluded that using the full Bethe-Bloch formula reduces the bias but only by a small amount. Therefore, the approximations made in Equation 1 are not the dominant cause of the observed effect.

5 Mass resolution

From the fits to the mass distribution the resolution was extracted. Fig. 15 shows the mass resolution found in four cases:

- A Gaussian fit to invariant mass distribution for the Monte Carlo sample from the DC 06 production reconstructed with the Brunel v30r14 track fit.

- A Gaussian fit to the invariant mass distribution for the Monte Carlo sample from the DC 06 production with the Brunel v32r1 track fit.
- A Crystal Ball fit to the J/ψ mass distribution for the Monte Carlo sample from the DC 06 production reconstructed with the Brunel v32r1 track fit.
- A Gaussian fit to the difference between the true and reconstructed invariant mass for data generated with the new version of Geant4 and reconstructed with Brunel v32r1.

The improvements to the track fit between DC 06 and the current version of the reconstruction [10] have led to a 5 % improvement in the mass resolution. In addition, a further 10 % improvement is seen with the data reconstructed with more recent versions of Geant4. A similar is observed in studies of the momentum resolution and is attributed to changes in the multiple scattering model used in Geant4 (see Appendix A).

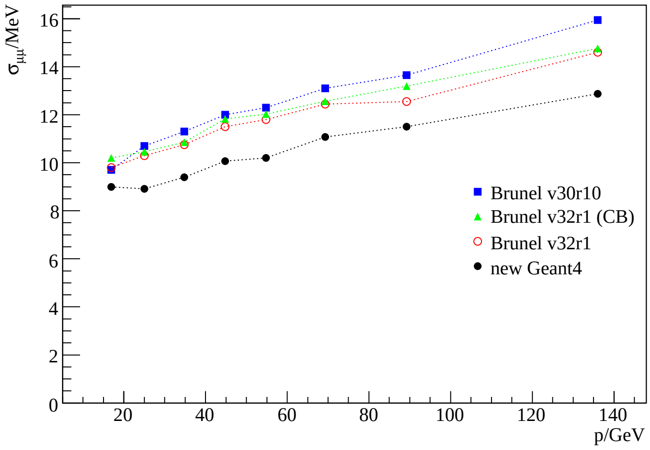

Figure 15: Di-muon mass resolution.

The dependence of the di-muon mass resolution on the number of visible interactions has also been studied. Fig. 16 shows the momentum resolution versus the number of interactions. No significant dependence is seen indicating that the mass resolution is robust against pile-up effects.

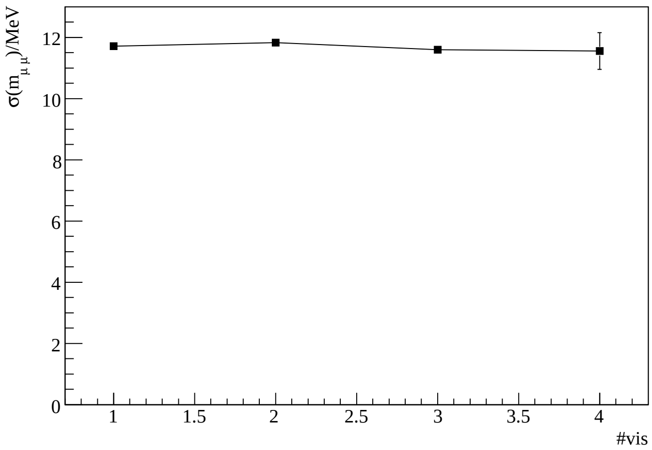

Figure 16: Di-muon mass resolution versus the number of visible interactions.

6 Summary

In this note the use of J/ψ decays for calibrating the momentum scale has been discussed. Studies with simulated data from the DC' 06 production have shown that a bias is present in the Monte Carlo. The majority of the bias has been shown to be due to the energy loss correction (Table 3). Most of this bias remains even if the full Bethe-Bloch formula is used to correct for energy loss in the reconstruction and with more recent versions of Geant4. More work is needed to understand how energy loss leads to this bias. For example, the material properties assumed in the reconstruction and Geant4 should be cross-checked. In addition, the effect of the tail of the straggling function needs to be better quantified.

A bias is also observed when fitting a Crystal Ball shape to account for the effect of QED radiative corrections. Further studies are needed to understand if fitting a function that better models the shape of the mass distribution can reduce this bias. For example the approach described in [14] of modeling the radiative tail using a polynomial with parameters that are fixed based on Monte Carlo studies will be tried. With real data to correctly disentangle the effects of radiative corrections, energy loss, the scale of the field and also background will be a challenge.

As a by-product of the studies made above the J/ψ mass resolution has also been investigated. It has been shown that there is a significant improvement

Effect	**Bias**/MeV
Radiative corrections	-0.4
dE/dx correction	-0.6
Field propagation	-0.2
Total	-1.2

Table 3: Summary of the origin of the observed bias on the J/ψ mass with the standard track fit settings used in the DC 06 production.

in the mass resolution between DC 06 and the reprocessing. In addition, a further 10 % improvement is seen with the latest version of Geant4.

A Momentum Resolution and Geant4 version

For DC 06 production the LHCb simulation, Gauss, was based on Geant4 version 7.1. This version of Geant4 is known to have several bugs in the treatment of multiple scattering. These are fixed from Geant4 8.1 onwards. To understand the effect of these changes on the track fit 1000 minimum bias events were generated with Gauss v30r5, digitized with Boole v14r7 and reconstructed with Brunel v32r1. For comparison 1000 minimum bias events generated for DC 06 with Gauss v25r7 were digitized and reconstructed with the same versions of Boole and Brunel.

To quantify the performance of the track fit in the two samples the probability of χ^2 and the momentum resolution have been studied. Fig. 17 shows the probability of χ^2 distributions in the two cases. It can be seen that the accumulation of events at low $p(\chi^2)$ is reduced with the later version of Gauss. From previous studies it is known that one contribution to this accumulation is tracks that have a 'large' kink due to multiple scattering around the entrance or exit of the magnet. Therefore, the improvement in this distribution indicates that the effect of multiple scattering is reduced and also closer to the model assumed in the track fit.

Fig 18 shows the momentum resolution. An improvement of 10 % is seen with the later version of Gauss. Since the momentum resolution is known to be dominated by multiple scattering the improvement indicates that the

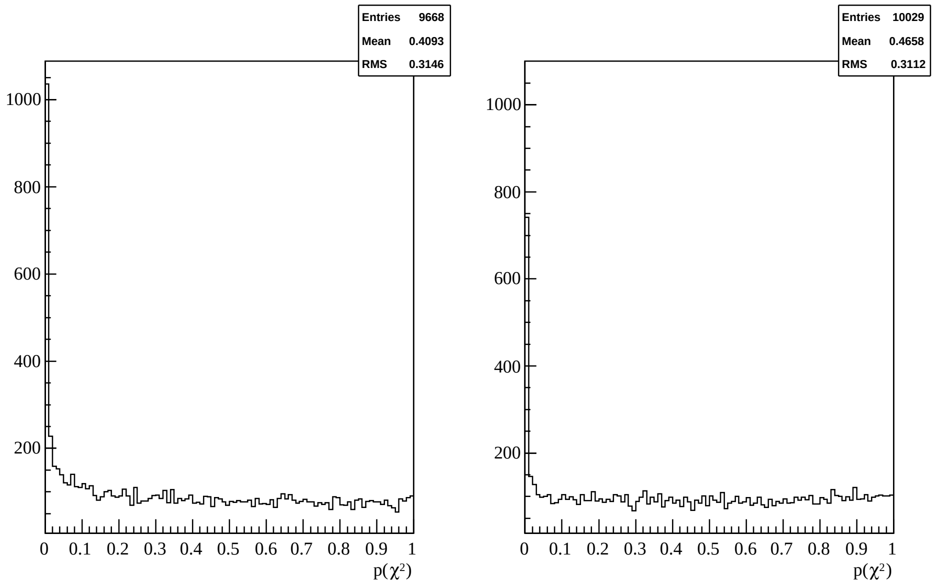

Figure 17: Probability of χ^2 for data generated with Gauss v25r7 (left) and v30r5 (right).

effect of this is reduced. The size of the improvement is somewhat surprising. From previous studies [15] it is known that the momentum resolution scales with $\sqrt{X_0}$. Therefore, a 10 % improvement in the momentum resolution is equivalent to a 20 % reduction in the amount of detector material.

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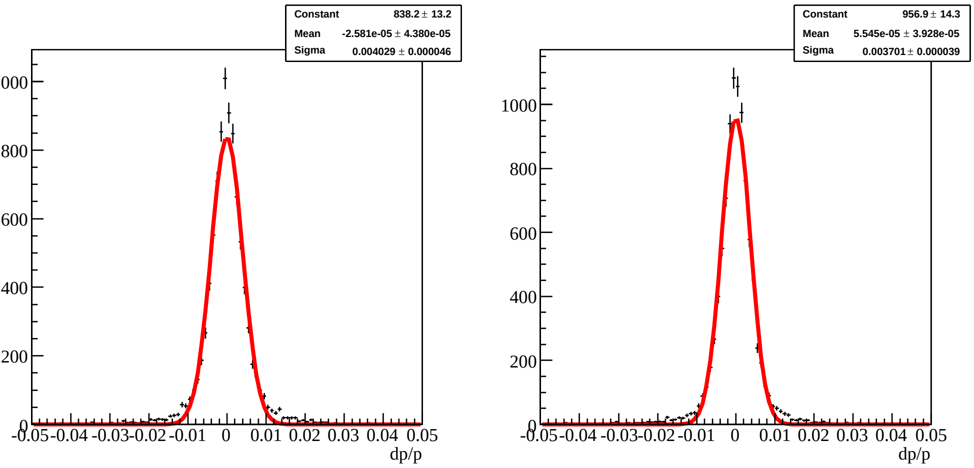

Figure 18: Momentum resolution for data generated with Gauss v25r7 (left) and v30r5 (right). The result of a Gaussian fit is superimposed on each histogram.

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