Online and offline monitoring of the relative luminosity at LHCb

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Abstract
This note presents a proposal for the monitoring of the relative luminosity based on “minimum bias” interaction triggers, as well as some ideas about the possible use of the relative integrated luminosity in the offline environment.

1 Introduction
The LHC will provide $pp$ interactions at energies never reached before at a hadron collider. Experimentally, most production cross-sections in such interactions are basically unknown (despite a few low-statistics studies performed using cosmic rays), and their measurement in the forward region is an interesting part of the LHCb physics programme with the first data.

Production cross-section measurements require the knowledge of the time-integrated luminosity corresponding to the analyzed sample of events. Several methods can be envisaged to determine absolute luminosity of certain specific samples of events. However, this absolute luminosity calibration needs to be “transferred” to any sample used subsequently for cross-section measurements. This implies that the relative luminosity be monitored in a systematic way during the LHCb data-taking, and that this information be properly propagated in the offline environment for use by the people working on these measurements.

In contrast, the knowledge of the luminosity will not be necessary for many other measurements planned at LHCb. However, several of these measurements will be statistics limited, and it will therefore be important to make sure the integrated luminosity recorded on tape is maximized. This also calls for a monitoring of the luminosity during data-taking, such as to be able to give fast feedback to the shift crews operating the LHCb detector and the LHC machine.

This note is organized as follows. Section 2 outlines general requirements. Section 3 presents a proposal for the monitoring of relative luminosity variations during data-taking, i.e. in the online environment. Section 4 discusses some issues related to propagation of these monitoring results to the offline environment.
2 Requirements and general considerations

2.1 Physics requirements

The first main requirement is that a physicist performing production studies with any LHCb dataset should be able to easily find out the absolute integrated luminosity $L_{\text{int}}$ corresponding to this dataset. This will enable the measurement of a production cross-section as

$$\sigma_x^{\text{measured}} = \frac{N_x}{\varepsilon_x L_{\text{int}}} ,$$

where $N_x$ is the number of signal events of a certain type $x$ observed in the dataset, and $\varepsilon_x$ is the total efficiency of the trigger, reconstruction and offline selection for those signal events. The fact that the size of the dataset is not known in advance, because some data may get lost during offline processing and analysis (e.g. one finds out after the fact that some fraction of the data has problems and therefore needs to be discarded in the final analysis), leads to the requirement that $L_{\text{int}}$ should be accessible for any subset of the original data sample, which in turn implies that the luminosity variations need to be monitored within the dataset.

This monitoring implies that some reference rate $R_i$, known to be proportional to the luminosity, be recorded in many short time intervals $\Delta t_i$ covering the whole data-taking period (without overlap). In practice, it is unlikely that the relation between the reference rate and the absolute luminosity be known at the time of data-taking. So this monitoring would only provide a relative measurement of the integrated luminosity, i.e.

$$\frac{L_{\text{int}}}{U} = R_{\text{int}} = \sum_i R_i \Delta t_i^{\text{live}},$$

where $U$ is an unknown (but hopefully constant !) luminosity unit and the sum runs over all deadtime-subtracted time intervals

$$\Delta t_i^{\text{live}} = \Delta t_i - \Delta t_i^{\text{dead}},$$

$\Delta t_i^{\text{dead}}$ being the dead time during which no data was recorded.

The determination of the absolute “calibration” of the luminosity, i.e. of the value of the constant $U$, will require dedicated analyses, which may have to be performed on dedicated datasets. Several methods to determine absolute luminosity are desirable. Some of them might be fast and imprecise, whereas other more precise ones may require a lot more statistics. As usual, systematic uncertainties may differ between methods, which makes it a useful check to compare the results of several methods. Each of these methods will provide, based on a certain calibration data sample, an absolute luminosity estimate $L_{\text{int}}^{\text{calib}}$ from which the luminosity unit

$$U = \frac{L_{\text{int}}^{\text{calib}}}{R_{\text{int}}^{\text{calib}}}$$

can be determined using the integrated rate $R_{\text{int}}^{\text{calib}}$ recorded for this calibration sample. Once $U$ is known, any production cross-section measurement can be performed as

$$\sigma_x^{\text{measured}} = \frac{N_x}{\varepsilon_x R_{\text{int}} U} ,$$
<table>
<thead>
<tr>
<th></th>
<th>2008 (5 pb(^{-1}))</th>
<th>2009 (0.5 fb(^{-1}))</th>
<th>2010 (2 fb(^{-1}))</th>
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</thead>
<tbody>
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<td>Van der Meer scan</td>
<td>20%</td>
<td>5–10%</td>
<td>5–10%</td>
</tr>
<tr>
<td>Beam-gas interactions</td>
<td>10%</td>
<td>&lt;5%</td>
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<tr>
<td>(pp \rightarrow Z(\mu^+\mu^-)X)</td>
<td>5%</td>
<td>4%</td>
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<tr>
<td>(pp \rightarrow ppp^+\mu^-)</td>
<td>20%</td>
<td>2.5%</td>
<td>1.5%</td>
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Table 1: Methods to determine the absolute luminosity and preliminary estimates [1] of the relative precision to be expected as a function of time and/or integrated luminosity.

where the integrated rate \(R_{\text{int}}\) is of course the one obtained during the monitoring of the data used for the cross-section measurement itself.

Table 1 gives a list of methods foreseen to measure absolute luminosities in LHCb, as well as rough estimates of the relative precision expected on \(L_{\text{calib}}\) in 2008, 2009 and 2010. Direct methods, such as the beam-gas method [2] or the Van der Meer separation [3], will aim at the determination of beam parameters in particular the bunch charges, bunches densities (in time and space) and crossing angle, from which \(L_{\text{int}}\) can be calculated as an “overlap integral”. Indirect methods will measure physics processes \(y\), such as \(pp \rightarrow Z(\mu^+\mu^-)X\) [4] or \(pp \rightarrow ppp^+\mu^-\) [5] for which the cross section can be reliably predicted by theory, and will provide

\[
L_{\text{calib}} = \frac{N_y}{\varepsilon_y \sigma^{\text{theory}}_y}. \tag{6}
\]

The second main requirement is about the precision \(L_{\text{int}} = R_{\text{int}} U = L_{\text{calib}} R_{\text{int}} / R_{\text{calib}}\). This is determined by the physics analyses where precision is most critical and driven by the knowledge of the luminosity. Such analyses include the measurement of the parton density functions (PDFs) using \(Z \rightarrow \mu^+\mu^-\) events. At present these PDFs are known at the 4% level (limiting the precision of the luminosity calibration using \(Z \rightarrow \mu^+\mu^-\) events). However, a determination of the luminosity at the 1–2% level (as seem to be possible with “elastic” dimuon production which can be precisely predicted by theory) would allow a significant improvement of our knowledge of the PDFs [4]. It is therefore desirable that the determination of the ratio \(R_{\text{int}} / R_{\text{calib}}\) have an uncertainty significantly smaller than 1%.

A third requirement is that relative luminosity measurements \(R_{\text{int}}\) should be available from the beginning of the data taking, so that cross section measurements can already be performed with the first data. This means that a simple and robust concept should be developed, which is also immune to machine-induced backgrounds which are likely to not be stable and too well controlled in the initial phase of the experiment.

A final general requirement from all statistically-limited physics analyses (even the ones for which the integrated luminosity does not need to be known), is that the integrated luminosity be as large as possible. This implies that the events recorded by the experiment should then be carefully tracked through all phases of the processing and analysis, to make sure that none is lost. The size of the samples, as well as possible losses, should be quantified in terms of integrated luminosity.
2.2 Online requirements

The members of the shift crew has the responsibility to record as much good data as possible, quantified by the total integrated luminosity. Hence they should be able to monitor the luminosity delivered by the machine while the experiment is running and the fraction of this luminosity which is logged to permanent storage. In practice this implies that they be provided with regular updates of the measurement of the relative instantaneous luminosity $R_i$ as well as of an estimate of the trigger live time fraction $\Delta t_i^{\text{live}}/\Delta t_i$ (which should of course be as close to unity as possible). A reasonable requirement on the frequency and precision of these updates is to ask every 5–10 seconds for a new measurement with 5–10% relative precision. In this way, action could be taken without delay in case, for example, of a sudden luminosity drop (of 20% or more) or dead time increase.

Again, this information should be available from the start of data-taking and should not be biased due to the presence of machine-induced background. In addition, information on machine-induced background, with similar precision and frequency, will be most welcome by the people on shift.

3 Online monitoring of the relative luminosity

The online monitoring proposed here consists of measuring the rates of simple “minimum bias” interaction triggers. These triggers should be High-Level Trigger (HLT) algorithms running on random crossings provided by the Readout Supervisor (ODIN) based exclusively on the knowledge of the bunch structure of the colliding beams. The data used by such triggers could therefore come from any sub-detector. For example, one could trigger on a minimal transverse energy deposited in the hadron calorimeter, or on a minimal number of 2D tracks found in the VELO.

We first describe the concept (which is independent of the actual implementation), then discuss the statistical precision and show how the requirements set in the previous section can be reached in a few explicit (but arbitrary) scenarios. Finally we give some examples of interaction triggers and discuss some implementation issues.

3.1 Concept

3.1.1 Simplest case without background

The rates $R_i$ mentioned in Eq. 2 are nothing else than an estimate of the relative instantaneous luminosity. Naively, these $R_i$ could just be measurements of the output rate of the interaction trigger performed over the time intervals $\Delta t_i$. However, this would only be true in absence of pileup,¹ i.e. at very low luminosity and high bunch-crossing frequency. In practice, pileup cannot be avoided in LHCb, even in the early phase of the experiment where the low luminosity will be “compensated” by a small bunch-crossing frequency. This means that the interaction trigger rate does not depend linearly on the instantaneous luminosity. However, this dependence is known and an appropriate transformation can be applied on the trigger rate to obtain values of $R_i$ that do scale with luminosity.

¹We call “pileup” the possibility that several $pp$ interactions may occur in the same crossing of two bunches at IP8. A “bunch” is defined as an RF bucket filled with protons.
The LHC machine consists of a total of $N_{\text{buckets}} = 3564$ RF buckets per beam, separated by regular time intervals $\Delta t_{\text{buckets}} = 24.95$ ns (corresponding to the $1/\Delta t_{\text{buckets}} = 40.08$ MHz clock), which may or may not be filled with a bunch of protons. Let $N_{\text{bunches}}$ be the number of bunches per beam, and $N_{\text{bb}}$ the number of those that do intersect at IP8 with a bunch from the other beam (note that $N_{\text{bb}} < N_{\text{bunches}}$ since not all bunches collide in IP8). The average bunch crossing frequency at IP8 is then given by

$$f_{\text{bb}} = \frac{N_{\text{bb}}}{N_{\text{bunches}} \Delta t_{\text{buckets}}} = \frac{N_{\text{bb}}}{\ell_{\text{ring}}/c},$$

where $\ell_{\text{ring}} = 26.66$ km is the circumference of the LHC ring and $c$ is the speed of light.

We define a visible interaction as a $pp$ collision which does fire the interaction trigger. The mean number of such interactions in a single bunch crossing is given by

$$m = L\sigma_{\text{vis}}/f_{\text{bb}},$$

where $L$ is the instantaneous luminosity and $\sigma_{\text{vis}}$ the fraction of the total $pp$ cross section producing an interaction firing the interaction trigger. From the above equation, it is easy to see that the rate

$$R = f_{\text{bb}}m$$

is equal to $L\sigma_{\text{vis}}$, and hence is a measure of the relative luminosity in units of $U = 1/\sigma_{\text{vis}}$. The number $n$ of visible interactions in a bunch crossing is a Poisson variable with mean $m$, i.e.,

$$\text{prob}(n) = \frac{m^ne^{-m}}{n!},$$

implying that the probability that the interaction trigger will fire on a bunch crossing is

$$p_{\text{bb}} = 1 - \text{prob}(0) = 1 - e^{-m}.$$

From the latter, we can extract

$$m = -\ln(1 - p_{\text{bb}})$$

and re-write Eq. 9 as

$$R = f_{\text{bb}}\ln(1 - p_{\text{bb}}).$$

Note that, in the limit of no pileup, i.e., when $m \ll 1$ (or equivalently $p_{\text{bb}} \ll 1$), the above expression of $R$ becomes equal to $f_{\text{bb}}/p_{\text{bb}}$, which is nothing else than the output rate of the interaction trigger due to the $pp$ collisions.

It is rather amusing to think that the proposed rate $R$ is proportional to $-\ln(\text{prob}(0))$, i.e., that the luminosity can be monitored by measuring “emptiness”, or more precisely by measuring the probability of the absence of interaction in a bunch crossing. This is actually a very nice way to take pileup into account, because it avoids the difficulty of having to count the number of interactions in a bunch crossing or disentangle single-interaction events from multiple-interaction events. As far as we know, such idea was originally put forward (within LHCb) in Ref. [6], which contains a chapter discussing the pileup system as a luminosity monitor.
3.1.2 Simple case in presence of background

In real life, the interaction trigger may fire not only due to a $pp$ interaction occurring in a bunch crossing, but also because of

- machine-induced background, such as beam-gas interactions and particles from the beam halo, and
- background unrelated to the machine, e.g., cosmic rays and electronic noise in the detector used for the interaction trigger.

We describe below how these effects can be measured and “subtracted” from the total interaction trigger rate.

The idea is to use, in addition to the normal bunch-crossings, crossings where one or both buckets are empty. Calling “beam1” the beam circulating clockwise in the LHC (i.e., traversing LHCb from the VELO to the muon stations) and “beam2” the anticlockwise beam, we define the following four different types of crossings:

- **bb**: crossing between a filled bucket of beam1 and a filled bucket of beam2;
- **be**: crossing between a filled bucket of beam1 and an empty bucket of beam2;
- **eb**: crossing between an empty bucket of beam1 and a filled bucket of beam2;
- **ee**: crossing between an empty bucket of beam1 and an empty bucket of beam2.

It should be noted that the four types of crossings will always be present, whether in the initial phase of data-taking in 2008 or in the nominal operating mode. The number of crossings $N_{\alpha\beta}$ in a machine turn for a given type $\alpha\beta$ ($\alpha\beta = bb, be, eb, ee$) and the corresponding average frequency

$$f_{\alpha\beta} = \frac{N_{\alpha\beta}}{N_{\text{buckets}} \Delta t_{\text{buckets}}} = \frac{N_{\alpha\beta}}{\ell_{\text{ring}}/c},$$

will depend on the exact bunch structure. We give some examples in Table 2, Section 3.2. Note that we always have $N_{\text{be}} = N_{\text{eb}} = N_{\text{bunches}} - N_{\text{bb}}$ and $N_{\text{ee}} = N_{\text{buckets}} - 2N_{\text{bunches}} + N_{\text{bb}}$.

Let us define, for a bb crossing:

$$p_0 = \text{prob. that interaction trigger fires due to background unrelated to the beams},$$

$$p_1 = \text{prob. that interaction trigger fires due to beam1-induced background},$$

$$p_2 = \text{prob. that interaction trigger fires due to beam2-induced background},$$

$$p_{bb} = \text{prob. that interaction trigger fires due to } pp \text{ interactions}.$$  

Under the assumptions that $p_0$ is the same in bb, be, eb and ee crossings, that $p_1$ is the same in bb and be crossings (but 0 in eb and ee crossings), and that $p_2$ is the same in bb and eb crossings (but 0 in be and ee crossings), we can write

$$1 - q_{ee} = 1 - p_0,$$

$$1 - q_{be} = (1 - p_0)(1 - p_1),$$

$$1 - q_{eb} = (1 - p_0)(1 - p_2),$$

$$1 - q_{bb} = (1 - p_0)(1 - p_1)(1 - p_2)(1 - p_{bb}).$$
where \( q_{\alpha\beta} \) is the probability that the interaction trigger fires on a crossing of type \( \alpha\beta \). Because the DAQ system (more precisely the readout supervisor) knows the true nature of each crossing, all four quantities \( q_{\alpha\beta} \) can be measured and the above equations solved to obtain

\[
\begin{align*}
1 - p_0 &= 1 - q_{ee}, \\
1 - p_1 &= (1 - q_{be})/(1 - q_{ee}), \\
1 - p_2 &= (1 - q_{eb})/(1 - q_{ee}), \\
1 - p_{bb} &= (1 - q_{eb})(1 - q_{ee})/(1 - q_{eb}).
\end{align*}
\]

The quantities \( m \) of Eq. 12 and \( R \) of Eq. 13 will still be proportional to the luminosity and can be estimated as

\[
m = -\ln \frac{(1 - q_{bb})(1 - q_{ee})}{(1 - q_{be})(1 - q_{eb})}
\]

and

\[
R = -f_{bb} \left[ \ln(1 - q_{bb}) + \ln(1 - q_{ee}) - \ln(1 - q_{be}) - \ln(1 - q_{eb}) \right].
\]

In the limit of negligible pileup and very small background rates \( (p_{bb}, p_0, p_1, p_2 \ll 1) \), i.e. when \( q_{\alpha\beta} \ll 1 \), \( R \) reduces to the expression

\[
\lim_{q_{\alpha\beta} \ll 1} R = f_{bb} \left[ q_{bb} + q_{ee} - q_{be} - q_{eb} \right] = f_{bb} \left[ (q_{bb} - q_{ee}) - (q_{be} - q_{ee}) - (q_{eb} - q_{ee}) \right],
\]

which merely illustrates how the background is subtracted: the noise measured in the ee crossings is first subtracted from the measurements in the three other types of crossings, and then the noise-subtracted beam-background probabilities \( (q_{be} - q_{ee} \) and \( q_{eb} - q_{ee} \) are subtracted from the noise-subtracted interaction probability seen in bb crossings \( (q_{bb} - q_{ee}) \).

At this stage we can note that the \( R \) expression of Eq. 28 is somewhat simpler when expressed in terms of the probabilities \( \zeta_{\alpha\beta} = 1 - q_{\alpha\beta} \) that the interaction trigger does not fire on a crossing of type \( \alpha\beta \),

\[
R = -f_{bb} \left[ \ln(\zeta_{bb}) + \ln(\zeta_{ee}) - \ln(\zeta_{be}) - \ln(\zeta_{eb}) \right],
\]

so from now on we will express everything in terms of \( \zeta_{\alpha\beta} \) (which are the probabilities of “emptiness” measured for each type of crossings) rather than \( q_{\alpha\beta} \).

3.1.3 More realistic case dealing with bunch non-uniformities

The assumptions formulated in the previous section imply that all bunches should be identical, in particular have the same charge. In practice, this may not be true, and corrections will need to be applied, depending on the individual bunch currents. Indeed, if the average current per bunch participating in be (eb) crossings is different from the average current per beam-1 (beam-2) bunch participating in bb crossings, the beam-background probabilities measured in be (eb) crossings will differ from those present in bb crossings. So, before performing the beam-background subtraction, we should normalize the former to the bunch currents in the latter.

We will assume that the mean number of beam-background “interactions” in a crossing is proportional to the current (in analogy with the fact that the mean number of visible pp
interactions in a bunch crossing, \( m_i \) is proportional to the instantaneous luminosity). This is certainly true for beam-gas interactions, and very plausible for beam halo effects. Therefore, the beam-background rate in a given crossing or group of crossings will be written as

\[
B_i = I_i b_i \quad i = 1, 2 \text{ for beam1, beam2,}
\]

where \( I_i \) is the current producing this background and \( b_i \) is defined as the “specific” beam-background level, assumed to be independent of the current.

Using an expression similar to that of Eq. 13, the beam1-background rates observed in be crossings can be written as

\[
B_{1, be} = -f_{be} \ln(1 - p_{1, be}),
\]

where \( p_{1, be} \) is the probability that the interaction trigger fires due to beam1-induced background in be crossings. Since the noise probability \( p_{1, be} \) is by definition the same in all types of crossings, we have \( 1 - p_{1, be} = \zeta_{be}/\zeta_{ee} \) (from Eq. 24), hence

\[
B_{1, be} = -f_{be} [\ln \zeta_{be} - \ln \zeta_{ee}].
\]

The specific background level can then be estimated as

\[
b_1 = \frac{B_{1, be}}{I_{1, be}} = -\frac{f_{be}}{I_{1, be}} [\ln \zeta_{be} - \ln \zeta_{ee}] = -\frac{N_{be}}{Q_{1, be}} [\ln \zeta_{be} - \ln \zeta_{ee}],
\]

where \( I_{1, be} (Q_{1, be}) \) is the total current (charge) of the beam1 bunches participating in be crossings. Similarly, the specific background level for beam2 can be obtained as

\[
b_2 = \frac{B_{2, eb}}{I_{2, eb}} = -\frac{f_{eb}}{I_{2, eb}} [\ln \zeta_{eb} - \ln \zeta_{ee}] = -\frac{N_{eb}}{Q_{2, eb}} [\ln \zeta_{eb} - \ln \zeta_{ee}].
\]

The beam-background rate in bb crossings is then, for each beam \( i \),

\[
B_{i, bb} = I_{i, bb} b_i = f_{bb} \frac{Q_{i, bb}}{N_{bb}} b_i = \begin{cases} -f_{bb} r_i [\ln \zeta_{be} - \ln \zeta_{ee}] & \text{for } i = 1 \\ -f_{bb} r_i [\ln \zeta_{eb} - \ln \zeta_{ee}] & \text{for } i = 2 \end{cases},
\]

where

\[
r_1 = \frac{Q_{1, bb}/N_{bb}}{Q_{1, be}/N_{be}} = \frac{I_{1, bb}/f_{bb}}{I_{1, be}/f_{be}} \quad \text{and} \quad r_2 = \frac{Q_{2, bb}/N_{bb}}{Q_{2, eb}/N_{eb}} = \frac{I_{2, bb}/f_{bb}}{I_{2, eb}/f_{eb}}
\]

are the ratios of the average bunch charges \( (Q/N) \) in bb crossings with respect to be and eb crossings. Finally, the rate \( R \) proportional to the luminosity is given by the expression

\[
R = -f_{bb} [\ln \zeta_{be} - \ln \zeta_{ee}] - B_{1, bb} - B_{2, bb} = -f_{bb} \{ [\ln \zeta_{bb} - \ln \zeta_{ee}] - r_1 [\ln \zeta_{be} - \ln \zeta_{ee}] - r_2 [\ln \zeta_{eb} - \ln \zeta_{ee}] \},
\]

which would obviously reduce to Eq. 30 if \( r_1 = r_2 = 1 \). It may be convenient to re-write it as

\[
R = -f_{bb} \sum_{\alpha, \beta = be} \rho_{\alpha \beta} \ln \zeta_{\alpha \beta},
\]

where the coefficients \( \rho_{\alpha \beta} \) contain all the bunch-current dependencies:

\[
\rho_{bb} = 1, \quad \rho_{be} = -r_1, \quad \rho_{eb} = -r_2, \quad \rho_{ee} = r_1 + r_2 - 1.
\]
3.1.4 Effect of spill-over

The interaction trigger may also fire due to signals registered in the detector during neighbouring (previous or next) crossings. This effect, known as spill-over, will be negligible in the early phase of the LHC operation with only a small number of bunches, but may have to be taken into account in nominal 25 ns operation. A possibility is to apply the procedure described here several times in parallel, once for each set of four types of crossings (bb, be, eb and ee) sharing the same “neighbouring environment”; in such a scheme any spill-over effect will be automatically embedded in the \( p_0 \) probabilities as an additional source of noise.

3.2 Statistical precision

The emptiness probabilities \( \zeta_{\alpha\beta} \) will be estimated by counting how many times the interaction trigger does not fire out of a known number \( M_{\alpha\beta} \) of crossings of type \( \alpha\beta \). In practice it may be desirable to sample randomly the crossings of each type; this can be done by the readout supervisor using prescale factors \( S_{\alpha\beta} \). After prescaling, the numbers of crossings which should become available in the HLT during an elapsed time interval \( \Delta t \) will be \( f_{\alpha\beta}\Delta t/S_{\alpha\beta} \). However, the DAQ system may saturate and the readout supervisor may be forced to randomly kill a certain fraction of all L0 triggers, introducing dead time, i.e. time during which the experiment is blind to luminosity. Defining the time during which the DAQ system is alive as

\[
\Delta t^{\text{live}} = \varepsilon^{\text{live}} \Delta t,
\]

where \( \varepsilon^{\text{live}} \) is the DAQ efficiency; the numbers of crossings used to estimate \( \zeta_{\alpha\beta} \) are given by

\[
M_{\alpha\beta} = \frac{f_{\alpha\beta}\Delta t^{\text{live}}}{S_{\alpha\beta}},
\]

corresponding to a total average rate of used crossings of any type equal to

\[
f_{\text{used}} = \frac{1}{\Delta t} \sum_{\alpha\beta=bb,be,eb,ee} M_{\alpha\beta} = \varepsilon^{\text{live}} \sum_{\alpha\beta=bb,be,eb,ee} \frac{f_{\alpha\beta}}{S_{\alpha\beta}}.
\]

Using binomial statistics, the variance of \( \zeta_{\alpha\beta} \) is

\[
\sigma^2_{\zeta_{\alpha\beta}} = \frac{\zeta_{\alpha\beta}(1 - \zeta_{\alpha\beta})}{M_{\alpha\beta}},
\]

and after propagating all four independent statistical errors on Eq. 39 we get

\[
\sigma^2_R = f_{bb}^2 \sum_{\alpha\beta=bb,be,eb,ee} \frac{\rho^2_{\alpha\beta}}{M_{\alpha\beta}} \left( \frac{1 - \zeta_{\alpha\beta}}{\zeta_{\alpha\beta}} \right),
\]

Here we have assumed that the coefficients \( \rho_{\alpha\beta} \) are known without error. In practice, beam currents will not be known with infinite precision, leading to some systematic error to be added to the above statistical error. Combining this with Eq. 9, we get the relative statistical error on \( R \) as

\[
\frac{\sigma_R}{R} = \frac{1}{m} \sqrt{\sum_{\alpha\beta=bb,be,eb,ee} \frac{f_{\alpha\beta}^2}{M_{\alpha\beta}} \left( \frac{1}{\zeta_{\alpha\beta}} - 1 \right)} = \frac{1}{m} \sqrt{\sum_{\alpha\beta=bb,be,eb,ee} \frac{S_{\alpha\beta}f_{\alpha\beta}^2}{f_{\alpha\beta}\Delta t^{\text{live}}} \left( \frac{1}{\zeta_{\alpha\beta}} - 1 \right)},
\]

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where, following Eq. 39, \( m \) is estimated as

\[
m = - \sum_{\alpha, \beta = \text{be}} \rho_{\alpha\beta} \ln \zeta_{\alpha\beta}.
\] (47)

If one of the \( \zeta_{\alpha\beta} \) values is close to 0, both the central value of \( R \) given in Eq. 39 and its error given in Eq. 45 will blow up. Should this be the case for \( \zeta_{\text{ee}} \), \( \zeta_{\text{eb}} \) or \( \zeta_{\text{ee}} \), then the experiment probably wouldn’t be able to run because of unbearable noise or machine-induced background conditions, or the interaction trigger threshold is too low and needs to be raised. Also, under reasonable background conditions, this will never be the case for \( \zeta_{\text{bb}} \), because \( \rho_{\text{bb}} \) is never expected to reach close to 1. Values of \( \rho_{\text{bb}} \) larger than 0.9 with the nominal bunch structure would indeed only be reached at luminosities in excess of \( 10^{33} \, \text{cm}^{-2}\text{s}^{-1} \). In practice, time intervals where one of the \( \zeta_{\alpha\beta} \) is measured to be too close to 0 should probably be discarded in any analysis dealing with luminosity information based on the method proposed here.

The variance of the relative integrated luminosity \( R_{\text{int}} \) given in Eq. 2, for an entire dataset collected with constant crossing frequencies of each type and constant prescale factors, reads\(^2\)

\[
\sigma_{R_{\text{int}}}^2 = \sum_i (\Delta t_{i}^{\text{live}})^2 \sigma_{R_i}^2 = f_{\text{bb}} \sum_{\alpha, \beta = \text{be}} \frac{S_{\alpha\beta}}{f_{\alpha\beta}} \sum_i \Delta t_{i}^{\text{live}} (\rho_{\alpha\beta})_i^2 \left( \frac{1}{(\zeta_{\alpha\beta})_i} - 1 \right),
\] (48)

and the corresponding relative error is

\[
\frac{\sigma_{R_{\text{int}}}}{R_{\text{int}}} \approx \frac{1}{\sum_i \Delta t_{i}^{\text{live}} m_i} \sqrt{\sum_{\alpha, \beta = \text{be}} \frac{S_{\alpha\beta}}{f_{\alpha\beta}} \sum_i \Delta t_{i}^{\text{live}} (\rho_{\alpha\beta})_i^2 \left( \frac{1}{(\zeta_{\alpha\beta})_i} - 1 \right)}.
\] (49)

In scenarios with an approximately constant instantaneous luminosity and approximately constant bunch currents, the probability estimates \( (\rho_{\alpha\beta})_i \) are close to their average values \( \bar{\rho}_{\alpha\beta} \), the coefficients \( (\rho_{\alpha\beta})_i \) are close to their average values \( \bar{\rho}_{\alpha\beta} \), and the mean pileup values \( m_i \) are close to

\[
\bar{m} = - \sum_{\alpha, \beta = \text{be}} \bar{\rho}_{\alpha\beta} \ln \bar{\zeta}_{\alpha\beta}.
\] (50)

Then, defining the total numbers of used crossings \( M_{\alpha\beta}^{\text{tot}} = (f_{\alpha\beta}/S_{\alpha\beta}) \Delta T_{\text{live}} \) during the entire live time \( \Delta T_{\text{live}} = \sum_i \Delta t_{i}^{\text{live}} \), we get

\[
\frac{\sigma_{R_{\text{int}}}}{R_{\text{int}}} \sim \frac{1}{\bar{m}} \sqrt{\sum_{\alpha, \beta = \text{be}} \frac{\bar{\rho}_{\alpha\beta}^2}{M_{\alpha\beta}^{\text{tot}}} \left( \frac{1}{\bar{\zeta}_{\alpha\beta}} - 1 \right)}.
\] (51)

For the sake of illustration, Table 2 gives three different steady-state scenarios. In these scenarios, the assumed bunch crossing schemes are as outlined in [7,8] with uniform bunch currents \( r_1 = r_2 = 1 \Rightarrow \rho_{\alpha\beta}^2 = 1 \), and the assumed background probabilities \( p_0 = 10^{-3} \).

---

\(^2\)Because we only aim, for now, at rough estimates of the errors, Eq. 48 is derived under the simplifying assumption that the relative statistical error on \( \Delta t_{\text{live}} = \sum_{\alpha, \beta} M_{\alpha\beta}/\sum_{\alpha, \beta} (f_{\alpha\beta}/S_{\alpha\beta}) \) (see Eq. 52), equal to \( \sigma_{\Delta t_{\text{live}}} / \Delta t_{\text{live}} = 1 / \sqrt{\sum_{\alpha, \beta} M_{\alpha\beta}} \), can be neglected with respect to that on \( R \) given in Eq. 46. This assumption may be rather bold (in the examples of Table 2, \( \sigma(\Delta t_{\text{live}}) / \Delta t_{\text{live}} \) is found to be between 25% and 50% of \( \sigma_R / R \)) and will have to be dropped whenever precise estimates are needed.
<table>
<thead>
<tr>
<th>Assumptions (= inputs):</th>
<th>Startup scenario 1</th>
<th>Startup scenario 2</th>
<th>Nominal year</th>
</tr>
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<tbody>
<tr>
<td>instantaneous luminosity $L$</td>
<td>$4 \times 10^{30}$ cm$^{-2}$s$^{-1}$</td>
<td>$1.5 \times 10^{31}$ cm$^{-2}$s$^{-1}$</td>
<td>$2 \times 10^{32}$ cm$^{-2}$s$^{-1}$</td>
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<tr>
<td>integrated luminosity $L_{int}$</td>
<td>1 pb$^{-1}$</td>
<td>4 pb$^{-1}$</td>
<td>2 fb$^{-1}$</td>
</tr>
<tr>
<td>bunches/beam $N_{bunches}$</td>
<td>43</td>
<td>156</td>
<td>2808</td>
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<tr>
<td>bb crossings/turn $N_{bb}$</td>
<td>19</td>
<td>68</td>
<td>2622</td>
</tr>
<tr>
<td>visible cross section $\sigma_{vis}$</td>
<td>60 mb</td>
<td>60 mb</td>
<td>60 mb</td>
</tr>
<tr>
<td>noise probability $p_0$</td>
<td>$10^{-3}$</td>
<td>$10^{-3}$</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>beam1 background prob. $p_1$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>beam2 background prob. $p_2$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>measurement cycle $\Delta t$</td>
<td>5 s ; 5 min</td>
<td>5 s ; 5 min</td>
<td>10 s ; 10 min</td>
</tr>
<tr>
<td>bb prescale $S_{bb}$</td>
<td>900</td>
<td>3'000</td>
<td>90'000</td>
</tr>
<tr>
<td>be prescale $S_{be}$</td>
<td>5'000</td>
<td>20'000</td>
<td>20'000</td>
</tr>
<tr>
<td>eb prescale $S_{eb}$</td>
<td>8'000</td>
<td>30'000</td>
<td>30'000</td>
</tr>
<tr>
<td>ee prescale $S_{ee}$</td>
<td>8'000'000</td>
<td>9'000'000</td>
<td>50'000</td>
</tr>
<tr>
<td>DAQ efficiency $\xi$</td>
<td>95%</td>
<td>95%</td>
<td>95%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Derived quantities:</th>
<th>$\Delta T_{live}$</th>
<th>2.89 days</th>
<th>3.09 days</th>
<th>$10^7$ s = 115.74 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>total live time</td>
<td></td>
<td>2.89 days 3.09 days 10$^7$ s = 115.74 days</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of cycles</td>
<td>$N_{cycles}$</td>
<td>52'632 ; 877</td>
<td>56'140 ; 936</td>
<td>1'052'632 ; 17'544</td>
</tr>
<tr>
<td>be crossings/turn</td>
<td>$N_{be}$</td>
<td>24</td>
<td>88</td>
<td>186</td>
</tr>
<tr>
<td>ee crossings/turn</td>
<td>$N_{ee}$</td>
<td>3'497</td>
<td>3320</td>
<td>570</td>
</tr>
<tr>
<td>bb frequency</td>
<td>$f_{bb}$</td>
<td>0.21 MHz</td>
<td>0.76 MHz</td>
<td>29.49 MHz</td>
</tr>
<tr>
<td>be frequency</td>
<td>$f_{be}=f_{eb}$</td>
<td>0.27 MHz</td>
<td>0.99 MHz</td>
<td>2.09 MHz</td>
</tr>
<tr>
<td>ee frequency</td>
<td>$f_{ee}$</td>
<td>39.33 MHz</td>
<td>37.34 MHz</td>
<td>6.41 MHz</td>
</tr>
<tr>
<td>average pileup</td>
<td>$m$</td>
<td>1.123</td>
<td>1.177</td>
<td>0.407</td>
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<tr>
<td>beam1-beam2 probab.</td>
<td>$p_{bb}$</td>
<td>0.675</td>
<td>0.692</td>
<td>0.334</td>
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<tr>
<td>be emptiness probability $\xi_{bb}$</td>
<td>0.278</td>
<td>0.263</td>
<td>0.569</td>
<td></td>
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<tr>
<td>be emptiness probability $\xi_{be}$</td>
<td>0.899</td>
<td>0.899</td>
<td>0.899</td>
<td></td>
</tr>
<tr>
<td>eb emptiness probability $\xi_{eb}$</td>
<td>0.949</td>
<td>0.949</td>
<td>0.949</td>
<td></td>
</tr>
<tr>
<td>ee emptiness probability $\xi_{ee}$</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>bb crossings used/cycle $M_{bb}$</td>
<td>1'128 ; 67'662</td>
<td>1'211 ; 72'648</td>
<td>3'113 ; 186'748</td>
<td></td>
</tr>
<tr>
<td>be crossings used/cycle $M_{be}$</td>
<td>256 ; 15'384</td>
<td>235 ; 14'102</td>
<td>994 ; 59'614</td>
<td></td>
</tr>
<tr>
<td>eb crossings used/cycle $M_{eb}$</td>
<td>160 ; 9'615</td>
<td>157 ; 9'402</td>
<td>994 ; 59'614</td>
<td></td>
</tr>
<tr>
<td>ee crossings used/cycle $M_{ee}$</td>
<td>23 ; 1'401</td>
<td>20 ; 1'182</td>
<td>122 ; 7'308</td>
<td></td>
</tr>
<tr>
<td>used crossing rate</td>
<td>$f_{used}$</td>
<td>314 Hz</td>
<td>324 Hz</td>
<td>522 Hz</td>
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<table>
<thead>
<tr>
<th>Relative luminosity estimate:</th>
<th>$\frac{R - R_0}{R_0}$</th>
<th>$0.24$ MHz</th>
<th>$0.9$ MHz</th>
<th>12 MHz</th>
</tr>
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<tbody>
<tr>
<td>rate to be monitored</td>
<td>$\sigma_R/R$</td>
<td>5.0% ; 0.64%</td>
<td>4.8% ; 0.62%</td>
<td>5.0% ; 0.65%</td>
</tr>
<tr>
<td>$\sigma_{R_{int}}/R_{int}$</td>
<td>$2.2 \times 10^{-4}$</td>
<td>$5.0 \times 10^{-4}$</td>
<td>0.11%</td>
<td></td>
</tr>
<tr>
<td>unknown humi. unit $U$</td>
<td>16.7 $b^{-1}$</td>
<td>16.7 $b^{-1}$</td>
<td>16.7 $b^{-1}$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{bb}/b_1$</td>
<td>21% ; 2.7%</td>
<td>22% ; 2.7%</td>
<td>10% ; 1.3%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{be}/b_2$</td>
<td>38% ; 4.9%</td>
<td>39% ; 5.0%</td>
<td>15% ; 2.0%</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{ee}/\xi_{live}$</td>
<td>25% ; 0.3%</td>
<td>25% ; 0.3%</td>
<td>14% ; 0.2%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Optimization (see Sec. 3.3):</th>
<th>$(\sigma_R/R)\sqrt{f_{used}AT}$</th>
<th>1.9693</th>
<th>1.9305</th>
<th>3.6344</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1/m)\sum_{\alpha,\beta} \sqrt{1/\zeta_{\alpha,\beta} = 1}$</td>
<td>1.9682</td>
<td>1.9298</td>
<td>3.6107</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Rough illustrative examples corresponding to three different steady-state scenarios.
\( p_1 = 10\% \), \( p_2 = 5\% \) are rather arbitrary but hopefully pessimistic. As can be seen, a used crossing rate \( f_{\text{used}} \) of a few hundred Hz is needed to reach the requirement that a 5\% estimate of the relative luminosity \( R \) be produced every 5–10 seconds. This then leads to a precision of less than 1\% on the relative integrated luminosity after 5–10 minutes, which amply fulfills the physics requirements.

Besides the rate \( R \), other quantities will be interesting to monitor during data-taking. These include the specific beam-induced background levels \( b_1 \) and \( b_2 \) of Eqs. 34 and 35, as well as the DAQ efficiency

\[
\varepsilon_{\text{live}} = \frac{f_{\text{used}}}{\sum_{a,\beta=b\varepsilon} f_{a\beta}/S_{a\beta}} = \frac{1}{\Delta t} \frac{\sum_{a,\beta=b\varepsilon} M_{a\beta}}{\Delta t \sum_{a,\beta=b\varepsilon} f_{a\beta}/S_{a\beta}}.
\]

The statistical errors on these quantities are

\[
\sigma_{\varepsilon_{\text{live}}} = \frac{\varepsilon_{\text{live}}}{\sqrt{f_{\text{used}}/\Delta t}},
\]

and

\[
\sigma_{b_1} = \frac{N_{b\varepsilon}}{Q_{1b\varepsilon}} \sqrt{\sum_{a=b\varepsilon} \frac{1}{M_{a\varepsilon}} \left( \frac{1}{\zeta_{a\varepsilon}} - 1 \right)}, \quad \sigma_{b_2} = \frac{N_{b\varepsilon}}{Q_{2b\varepsilon}} \sqrt{\sum_{\beta=b\varepsilon} \frac{1}{M_{\varepsilon\beta}} \left( \frac{1}{\zeta_{b\beta}} - 1 \right)}.
\]

According to the (arbitrary) scenarios of Table 2, a few percent relative statistical uncertainty can be reached in 5–10 minutes for the beam-induced background levels and in 5–10 seconds for the DAQ efficiency.

### 3.3 Optimization of the prescale factors

As can be seen from Eqs. 43 and 46, both the rate of used crossings and the relative error on \( R \) depend on the prescale factors \( S_{a\beta} \). These can be set independently to any arbitrary value. In practice, one would like to get the best error while using the least resources in the trigger: this means minimizing both \( f_{\text{used}} \) and \( \sigma_R/R \). However these two requirements are in conflict, and one must decide either which precision is desired or which rate is desired. So two different questions can be asked:

- What are the values of the four prescale factors \( S_{a\beta} \) which will minimize the rate \( f_{\text{used}} \) under the condition that the relative error on \( R \) should be equal to a given precision \( \sigma_R/R \) after an elapsed time \( \Delta t \)?
- What are the values of the four prescale factors \( S_{a\beta} \) which will minimize \( \sigma_R/R \) obtained after an elapsed time \( \Delta t \) under the condition that the rate should be equal to a given value \( f_{\text{used}} \)?

The answers, which can be obtained using the Lagrange-multiplier method, are respectively

\[
S_{a\beta} = \frac{f_{a\beta} (\sigma_R/R)^2 m^2 \Delta t}{z_{a\beta}} \left( \sum_{a',\beta'} z_{a'\beta'} \right) \Rightarrow \text{minimized } f_{\text{used}} = \frac{1}{(\sigma_R/R)^2 m^2 \Delta t \left( \sum_{a,\beta=b\varepsilon} z_{a\beta} \right)^2}, \quad (55)
\]

\[
S_{a\beta} = \frac{f_{a\beta} \sum_{a,\beta=b\varepsilon} z_{a\beta} \varepsilon_{\text{live}}}{z_{a\beta} f_{\text{used}}/\varepsilon_{\text{live}}} \Rightarrow \text{minimized } \frac{\sigma_R}{R} = \frac{1}{m\sqrt{f_{\text{used}}/\Delta t} \sum_{a,\beta=b\varepsilon} z_{a\beta}}, \quad (56)
\]
where

\[
Z_{\alpha\beta} = |\rho_{\alpha\beta}| \leq \frac{1}{\sigma_{\alpha\beta}} - 1. \tag{57}
\]

It can easily be seen that these two optimizations are strictly equivalent. The common solution is also the one that minimizes the elapsed time \(\Delta t\) needed to reach a given precision \(\sigma_R/R\) with a given rate \(\dot{f}_{\text{used}}\). At the optimal operating point, the product \((\sigma_R/R)\sqrt{\text{used} \Delta t}\) reaches its minimum value equal to

\[
\left(\frac{\sigma_R}{R}\right) \sqrt{\text{used} \Delta t} = \frac{1}{m} \sum_{\alpha\beta=\text{b,e}} z_{\alpha\beta}. \tag{58}
\]

In the case of a steady-state scenario, a similar formula should hold approximately for the relative error on the integrated luminosity \(R_{\text{int}}\) over an elapsed time interval \(\Delta T = \sum_i \Delta t_i\),

\[
\left(\frac{\sigma_{R_{\text{int}}}}{R_{\text{int}}}\right) \sqrt{\text{used} \Delta T} \approx \frac{1}{m} \sum_{\alpha\beta=\text{b,e}} \bar{z}_{\alpha\beta}, \tag{59}
\]

where

\[
\bar{z}_{\alpha\beta} = |\bar{\rho}_{\alpha\beta}| \leq \frac{1}{\sigma_{\alpha\beta}} - 1. \tag{60}
\]

In the examples given in Table 2, the prescale factors have been chosen close to their optimal values, i.e., the product \((\sigma_R/R)\sqrt{\text{used} \Delta t}\) is fairly close to its minimum already, as can be seen in the last two lines of the table.

### 3.4 Considerations on online implementation

Figure 1 shows the largest \(E_T^{\text{hadron}}\) L0-cluster, the L0-SPD multiplicity, and the number of forward and backward RZ-tracks in the VELO for a few selected Pythia physics processes [9]. These distributions correspond to events with a single \(pp\) interaction, and no interaction in the previous crossing. Pythia process 91 does not leave any detectable particles inside the acceptance of the LHCb spectrometer, and hence represent the empty events. The distinction between empty events and events of other process types is clear, while the VELO forward/backward rates show a difference in forward and backward diffractive events as expected.

Figure 2 shows the same distributions, but now the previous crossing does have one or more \(pp\) interactions, which are generated corresponding to a luminosity of \(2 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}\). All sub-detectors do show a sensitivity to spill-over, which will only be relevant while running with 25 ns bunch separation. As mentioned in Section 3.1.4 one could extend the number of distributions by sub-dividing them according to the presence of beam in the previous crossing, or make use of the L0DU Sum\((E_T)\) variable of neighboring crossings.

In the HLT the above distributions, as well as the distributions of the bunch charges, will be accumulated throughout a run for random crossings provided by the Readout Supervisor (ODIN) based exclusively on the knowledge of the bunch structure of the colliding beams, and allow the extraction of \(R, b_1, b_2, \sigma_{\text{live}}\) for monitoring the relative luminosity, the specific beam-related background levels, and the dead time of the experiment respectively. For monitoring a snapshot is made of the histograms as they are accumulated on all nodes of the Event Filter Farm (EFF). These snapshots come in two types:

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Figure 1: Largest $E_t^{\text{hadron}}$ LO-cluster, LO SPD multiplicity, and number of forward and backward RZ-tracks in the Velo for Pythia physics processes 91 (full line, elastic scattering), 92 (dashed line, single diffractive (AB→XB)), 93 (dotted line, single diffractive (AB→AX)), and 68 (dash-dotted line, $gg \rightarrow gg$). Single interactions per crossing, no interaction in the previous crossing.
Figure 2: Largest $E_T^{\text{hadron}}$ L0-cluster, L0-SPD multiplicity, and number of forward and backward RZ-tracks in the VELO for Pythia physics processes 91 (full line, elastic scattering), 92 (dashed line, single diffractive ($AB\rightarrow XB$)), 93 (dotted line, single diffractive ($AB\rightarrow AX$)), and 68 (dash-dotted line, $gg \rightarrow gg$). Single interactions per crossing, and one or more $pp$ interactions in the previous crossing.
• Fast histogram collection: a limited set of histograms will be collected every few seconds to give a fast feedback to the shifters on trigger rates, and the quantities mentioned above.

• Slow histogram collection: all histograms filled on the EFF are saved every say 10–15 minutes, and also at the end of a run for future reference and to be analysed by dedicated analysis tasks. These histograms also contain the total number of events received and accepted by the HLT.

To allow to check that a set of collected histograms contains all EFF nodes, and that all belong to the same cycle, where a cycle refers to a given snapshot in time, the histograms are accompanied by their cycle number, and the number of cores contributing to a cycle is available after the summation of all histograms. The cycle number is derived from the wall-clock, which is synchronized between all nodes. The integrated number of events in a time interval between cycle snapshots is obtained by taking the difference between histograms collected in different cycles.

The saved histograms from the slow collection allow to recompute $R_{\text{in}}$ and its error for that cycle and for a given threshold (which can be changed offline, if needed). Rather than doing a counting experiment, more sophisticated analysis (fitting) will be possible. In addition, a few Hz of the random crossings can be accepted by the HLT, which allows to recompute these quantities after more sophisticated calibration all be it at a lower rate. Adding the cycle number to the event, allows a comparison with the number of events sent to storage per cycle by the EFF, and the number of events on for instance the rDSTs used for stripping per cycle.

4 Offline monitoring of the relative luminosity

The “offline monitoring” of the luminosity is essentially a book-keeping task. It consists of making sure that all events recorded by the experiment are being reconstructed, stripped and analyzed. However, even if all the steps are performed with the greatest care, we cannot exclude that some events will be missing in the final analysis. Losses can occur for various reasons, e.g. some part of the data may need to be declared unusable for a given analysis, some files may be corrupted or no longer available, or the central book-keeping database may have problems. These losses need to be identified, understood, and fixed wherever possible. The “irreducible” losses then need to be quantified and taken into account in the computation of the integrated luminosity of the sample effectively used in an analysis. One possibility would be to rely on the logical division of the data in cycles, as defined in Section 3.4, and keep track of what happens to each cycle in the offline environment. This section presents some first tentative ideas about how to do this. While no concrete implementation is proposed at this stage (and no study of the computing implications has been made), and while other solutions may exist, this section certainly has the virtue to spell out more concretely the requirements associated with the offline monitoring of the luminosity.

For each cycle, the DAQ system will save the necessary information to compute the corresponding relative integrated luminosity. After the data taking, a special “luminosity job” will read these data, compute the integrated luminosity and produce a “luminosity file” containing at least the following information about each cycle:

• the run number and the cycle number;
• the relative integrated luminosity $R_{\text{in}}^{\text{cycle}}$ of that cycle, and its statistical error $\sigma_{R_{\text{in}}^{\text{cycle}}}$;
- the total number of LHCb events $N_0^{\text{out}}$ produced by the DAQ during that cycle and sent for permanent storage in RAW data format.

All the luminosity files can then be merged in a single "master luminosity file" (or database), which will serve as a reference for offline luminosity monitoring. Whenever necessary, this reference file can be re-generated at any time from the saved histograms, for example in case a new (better) estimate of the luminosity has been designed.

The offline data processing organized centrally by the collaboration is logically divided in steps applied in sequence, in such a way that each step uses as input the output of the previous step. Such a sequence of steps could be the reconstruction with a certain version of Brunel, followed by the stripping with a certain version of Davinci, and finally a re-processing of the stripped events with a new version of Brunel. Of course, more than one sequence may be applied on the same original data, e.g., when it will be decided to do a re-processing from the RAW data with an improved version of the software and calibration constants.

For each sequence of steps, the master luminosity file (corresponding to the entire dataset to be processed) will be duplicated and the copy become the luminosity file associated with this sequence. Then for each step in the sequence, the jobs should count how many events they read on input and how many event they write on output for each individual cycle and store this as additional information in the luminosity file. At the end of the sequence, once all jobs have run, the luminosity file will contain at least the following information for each cycle:

- the run number and the cycle number;
- the relative integrated luminosity $R_{\text{int}}^{\text{cycle}}$ and its error $\sigma_{R_{\text{int}}^{\text{cycle}}}$;
- the total number of raw events $N_0^{\text{out}}$;
- the number of events $N_1^{\text{in}}$ processed as input of step 1 (reconstruction);
- the number of events $N_1^{\text{out}}$ output by step 1;
- the number of events $N_2^{\text{in}}$ processed as input of step 2 (stripping);
- the numbers of events $N_2^{\text{out,k}}$ output by step 2 for each stream $k$;
- etc.

It will certainly be convenient if the luminosity file also contains a "header" with all the relevant information about the sequence, such as the number of steps $n_{\text{steps}}$ (counting the DAQ production at the pit as a step, i.e., the index $j$ in $N_j^{\text{out}}$ takes the values $0, 1, 2, \ldots, n_{\text{steps}} - 1$), the Brunel or DaVinci version number used at each step, and the associated book-keeping configurations where the relevant datasets can be found. It is obvious that such a luminosity file would provide then an easy way to identify lost data, cycle by cycle.

Assuming 10-minute cycles, a nominal year of data taking will correspond to less than 20'000 cycles. A typical luminosity file for a normal sequence consisting of a reconstruction step and a stripping step producing 5 different streams would then contain a minimum of 11 integers and 2 floating point values per cycle, representing a total size of no more than 2 Mbytes.

Such luminosity file can then be used at the level of the user analysis. The user analysis jobs, which we consider here as an additional (and last) step in the processing, should count how many events $N_{n_{\text{steps}}}^{\text{in}}$ have been analyzed from each cycle and, based on the information found in the luminosity file, build the list of complete cycles and the list of incomplete cycles. A cycle is defined as complete for that analysis if

$$N_j^{\text{in}} = N_{j-1}^{\text{out}} \quad \text{for each step } j = 1, 2, \ldots, n_{\text{steps}}.$$  

(61)
The relative integrated luminosity of the analyzed sample is then equal to

\[ \frac{L_{\text{int}}}{U} = R_{\text{int}} = \sum_{i}^{\text{complete}} R_{\text{int},i}^{\text{cycle}} + \sum_{i}^{\text{incomplete}} (1 - \lambda_i) R_{\text{int},i}^{\text{cycle}}, \]  

(62)

where \( \lambda_i \) represents the unknown fraction of integrated luminosity lost in cycle \( i \) (0 ≤ \( \lambda_i \) < 1).

Assumptions can be made to estimate the fractions \( \lambda_i \), but this would introduce a systematic error which will need to be assessed and controlled. For analyses which depend crucially on a precise determination of the relative integrated luminosity (as well as for analyses aiming at the determination of the absolute luminosity scale \( U \)) it will be better to avoid such assumptions. In this case, events from incomplete cycles should be rejected from the final analysis. The integrated luminosity can then be computed from Eq. 62 where all \( \lambda_i \) have been set to 1.

A simple estimate of the luminosity losses in each incomplete cycle \( i \) is

\[ \lambda_i = 1 - \prod_j^{n_{\text{steps}}} \frac{N_j^{\text{in}}}{N_j^{\text{out}}}. \]  

(63)

This expression is valid if, for that cycle, the HLT output rate is proportional to the instantaneous luminosity, the probability to loose an event between any two consecutive steps is independent of the instantaneous luminosity, and the probability for a processed event to be selected by any step is independent of the instantaneous luminosity. In practice it may be difficult to quantify the validity of the assumptions on which Eq. 63 is based. In absence of quantitative understanding, a (perhaps conservative but hopefully reasonable) systematic uncertainty equal to \( \min(\lambda_i, 1 - \lambda_i) \) could be assigned to the estimate of \( \lambda_i \) given in Eq. 63.

Users will access centrally produced datasets (typically stripped samples of DST events) using Gaudi jobs. It would therefore make sense to foresee an algorithm, running first in the Gaudi sequence, which will count the events seen in each cycle and make the list of complete and incomplete cycles. At finalization, this algorithm could also print in the log file the value of \( L_{\text{int}}/U \) as well as its estimated uncertainty, with and without the inclusion of incomplete cycles. Additional checks should be performed in the user jobs, for example that no duplicate event (with same run and event numbers as another event) is present in the sample.

Another approach to the problem would be to write directly in the RAW data files, together with each event or groups of events, some luminosity records, which should then be copied (or merged) in any subsequent datasets, for example during stripping.\(^3\) This would then avoid the need to have to deal with separate luminosity files, and each dataset would automatically "know" its own integrated luminosity. However, external luminosity files would have the following two advantages: it should in principle be easy to regenerate or correct the luminosity information contained in these files, and it will be possible to quantify the luminosity losses in the offline environment.

\(^3\)The presentation of the ideas reported in this note [10] triggered several discussions in LHCb. A subsequent proposal emerged to include a "File Summary Record" [11] in each LHCb data file produced in the offline environment, in which all needed luminosity information for that particular file can be stored, and that could well play the role of what we have called the "luminosity files" in this document.
5 Summary

We have presented a proposal for the online and offline monitoring of the relative luminosity, using “minimum bias” interaction triggers running in the HLT farm on crossings randomly selected by the trigger supervisor based on the bunch structure of the LHC beams. The method is based on measuring the probability that no interaction takes place. We have shown that, with an HLT processing rate below 1 kHz, both the physics and online requirements on the precision of the relative luminosity estimates can be met. Measurements of the relative instantaneous luminosity, beam-induced background levels and DAQ dead time can be made available every few seconds to the shift crew for online monitoring. Once every few minutes, the integrated luminosity information can be saved in the form of histograms. From this information, a luminosity file could be constructed (independently of the files containing the RAW data events), which could then be used to propagate the luminosity information to the offline reconstruction, stripping and analysis framework, and provide the final relative integrated luminosity of each analyzed dataset. The proposed scheme can help tracking data lost in the offline processing and also provides a useful framework to cross-calibrate absolute luminosity measurements.

Further discussion and thinking is still needed to understand the best strategy for the treatment of luminosity information to be used in physics measurements, taking into account the concrete implementation aspects.

References


