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Measurement of EPR-type flavour entanglement in $\Upsilon(4S) \to B^0\overline{B}^0$ decays

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Abstract: The neutral B-meson pair produced at the $\Upsilon(4S)$ should exhibit a non-local correlation of the type discussed by Einstein, Podolski, and Rosen. The time-dependent flavour asymmetry of the B mesons decaying into flavour eigenstates is used to test such a correlation. The asymmetry obtained from semileptonic B^0 decays is in agreement with the prediction from quantum mechanics and far away from the predictions of the local realistic models tested.

The observation of partial decoherence in EPR systems could be a signal for New Physics: we have tested for such effects, and found our results are consistent with no decoherence.

Introduction

The concept of entangled states (i. e. states which cannot be represented as product states of their parts) was born in the '30s in the midst of several conceptual difficulties with Quantum Mechanics (QM). In 1935 Einstein, Podolski, and Rosen (EPR) wrote a paper which was an effort to clarify the conceptual basis of QM and arrived at the conclusion that QM could not be considered a "complete" theory [1]. EPR considered a pair of particles produced by the same interaction, subsequently freely propagating in space but still linked by momentum conservation. EPR found a contradiction when realism and locality are applied to the predictions of QM on a couple of non-commuting observables (position and momentum, in their paper). The conceptual problem is better understood considering the 1951 variant by David Bohm using spin correlations [2]. In the EPR-Bohm experiment the two-particle singlet state can be written as:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2] \quad (1)$$

where $|\uparrow\rangle_j$ ($|\downarrow\rangle_j$) describes the spin state of j^{th} particle (j=1,2) with spin up (down) respectively. Measurement of the spin on one particle, undetermined prior to the measurement, will "collapse" the wave function to one of the eigenstates and therefore predicts with certainty the outcome of the spin measurement on the second particle without actually doing any measurement. The important point is that the spin of the second particle in a given direction is defined by the choice of the polarizer orientation on the first particle. The orientation can be chosen at the "last moment", just prior to the arrival of the particle, and cannot be communicated to the second particle system unless superluminal signals are invoked. We should conclude that in a way or another the second particle carries the information needed to behave correctly for any possible choices of the measurement in the system of the first particle. Indeed, following EPR, one can define "elements of reality" for spin in S_x and S_y direction for the second particle, determined from the spin measurements done on the first particle. But according to QM the observables S_x and S_y do not commute and therefore cannot have definite values at the same time. Following the EPR-Bohm conclusion the description of reality given by QM is incomplete. This points to the need of extra information, "hidden variables" (HV) for instance, to complement QM. In 1964 J. S. Bell found a general scheme to test QM against HV theories: he showed that a certain inequality which is always satisfied by all local hidden variable models, can instead be violated by QM [3]. Following the demonstration by J. Clauser, M. Horne, A. Shimony, and R.Holt (CHSH), the correlation function for the measurements of the two particles with spin analyzers with orientation a and b is given by

$$E(a,b) = \int_\Gamma A(a,\lambda)B(b,\lambda)\rho(\lambda)\,d\lambda. \tag{2}$$

$A(a,\lambda)$, $B(b,\lambda)$ are the results of the measurements (+1 or −1, corresponding for instance to spin up and down in an experiment performed on spin 1/2 particles), and the HV are represented by the symbol λ. The HV follow a normalized probability distribution $\rho(\lambda)$. CHSH show that the inequality

$$|E(a,b) - E(a,c)| + |E(b',b) + E(b',c)| \leq 2 \tag{3}$$

is always satisfied by any local realistic theory featuring HV, while QM can violate it for some particular values of the analyzer orientations a, b, c, b'. The key point of the demonstration is the presence of products of the kind $B(b,\lambda)B(c,\lambda)$, representing the measurement of the same event by the same apparatus, but with different orientations. The additional information carried by λ is used to infer the result of such classical measurement, incompatible with QM when orthogonal observables are measured.

Several Bell-CHSH experiments have been performed by measuring the linear polarization of pairs of photons produced in a correlated state.

For this kind of experiment an optimized choice of angles brings to the following Bell-CHSH inequality:

$$|S(\phi)| = |3E(\phi) - E(3\phi)| \leq 2 \tag{4}$$

in which only two orientations of the analyzers need to be considered. In this case QM predicts $E(\phi) = cos(2\phi)$ for the two photons, giving a maximal value $S_{max} = 2\sqrt{2}$, when $\phi = \pi/8$. Note that in the case of a spin 1/2 system, we would have obtained $E(\phi) = cos(\phi)$, the maximum for S occurring at $\phi = \pi/4$.

In the experiment of A. Aspect, P. Grangier, and G. Roger [4] the photons with correlated linear polarization are produced from a ^{40}Ca source. The atom excited by a laser undergoes a cascade $(J = 1) \rightarrow (J = 0) \rightarrow (J = 1)$. A notable feature of this experiment is the usage of two two-channel polarizers allowing true dichotomic polarization measurements. On the other hand a normalization is introduced to account for the limited detector efficiency

$$E(\phi) = \frac{(R_{++} + R_{--}) - (R_{-+} + R_{-+})}{(R_{++} + R_{--}) + (R_{-+} + R_{-+})}(\phi) \tag{5}$$

where the $R_{\pm\pm}$ are the four coincidence rates for relative orientation ϕ of the two polarizers. For the optimal ϕ value, the results are in good agreement with QM, and violate LR by many standard deviations.

T. D. Lee and C. N. Yang suggested to use neutral kaon pairs as an entangled EPR system, the kaon flavour (the Strangeness S=1 or S=–1) playing the role of spin up and down in a 1/2 spin system. Strangeness-entangled pairs can be produced from the decay of a ϕ, for instance. The two experiments CPLear (at CERN) and KLOE (Frascati) have studied correlations in K^0-$\overline{K}^0$ pairs, and they obtained results in agreement with QM predictions [5, 17].

In this paper we present a study of EPR correlation in the flavour (the beauty +1 or –1) of neutral B-meson pairs from $\Upsilon(4S)$ decays. The system is described by a wavefunction analogous to (1) [6, 7]:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\left|B^0\right\rangle_1 \otimes \left|\overline{B}^0\right\rangle_2 - \left|\overline{B}^0\right\rangle_1 \otimes \left|B^0\right\rangle_2\right]. \tag{6}$$

An important difference with the photon experiment resides in the fact that the meson's flavour is known only when it decays and only if it decays into "flavour-specific" modes (like one of the semi-leptonic channels, in which the flavour can be inferred from the electric charge of the lepton). From 6 we deduce that decays occurring at the same proper time are fully correlated: the flavour-specific decay of one meson fixes the (previously undetermined) flavour (B^0 or $\overline{B}^0$) of the other meson. From 6 we can also deduce the time-dependent rate for decay into two flavour-specific states for opposite flavour (OF, $B^0\overline{B}^0$) and same flavour (SF, B^0B^0 or $\overline{B}^0\overline{B}^0$) decays, and the corresponding time-dependent asymmetry (see Fig 1, curve QM):

$$R_{\substack{OF \\ SF}}(\Delta t) = e^{-\Delta t/\tau_{B^0}}/(4\tau_{B^0})\{1 \pm \cos(\Delta m_d \Delta t)\}, \tag{7}$$

$$A_{\text{QM}}(\Delta t) \equiv \frac{R_{\text{OF}} - R_{\text{SF}}}{R_{\text{OF}} + R_{\text{SF}}}(\Delta t) = \cos(\Delta m_d \Delta t) \tag{8}$$

$\Delta t \equiv |t_1 - t_2|$ is the proper-time difference of the decays, and Δm_d the mass difference between the two B^0-$\overline{B}^0$ mass eigenstates. (We have assumed a lifetime difference $\Delta\Gamma_d = 0$ and neglected the $O(10^{-4})$ effects of CP violation in mixing.) The fact that the asymmetry depends only on Δt, and not on the absolute decay (measurement) times, t_1 and/or t_2, is a manifestation of EPR-type entanglement at a distance. It must be noticed that experimentally it is very difficult to measure the absolute times t_1 and t_2, hence only Δt is available.

Recently the question arose about the possibility to perform a Bell test with neutral K or B mesons. Several authors [7, 8] have suggested that the oscillation of a neutral mesons meson during a time Δt plays a role analogous to the choice of the angular orientation of a spin analyzer, while in a passive mode. If this is correct, then the time dependent asymmetry $A(\Delta t)$ (Eq. 8) can be thought as a B-meson version of the correlation $E(\phi)$, Eq. 5, with $\Delta m_d \Delta t \equiv \phi$. Like in the Aspect *et al.* experiment, the denominator of the expression for $A(\Delta t)$ accounts for the inefficiencies of the apparatus and, in addition, for the fast rate reduction due to the short B^0 lifetime. Following Ref. [7], the Bell-CHSH test can be performed considering an expression similar to Eq. 4. In this spin 1/2 system the maximal violation should appear for $\phi = \pi/4$ ($\Delta t \approx 2.6$ ps). The results presented in Ref. [9] are in agreement with QM and 3σ above the LR limit of 2. This approach was criticized by [10, 11]. These authors consider the Bell tests unaccessible due to the rapid decrease in time of the B-meson amplitudes, and because of the passive character of the flavour measurement.

Here we propose to demonstrate the existence of an intrinsic problem by exhibiting a model HV based capable of violating the proposed inequality. This is found to be the case for the Spontaneous and immediate Disentanglement model (SD), in which the B-meson pair separates into a B^0 and $\overline{B}^0$

with well-defined flavour immediately after the $\Upsilon(4S)$ decay, which then evolve independently [12].

In the SD model, the time-dependent asymmetry is

$$\begin{aligned} A_{\rm SD}(t_1, t_2) &= \cos(\Delta m_d t_1)\cos(\Delta m_d t_2) \\ &= \frac{1}{2}[\cos(\Delta m_d(t_1+t_2)) + \cos(\Delta m_d \Delta t)]. \end{aligned} \quad (9)$$

Note the additional $t_1 + t_2$ dependence, which can be removed by integrating the OF and SF functions for fixed values of Δt. The result is represented on Fig. 1 by the curve SD. From this result we can compute $|S|$ as function of Δt. It is found that this HV model violates LR limit of two, for $\Delta t \approx 1.3$ ps and $\Delta t \approx 4.2$. This proves that we are not in the presence of a genuine Bell-CHSH test and we are brought to abandon this approach.

To probe the non-local behaviour of the B^0 pair we can pragmatically limit ourselves to verify that, first, QM reproduces the experimental asymmetry, and, second, this is not the case for any other "reasonable" HV-based model. Within the definition of "reasonable" we include the capability to reproduce the B^0-$\overline{B}^0$ oscillation behaviour for each boson taken individually, after the $\Upsilon(4S)$ decay. In conclusion, we have chosen to compare our results with the predictions of QM and two other models. We stress the fact that to keep open the possibility of testing more models we also provide a fully corrected experimental time-dependent asymmetry, i. e. the background is subtracted and the detector effects corrected by a deconvolution method.

Our first candidate is the SD model seen before. The second is the local realistic model by Pompili and Selleri (PS) [13]. In PS, each B transports flavour information (B^0 or $\overline{B}^0$), and mass (corresponding to the heavy and light B_H, B_L eigenstates). There are thus four basic states: B^0_H, B^0_L, $\overline{B}^0_H$, $\overline{B}^0_L$. The model imposes mass and flavour anti-correlations at equal times $\Delta t = 0$; mass values are stable, but the system is programmed to allow random simultaneous jumps in flavour within the pair. The model is also required to reproduce the QM predictions for uncorrelated B-decays. No other assumptions are made: the result is an upper and a lower bound for the asymmetry. For instance, the upper bound is

$$\begin{aligned} \mathrm{A}^{\rm max}_{\rm PS}(t_1, t_2) = 1 - |\{1 \quad &- \cos(\Delta m_d \Delta t)\}\cos(\Delta m_d t_{\rm min}) \\ &+ \sin(\Delta m_d \Delta t)\sin(\Delta m_d t_{\rm min})|. \end{aligned} \quad (10)$$

The additional $t_{\rm min} = \min(t_1, t_2)$ dependence is again removed by integrating the OF and SF functions for fixed values of Δt. A similar function is given for$\mathrm{A}^{\rm min}_{\rm PS}$. We obtain the curves $\mathrm{PS}_{\rm max}$ and $\mathrm{PS}_{\rm min}$ shown in Fig. 1.

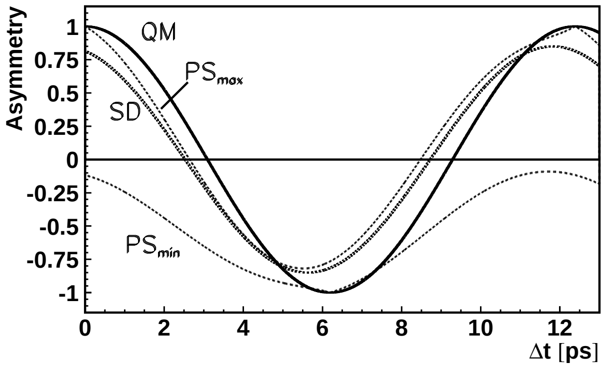

Figure 1: Time-dependent asymmetry predicted by (QM) quantum mechanics and (SD) spontaneous and immediate disentanglement of the B-pair, and (PS_{min} to PS_{max}) the range of asymmetries allowed by the Pompili and Selleri model. $\Delta m_d = 0.507\,\mathrm{ps}^{-1}$ is assumed.

Finally, assuming QM as the correct model, we can consider hypothetical effects which can disturb the propagation of the entangled wave function [14, 15], and affect the time-dependent asymmetry. For instance, decoherence can originate from the "interaction" with a foamy space-time. The time evolution of the system state represented by a density matrix $\rho(t)$ can be described by a modified Liouville equation

$$\frac{d\rho}{dt} = i[\rho, H] + D[\rho], \tag{11}$$

where $D[\rho]$ represents a dissipative term. In the most simple approach a single "decoherence parameter" $\lambda \leq 0$ is introduced, $D[\rho] \sim \lambda$. From Ref. [16] the order of λ cannot be larger than $E^2/m_P l$, with E the typical energy scale of the mass system, emphasizing the interest to consider systems with the largest possible energy. In this framework Eq. 7 becomes

$$R_{\substack{OF\\SF}} = \frac{e^{-\Delta t \tau_{B^0}}}{4\tau_{B^0}}(1 \pm (1 - e^{\lambda t_{min}}) \cos(\Delta m_d \Delta t)). \tag{12}$$

The effect on $A(\delta t)$ is a modulation of the amplitude. Not having access to t_{min}, a simplified parametrisation of the global effect has been adopted. Making the hypothesis of a partial disentanglement into mass or flavour eigenstates the asymmetry becomes, respectively

$$A = (1 - \zeta_{B_H B_L}) A_{\text{QM}}, \text{ and} \tag{13}$$

$$A = (1 - \zeta_{B^0\overline{B}^0}) A_{\text{QM}} + \zeta_{B^0\overline{B}^0} A_{\text{SD}} \tag{14}$$

(Eq. (13) corresponds to formula 3.5 in Ref. [17], for $\Delta\Gamma = 0$).

Data analysis

To determine the asymmetry we use 152×10^6 $B\overline{B}$ pairs collected by the Belle detector at the $\Upsilon(4S)$ resonance at the KEKB asymmetric-energy (3.5 GeV on 8.0 GeV) e^+e^- collider [18].

The Belle detector [19] is a large-solid-angle spectrometer consisting of a silicon vertex detector (SVD), central drift chamber (CDC), aerogel Cherenkov counters (ACC), time-of-flight counters (TOF), and a CsI(Tl) electromagnetic calorimeter (ECL) inside a 1.5T superconducting solenoid. The flux return is instrumented to detect K_L^0 and identify muons (KLM).

The $\Upsilon(4S)$ is produced with $\beta\gamma = 0.425$ close to the z axis. As the B momentum is low in the $\Upsilon(4S)$ center-of-mass system (CMS), Δt can be determined from the z-displacement of B-decay vertices: $\Delta t \approx \Delta z/\beta\gamma c$. The SVD provides Δz with a precision of about 100 μm.

The event selection for this study (see Ref. [20] for details) was optimized for theoretical model discrimination. To enable direct comparison of the result with different models, we subtract both background and mistagged-flavour events from the data, and then correct for detector effects by deconvolution. The flavour of one neutral B was obtained by reconstructing the decay $B^0 \to D^{*-}\ell^+\nu$, with $D^{*-} \to \overline{D}^0\pi_s^-$ and $\overline{D}^0 \to K^+\pi^-(\pi^0)$ or $K^+\pi^-\pi^+\pi^-$ (charge-conjugate modes are included throughout this paper). The D^0 candidates must have a reconstructed mass compatible with the known value. A D^* is formed by constraining a D^0 and a slow pion to a common vertex. We require a mass difference $M_{\text{diff}} = M_{Kn\pi\pi_s} - M_{Kn\pi} \in [144.4, 146.4]$ MeV$/c^2$ (Fig. 2, left), and CMS momentum $p^*_{D^*} < 2.6$ GeV$/c$, consistent with B-decay.

The CMS angle between the D^* and lepton needs to be greater than $90°$. From the relation $M_\nu^2 = (E_B^* - E_{D^*\ell}^*)^2 - |\vec{p}_B^*|^2 - |\vec{p}_{D^*\ell}^*|^2 + 2|\vec{p}_B^*||\vec{p}_{D^*\ell}^*|\cos(\theta_{B,D^*\ell})$, where $\theta_{B,D^*\ell}$ is the angle between $\vec{p}_B^*$ and $\vec{p}_{D^*\ell}^*$, we can reconstruct $\cos(\theta_{B,D^*\ell})$ by assuming a vanishing neutrino mass. We require $|\cos(\theta_{B,D^*\ell})| < 1.1$. The neutral B decay position is determined by fitting the lepton track and D^0 trajectory to a vertex, constrained to lie in the e^+e^- interaction region. The remaining tracks are used to determine

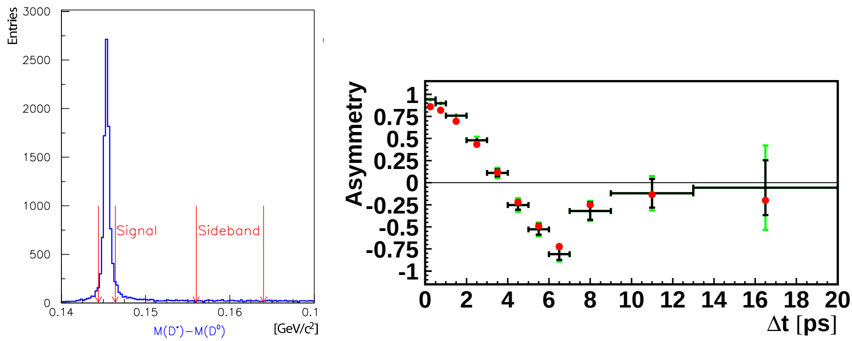

Figure 2: Left: M_{diff} distribution. Right: asymmetries before (red dots) and after (crosses) the corrections for the background and wrong flavour events. Statistical (black) and total errors (green) are superimposed.

the second B decay vertex and flavour(see Ref. [21]; in this analysis we use only the high purity leptonic tags).

In total 8565 events are selected (6718 OF, 1847 SF). To compensate for the rapid fall in event rate with Δt, the time-dependent distributions are histogrammed in 11 variable-size bins (see Table 1). The raw asymmetry is shown in Fig. 2, right (dots). Background subtraction is then performed bin-by-bin; systematic errors are likewise determined by estimating variations in the OF and SF distributions, and calculating the effect on the asymmetry.

A GEANT-based Monte Carlo (MC) sample was analysed with identical criteria, and used for consistency checks, background estimates and subtraction, and to build deconvolution matrices.

Four types of background events have been considered: $e^+e^- \to q\bar{q}$ continuum, fake D^*, wrong D^*–lepton combinations, and $B^+ \to \overline{D}^{**0}\ell\nu$ events. Off-resonance data (8.3 fb^{-1}) were used to estimate the continuum background, which was found to be negligible. Fake D^0 reconstruction and misassigned slow pions producing a fake D^* background were estimated from the sideband in $M_{\rm diff}$ (Fig. 2, left). The contamination from wrong D^*–lepton combinations was obtained by a reverse lepton momentum method, the validity of which was confirmed by MC studies. A fit of the $\cos(\theta_{B,D^*\ell})$ distribution allows the extraction of the D^{**-} component. The MC is then used to compute the fraction from charged B mesons which must be subtracted (as it has no mixing).

Table 1: Time-dependent asymmetry in Δt bins, corrected for experimental effects, with total uncertainties.

bin	window [ps]	A and total error	bin	window [ps]	A and total error
1	0.0 – 0.5	1.013 ± 0.028	7	5.0 – 6.0	-0.961 ± 0.077
2	0.5 – 1.0	0.916 ± 0.022	8	6.0 – 7.0	-0.974 ± 0.080
3	1.0 – 2.0	0.699 ± 0.038	9	7.0 – 9.0	-0.675 ± 0.109
4	2.0 – 3.0	0.339 ± 0.056	10	9.0 – 13.0	0.089 ± 0.193
5	3.0 – 4.0	-0.136 ± 0.075	11	13.0 – 20.0	0.243 ± 0.435
6	4.0 – 5.0	-0.634 ± 0.084			

After correction for wrong flavour assignments (an event fraction of 0.015 ± 0.005) using OF and SF distributions from wrongly-tagged MC events, we obtain the time-dependent asymmetry shown in Fig. 2, right (crosses).

Remaining experimental effects (e.g. resolution in Δt, selection efficiency) are corrected by a deconvolution procedure based on the singular value decomposition method described in Ref. [22]. 11×11 response matrices are built separately for SF and OF events, using MC $D^*\ell\nu$ events indexed by generated and reconstructed Δt values. The procedure has been optimised, and its associated systematic errors inferred by a toy Monte Carlo where sets of several hundred simulated experiments are generated with data and MC samples identical in size to those of the real experiment, but assuming the three theoretical models. We test the consistency of the method applied to our data by fitting the B^0 decay time distribution (summing OF and SF samples), leaving the B^0 lifetime as a free parameter. We obtain 1.532 ± 0.017(stat) ps, consistent with the world average [23]. We have also repeated the deconvolution procedure using a subset of events with better vertex fit quality, and hence more precise Δt values: consistent results are obtained. The final results are shown in Table 1 and Fig. 3.

Comparison with the theoretical models

The model testing is done by a least-square fit to $A(\Delta t)$, leaving Δm_d free, but taking the world-average Δm_d into account. To avoid bias, we discard BaBar and Belle measurements, which assume QM correlations: this yields [24] $\langle \Delta m_d \rangle = (0.496 \pm 0.014)\,\text{ps}^{-1}$.

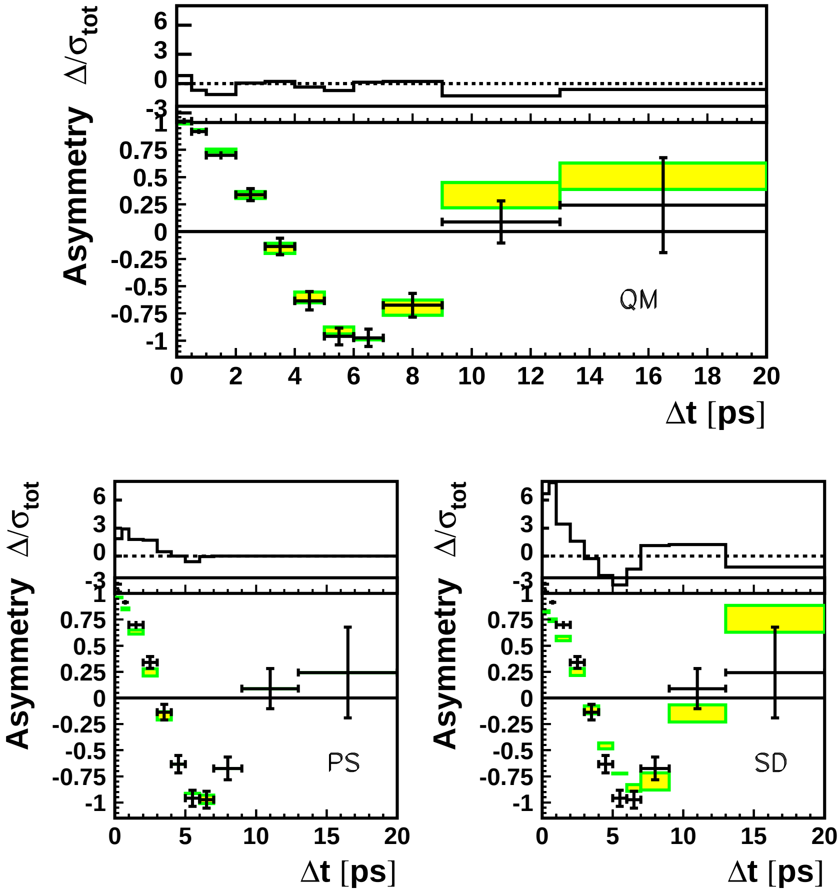

Figure 3: The bottom part of each plot represents the time-dependent flavour asymmetry (crosses) and the results of weighted least-squares fits to the model (the rectangles, showing $\pm 1\sigma$ errors on Δm_d). The top part show the differences $\Delta \equiv A_{\rm data} - A_{\rm model}$ in each bin, divided by the total experimental error $\sigma_{\rm tot}$. In the case of the PS model (bottom left) bins where $\mathrm{A}_{\rm PS}^{\rm min} < A_{\rm data} < \mathrm{A}_{\rm PS}^{\rm max}$ have been assigned a null deviation: see the text.

Our data is in agreement with the prediction of QM: we obtain $\Delta m_d = 0.501 \pm 0.009$ ps^{-1} with $\chi^2 = 5.2$ for 11 dof (see Fig. 3). SD is rejected by $\chi^2 = 174$ ($\Delta m_d = 0.419 \pm 0.008$). To fit PS we have used the closest boundary to our data A_{PS}^{max}, Eq. (11), or A_{PS}^{min}, but assumed a null deviation for data falling inside the boundaries. We obtain $\chi^2 = 31.3$ ($\Delta m_d = 0.447 \pm 0.010$ ps^{-1}): the data favour QM over PS at the 5.1σ level.

As noted above, *CP* violation in mixing can be neglected. Introducing a lifetime difference $\frac{\Delta\Gamma_d}{\Gamma_d} = 0.009 \pm 0.037$ [24] has a negligible effect on the fit.

Decoherence studies

We have examined the possibility of a partial loss of coherence just after the decay of the $\Upsilon(4S)$ resonance.

The fraction of events with disentangled B^0 and a $\overline{B}^0$ can be estimated by fitting our asymmetry with the mixture of Eq. (14), leaving $\zeta_{B^0\overline{B}^0}$ free. The fit finds $\zeta_{B^0\overline{B}^0} = 0.029 \pm 0.057$, consistent with no decoherence.

The second possibility considered is a partial decoherence into mass eigenstate, for which we expect a reduction in the amplitude of $A(\Delta t)$, as given by Eq. (13). The result of a fit gives a value of $\zeta_{B_H B_L} = 0.004 \pm 0.017$ (preliminary), also compatible with zero.

Conclusion

The neutral B-meson pair produced at the $\Upsilon(4S)$ should exhibit a non-local correlation of the EPR type. Ideally a Bell test should be performed to check that hidden variables are not active in the system. On the other hand we have seen that there is little hope to perform such a test. *Ultima ratio* we have decided to compare our data to QM and to two local realistic models: the model of Pompili and Selleri and a model with spontaneous and immediate disentanglement in which definite-flavour B^0 and $\overline{B}^0$ evolve independently.

We have measured with the Belle apparatus neutral B pairs from $\Upsilon(4S)$ decay. We have determined the time-dependent asymmetry due to flavour oscillations. The distribution has been corrected for experimental effects: the background has been subtracted and the resolution effects corrected by a deconvolution model, in such a way that the resulting distribution can be directly compared to theoretical models. We have found that QM reproduces our results well, while the two other models are strongly disfavoured.

The observation of partial decoherence in EPR systems could be a signal for New Physics. We have studied a possible disentanglement into mass or flavour eigenstates. We have found that our data is consistent with a null fraction of events with a loss of entanglement.

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