Masters Thesis in High Energy Physics

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Study for CP-violation in the $\psi' \to \pi^+\pi^- J/\psi$ transition

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Contents

1 Introduction

This master thesis studies a possible charge-parity violation in the $\psi' \rightarrow \pi^+\pi^- J/\psi$ transition, through the presence of an non-zero chromatic and electric dipole moment in the c-quark bound state. Since CP violating effects are not yet fully understood, any new search for CP violating source will is important. Yet, CP violating effects have only been observed in neutral mesons systems like for exemple, the $K^0 - \bar{K}^0$. In this latter, the CP violation appears through the CKM matrix, which causes quark generations mixing. The 14 millions ψ' events collected by the BESII experiment will be used in this experiment. Given the branching ratio, this gives $261 \cdot 10^3$ events of interest in the chosen channel where J/ψ decays into two muons.

The observed universe is mainly composed of matter, but it is known that in its first stage, matter and antimatter should have been in equilibrium for a short time. However, one need a process favoring matter over antimatter in some elementary processes. Such processes are called CP violating interactions. In the standard model theory of particles, CP violation appears with the CKM matrix which causes quark generations mixing. This quark generation mixing allows some mesons decays to violate the CP symmetry. However, those observed effects are way to small to sustain the observed asymmetry in nature. One must then suppose the existence of some other source of CP violation. A possibility is related to the Strong CP problem. In the standard model, electric dipole moment for baryons cannot be excluded. Such non-zero electric dipole moments for baryons will make some CP violation to appear in the decay of those particles. In the case of the neutrons, several very precise measurements have excluded a non-zero electric dipole moment. The theory is unable to explain such a cancelation without adding some ad-hoc symmetry arguments. Yet, electric dipole moment in other baryons have not been deeply investigated. This is mainly due to impossibility to make direct measurements on fast decaying particles. Still, it is possible to gain informations on the electric dipole moment through its effect on the decay by building a CP odd observable.

2 The BEPC and the BESII detector

In this section, the main informations about the BESII detector and the Beijing electron-positron collider are presented. A brief introduction on the BEPC facility is followed by a detailled presentation of the main parts of the BESII detector. For each of these parts, the upgrades of BESIII are presented with their new resolutions.

2.1 The BEPC facility

The Beijing electron-positron collider is a linear accelerator of 202 m long. This accelerator is connected with two storage rings of 237.5 m of circumference, one for electrons and one for positron. The BESII detector is placed on the interaction point. The beam energy is around 1.5 GeV, but can be pushed up to achieve an center of mass energy high enough to produce τ lepton pairs. This acclerator is mainly dedicaced to the production of J/ψ and ψ' mesons and also of τ leptons. The luminosity at a beam energy of 1.45 GeV is of $5 \cdot 10^{30}$ cm^{-2}s^{-1}.

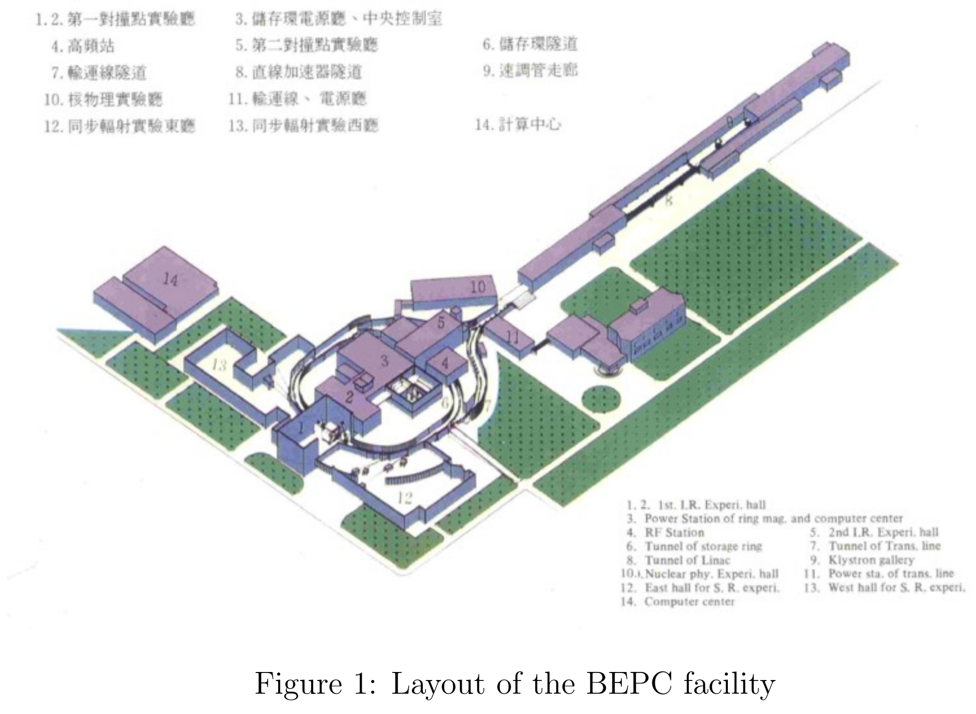

Figure 1: Layout of the BEPC facility

2.2 The BESII detector

The BESII spectrometer is composed of five detectors, arranged in an onion configuration. The total size is 6 meters long for 7 meters width and height. Starting from the beam, those detectors are the following :

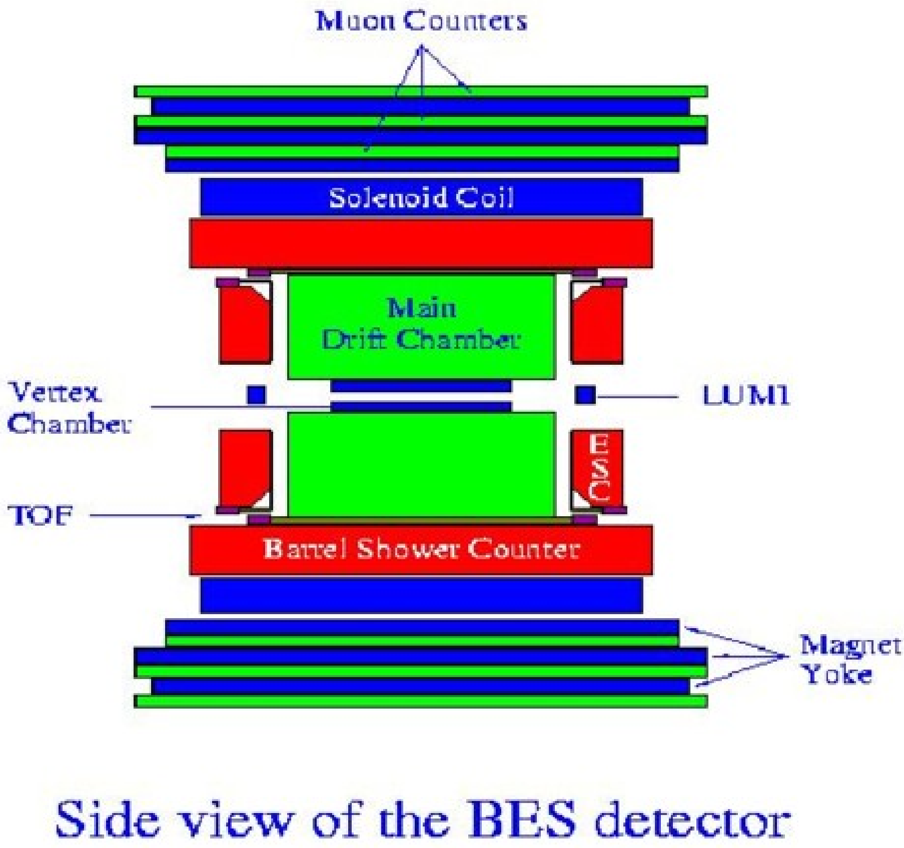

Figure 2: Sketch of the BESII detector

2.2.1 Vertex chamber

This device has been obtain from the MarkIII detector at SLAC. It is build arroud the 9.8 cm beryllium beampipe. It is composed twelve layers of 8 mm in diameter straws tubes, assembled in three groups of four layers. The first and third layers group are axial, while the second one is making a 3 ° angle with others. This configuration allows to know the z coordinates of the tracks without ambiguity. The vertex chamber has a total of 640 straw tubes made of mylar. The inner wires are made of tungsten plated with gold and are $50\,\mu$m in diameter. The total radius of this device is 13.5 cm. It works under a presure of 3 atm with a gas mix of 50% ethan 50% argon. The design voltage is 3.7 kV.

This vertex chamber uses gas ionization eectrons produced by particles crossing the tube. The strong electric potential near the anode wire allow the electrons to produce eletronic showers by secondary gas ionization. The potential is set to work in proportional regime. This allow to collect an amplified signal proportional to the primary gas ionization. One can then reconstruct the trajectory of the particle using the pulses and shapes of the collected signals.

This detector is measuring the trajectories of charged particles with a very high precision (resolution of $90\,\mu$m).

2.2.2 Main drift chamber

It is the main tracking system that is able to measure the trajectory, the energy loss and the momentum of particles. The BESII main drift chamber contains 804 drift cells and 3216 sensing wires. They are grouped in 10 super layers which have 4 axial or stereo signal wires. The detector has a radius of 1.1 m

In order to reconstruct trajectories, the drift chamber uses a principle analogous to the one used in the vertex detector. The main difference is that here the ionized molecules are collected to provide the informations about the trajectory. These ionized molecules are drifting at their limit velocity to the sense wires. Using the knowledge of the time required by the drifting molecules to reach the sense wire, one can compute precisely the point of the initial ionization. Using several measurements, the trajectory of the particle can be fully reconstructed. The magnetic field in the chamber will bend the trajectory of the particle depending on its charge, thus its curve radius provide a mesurment of the particle momentum. The dE/dX of the particle can also be mesured, as it is proportional to the primary ionization.

Signal wires also provide θ and ϕ coordinates, and position ambiguity on the position is avoided by using the field wires as gate and other signal wires as shields. The z position is obtained a stero configuration of the wire layers.

The resolution the BESII drift chamber has a precision between 200 and 250 μm for σ_{xy} and $1.78\% \cdot (1 + P^2)^2$ for $\frac{\Delta P}{P}$. The resolution on $\frac{dE}{dx}$ is about 8%

In the BESIII upgrade, the drift chamber resolution is increased to 130 μm for σ_{xy} and 0.5% for $\frac{\Delta P}{P}$. For thr $\frac{dE}{dx}$ measurment, the precision will be between 6% and 7%.

2.2.3 Time of flight

This device is placed just after the main drift chamber. The barrel part of this detector is build with 48 scintillator bars, arranged in a cylinder configuration. Each sides of those bars are conneted with photomultipliers. The bars are $284\,cm$ long, $15.6\,cm$ wide and $5\,cm$ thick. The scintillator has a decay time of $2.3\,ns$ and an attenuation lenght of $4.4\,m$. The time resolution of this device is $150\,ps$ for cosmic rays. The endcaps are also covered by the time of flight counter, with scintillators in cake slice shape covering all the main drift chamber endcaps, the main difference with the barrel is they are only connected with one photomultiplier.

In BESIII, the time of flight will have a temporal resolution improved to 90 ps for the axial part and 110 ps for the endcaps.

2.2.4 Shower counter

The shower counter consists of two parts, the two end-caps (ESC) and the barrel (BSC). This detector is an electromagntic calorimeter which goal is to mesure the energy of electrons and photons. The barrel part is made of 24 layers of gas tubes interleaved with 23 lead absorber layers. Each gas layer has a total of 560 cells made of aluminium. The resolution of this detector is 21% for $\frac{\Delta E}{E}$ with a σ_z of 3 cm.

With the BESIII upgrade, the precision of the shower counter will be greatly improved. For $\frac{\Delta E}{E}$, the resolution will reach 2.5%. The upgrade will also lower σ_z to 6 mm

2.2.5 Muon chambers

The muons chambers are the outermaost part of BESII and are made of three layers of iron absorber and three layers of proportionnal chambers. There is a total of 189 chambers disposed in an octogonal configuration. Each chamber consists in eight proportional gas tubes diposed in two layers. Its purpose is to measure tracks of muons and K_L, as they are the only particles able to go out from the rest of the detector.

The BESIII muons chambers will be improved as it will have nine layers instead of the actual three.

2.3 Weakness of the BESII detector

The BESII detector faces a problematic issue due to the absence of endcaps. This leads to particle 'loss' when their trajectories make a small angle with the beam pipe. The loss of usefull data implied by this problem is important in the studied channel. For the muons, the limit angle is of 53° for the muon chamber and 37° for the other tracks.

BESIII will make use of end-caps, thus will cover 90% of the total 4π solid angle. This increased coverage will greatly improve the total amount of usable data. Along with the 100-fold increased beam luminosity, one can expect a realy huge gain in the amount of the future collected data.

3 CP violation

This section shows how the CP violation appears in the Standard Model of particles. The main steps of the *Cabibbo-Kobayashi-Maskawa (CKM)* matrix derivation are presented, along with some remarks on its consequences. The last part is a short presentation of the CP-strong problem, which is tied to the search of a chromo-electric dipole moment for the ψ' particle as a possibility to generate this dipole moment.

3.1 CKM matrix

For now CP violating interactions have only been seen in the following neutral mesons systems : $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $B_s - \bar{B}_s$. In those cases the CP violating effect seems to come out from the CKM matrix which describes the mixing between the different quark generations. This matrix naturally comes out from the Standard Model theory. Experiments have explored many channels in neutral meson systems. Up to now, the amplitude of observed CP violating effects are rather small, and only concerning a little number of decay channels. There is some possibilities of CP violating effects out of CKM, for example in the strong interactions. Yet, none has been experimentally found. One can start be building the most general renormalizable

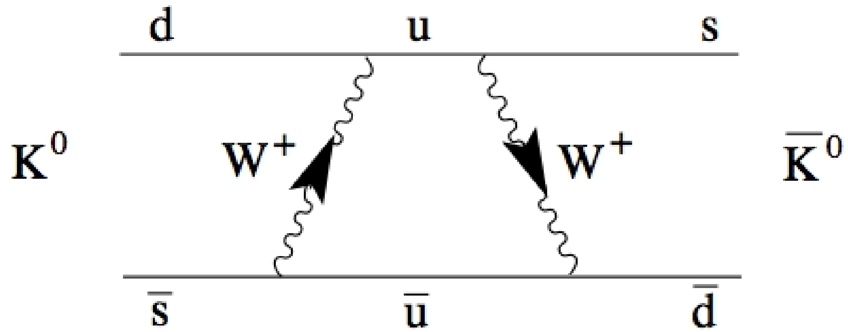

Figure 3: Box diagram in the $K^0 - \bar{K}^0$ oscillation

Lagrangian consistent with the $SU(3) \times SU(2) \times U(1)$ gauges symmetries of the strong, weak and electromagnetic interactions is the starting point. Then the problem is what further global symmetries are to be imposed on this theory to give the global symmetries observed in nature. First the Higgs scalar field is ignored as well as the quarks, leptons and gauge bosons mass terms. The Lagrangian is then fully specified by renormalizability and gauge invariance.

$$\mathcal{L} = -\frac{1}{4}\sum_i (F^a_{i\mu\nu})^2 + \sum J\bar{\Psi}_J(\imath\gamma^\mu D_\mu)\Psi_J \tag{1}$$

The index J index runs over the different chiral fermions multiplets, and i runs over the three gauge group factors. A pseudo-scalar gauge factor $\Delta\mathcal{L}_{theta}$ can be added to (1).

$$\Delta\mathcal{L}_{theta} = \sum_i \frac{\theta_i g_i^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^a_{i\mu\nu}F^a_{i\rho\sigma} \tag{2}$$

These terms are at first sight odd under P and T. Terms of this form can be generated or canceled by making a change of variables in the effective action. A change of variables on the right-handed electron field of the following form can be performed :

$$e_R \rightarrow e^{i\alpha}e_R \tag{3}$$

This produce a correction to the Lagrangian which is :

$$\delta\mathcal{L} = \alpha\frac{g'^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \tag{4}$$

For each fermions of hyper-charge Y we transform in that way, a similar correction to the Lagrangian appears, proportional to Y^2. If the fermion couple to the $SU(2)$ or $SU(3)$ gauge fields, terms proportional to those fields strengths are also created. If one performs the variables change with $\alpha = -\frac{1}{2}\theta_1$ all terms of the form (2) can be suppressed by doing appropriate chiral rotations on all fermions multiplets.

Considering (1) one can look further on its properties under T, P and C. The coupling of the QCD bosons are invariant under each of those symmetries. The couplings of the $SU(2)$ gauge bosons are not P and C invariant. Indeed, a close look at them shows that they violate P and C completely. The P operation convert a left-handed electron to a right-handed one. The C operation convert a left-handed electron to a left-handed positron. Those two operations convert a particle that couple to $SU(2)$ gauge bosons to another one who does not. However if those two operations are combined then a left-handed electron transforms into a right-handed positron which couple to the $SU(2)$. So, the CP operation is a symmetry of 1, which is also invariant under time reversal.

At this point one can conclude that every theory of massless fermions and gauge bosons is invariant under CP and T, even if any chiral theory violate both C and P separately. It is known experimentally that some CP violating interaction exists, like in some neutral mesons systems. In order to know where it comes out the theory, one can try to add terms to the base Lagrangian (1). It is possible to do so by adding the dynamics causing the breaking of $SU(2)\times U(1)$, using a simple scalar field for the Higgs. The most general Lagrangian is :

$$\mathcal{L}_\phi = |D_\mu \phi|^2 + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 \tag{5}$$

As $\mathcal{L}_\phi$ is hermitic, the parameters μ^2 and λ must be real. Then this $\mathcal{L}_\phi$ will respect C, P and T symmetries. Now, the last thing which can be done is to add the Higgs field coupling to the fermions. In the case of the quarks, one can remember that we have three generations. Thus three left-handed quarks doublets are to be considered. This series of doublets is here called Q_L. There are six right-handed quarks with either $Y = \frac{2}{3}$ or $Y = -\frac{1}{3}$. When the dynamics of the coupling of these quarks to the gauges fields is added, the derivatives are replaced by covariants derivatives. This give all quarks the same QCD coupling. It also give the same coupling to weak interactions for each quarks of the same type. These couplings does not allow any mixing between the different quark flavors. However, the coupling to the Higgs field is not constrained by such restriction as it does not follow gauge principle. One can postulate a new symmetry which consists in flavor conservation, but if we do not, the Higgs coupling will mix the different kinds of quarks. So excluding to put any additional constraints on our theory, imposes to write the most general general gauge invariant and renormalizable coupling, which is :

$$\mathcal{L}_c = -\lambda_d^{ij} \bar{Q}_L^i \cdot \phi d_R^j - \lambda_u^{ij} \epsilon^{ab} \bar{Q}_L^i a \phi_b^\dagger u_R^j + h.c. \tag{6}$$

Where Q_L^i are the three left-handed quarks doublets, and d_R^j, u_R^j respectively the right-handed quark triplets of hyper-charge $Y = \frac{2}{3}$ and $Y = -\frac{1}{3}$. The λ_d^{ij} and λ_u^{ij} matrix are general. They do not need to be symmetric nor Hermitian complex-valued matrix. Operating CP on those matrix, the operators λ_u^{ij} and λ_d^{ij} are exchanged with their Hermitian conjugate. So the effect of CP operation on these operators is :

$$\lambda_u^{ij} \to (\lambda_u^{ij})^\star, \qquad \lambda_d^{ij} \to (\lambda_d^{ij})^\star \tag{7}$$

In the case of real valued matrix, CP is a symmetry of 6. But no symmetry principles ask one to do so, meaning we have a coupling that seems to cause a maximal violation of all flavors and C, P and T.

The same idea as in 2 can be used to eliminate the T violating term by making the same kind of chiral rotation [1]. Doing so leads to the following coupling for the Higgs :

$$\mathcal{L}_c = -m_d^i \bar{d}_L^i d_R^i (1 + \frac{h}{v}) - m_u^i \bar{u}_L^i u_R^i (1 + \frac{h}{v}) + h.c. \tag{8}$$

It has a quark mass term and a Higgs boson coupling term. The chiral rotation changes the quark fields to the basis of the mass eigenstates. As in this basis the mass and Higgs coupling terms are diagonal in flavor, then they conserve C, P and T. Looking closer to the different interactions, it appears that the rotation matrix involved vanishes in all interactions, except in the current that couple to the W boson.

In this case the rotation matrices do not commutate with the covariant derivative. Thus the charged weak interactions mixes the different quarks by linking the u_L^i and d_L^i quark triplets with a unitary rotation. This unitary matrix V is called the *Cabibbo-Kobayashi-Maskawa (CKM)* mixing matrix. This matrix has 9 parameters, 3 angles and 6 phases. All phases can be canceled out by phase rotation, except one. Thus we have 3 free angles and one phase. This phase is the parameter causing CP violation in the weak interactions mediated by the W boson. However, the amplitude of such CP violating effects is very small as in order to get the CP violating phase we must have decays involving at least two generations. This means it can only contribute in loop corrections involving W boson, or in complicated exclusive decays. This explain the smallness of the CP violating effects in the neutral kaons systems, for example.

[1] If one is interested in the details of this transformation see [6]

3.2 Strong CP problem

In the previous section, the Lagrangian of the gauge theory as been studied with the goal to simplify it. To achieve this, one needed to make some chiral change of variables, like Eq.(2). It has been shown that such change of variables produce P and T violating terms, as it can be seen in Eq.(4). One can show that in the case of $SU(1)$ and $SU(2)$, this leads to no physical effects because of cancellations. However, this is not the happening with $SU(3)$. In this latter case, observable effects could take place. This observable effects can be the cause of an electric dipole moment for the neutron and other baryons. However, there is also other possibility to generate chromo-electric dipole moments, but they will not be presented in the frame of this work. Yet, electric dipole moment in the neutron has been excluded by all experiments. The theory cannot explain a zero value if one does not add a new symmetry principle linked to QCD. If we keep the theory with the most general higgs coupling matrices to the quarks, we cannot exclude the possibility of non-zero values for the electric dipole moment of some baryons.

3.3 Effective CP Violating Lagrangian

In this experiment, we will use a CP-odd observable to test the CP symmetry. Here, the relevant CP violating interaction can be the electric dipole moment and the chromo dipole moment. This CP violating interaction can be defined by an effective Lagrangian :

$$\mathcal{L}_{CP} = -id_c\bar{c}\gamma_5\sigma_{\mu\nu}F^{\mu\nu}c - i\tilde{d}_c\bar{c}\gamma_5\sigma_{\mu\nu}G^{\mu\nu}c \tag{9}$$

In this effective Lagrangian d_c is the electric dipole moment and $\tilde{d}_c$ is the chromo dipole moment. We will test the quarkonium transition $\psi' \to J/\psi\ \pi^+\pi^-$. The process considered is the following :

$$e^+(\vec{p}) + e^-(-\vec{p}) \to \psi' \to J/\psi\ (\vec{k}\,) + \pi^+(\vec{k}_+) + \pi^-(\vec{k}_-) \tag{10}$$

Here the ψ' is produced at rest. Both ψ' and J/ψ are polarized, and we will use $\epsilon_{\psi'}$ and $\epsilon_{J/\psi}$ respectively to denote them. The decay amplitude in the rest frame of the ψ' can be written :

$$\mathcal{A}\,(\psi' \to J/\psi\ \pi^+\pi^-) = \epsilon^i_{\psi'}A_{ij}\epsilon^{\star j}_{J/\psi} = \epsilon^i\tilde{A}_{ij}\tilde{\epsilon}^j_{J/\psi} \tag{11}$$

We will assume that we can observe the polarization of the J/ψ but not the polarization of ψ'. We have access to this information using the leptonic decays $J/\psi \to \mu^+\mu^-$ and $J/\psi \to e^+e^-$. Let p_{μ^+} and p_{μ^-} be the momentum of the μ^+ and μ^- in the ψ' rest frame. In the same way, let k_+ and k_- be the momentum of the π^+ and π^- also in the ψ' rest frame. First we make a Lorenz boost to go into the J/ψ rest frame. In this frame the polarization vector is :

$$\epsilon^i_{J/\psi} = \tilde{\epsilon}^i_{J/\psi} + k^i\big(\frac{k^0}{m_{J/\psi}} - 1\big)\frac{\vec{k}\cdot\vec{\tilde{\epsilon}}_{J/\psi}}{\vec{k}\cdot\vec{k}}, \quad \epsilon^0_{J/\psi} = \frac{1}{m_{J/\psi}}\vec{k}\cdot\vec{\tilde{\epsilon}}_{J/\psi} \tag{12}$$

Using this relation, we can write the decay amplitude :

$$\mathcal{A}(\psi' \to J/\psi\ \pi^+\pi^-) = \epsilon^i_{\psi'}A_{ij}\epsilon^{\star j}_{J/\psi} = \epsilon^i\tilde{A}_{ij}\tilde{\epsilon}^j_{J/\psi}$$

In the experiment we observe the 4-momenta of the decay products, thus any CP observable must be built only with them. The density matrix for the decay is defined as :

$$R(\vec{k}_+, \vec{k}_-, \vec{p}_{\mu^+}, \vec{p}) = \rho_{ik}(\vec{p})\tilde{A}_{ij}\tilde{A}^\dagger_{kl}\tilde{\rho}_{jl}(\vec{p}_{\mu^+}) \tag{13}$$

with $\tilde{\rho}_{ij}(\vec{p}_{\mu^+}) = \rho_{ij}(\vec{p}_{\mu^+})$. Any observable build with $\vec{p}_{\mu^+}, \vec{p}_{\mu^-}, \vec{k}_-, \vec{k}_+$ can be predicted to be :

$$\langle\mathcal{O}\rangle = \frac{1}{\mathcal{N}}\int d\Gamma d\Gamma_{\mu^+\mu^-}\mathcal{O}R(\vec{k}_+, \vec{k}_-, \vec{p}_{\mu^+}, \vec{p}) \tag{14}$$

Where $\mathcal{N}$ is the normalization :

$$\langle 1 \rangle = \frac{1}{\mathcal{N}} \int d\Gamma d\Gamma_{\mu^+\mu^-} R(\vec{k}_+, \vec{k}_-, \vec{p}_{\mu^+}, \vec{p}) \tag{15}$$

The $d\Gamma$ and $d\Gamma_{\mu^+\mu^-}$ are the phase space terms for the intermediate and final state of the reaction. If we have CP as symmetry, then R is constrained. Those constraints are linked to the structure of R which is build on CP sensitive coefficients. It means that using an appropriate observable based on kinematical data will allow us to properly test CP. One valid choice of such a CP observable among a lot of other possibilities is :

$$\mathcal{O} = \frac{1}{2} \hat{p}^i_{\mu^+} \hat{p}^j_{\mu^+} \left(\hat{k}^i_+ \hat{n}^j + \hat{n}^i \hat{k}^j_+ - \hat{k}^i_- \hat{n}^j - \hat{n}^i \hat{k}^j_- \right) = \hat{p}_{\mu^+} \cdot (\hat{k}_+ - \hat{k}_-)\, \hat{p}_{\mu^+} \cdot \hat{n} \tag{16}$$

with :

$$\hat{n} = \frac{\vec{k}_+ \times \vec{k}_-}{|\vec{k}_+ \times \vec{k}_-|} \quad \hat{k}_+ = \frac{\vec{k}_+}{|\vec{k}_+|} \quad \hat{k}_- = \frac{\vec{k}_-}{|\vec{k}_-|}$$

In the case of a CP violation, $\langle \mathcal{O} \rangle$ will have a non zero value. The statistical error on $\langle \mathcal{O} \rangle$ is given by :

$$\delta \mathcal{O} = \sqrt{\frac{\langle \mathcal{O}^2 \rangle}{N_{event}}} \tag{17}$$

4 Decay channel and event selection

This part focuses on the data analysis. It begins by a description of the decay and the particle selection. Possible backgrounds and how to reduce them is detailed. After this, the results are presented along with their errors and comments. The last part of the section presents some future perspectives and possible improvements. All the indicated branching ratio are taken form the particle data group booklet.

4.1 The $\psi\,'$ decay

The $\psi\,'$ resonance resonance is an excited state of J/ψ. It has a mass of $3686.1 \pm 0.034\text{M}eV$ with a full width of $\Gamma = 281 \pm 17$ keV. In this experiment the decay of interest is a transition from the $\psi\,'$ particle to the J/ψ particle, through the emission of two pions. The J/ψ particle has a mass of 3096.9 ± 0.011 MeV with a full width of $\Gamma = 91.0 \pm 3.2$ keV.

A very clear channel is also needed for the J/ψ decay, especially in the scope off avoid problems of particle miss identification. For this reason the leptonic decays in two muons is chosen, as they are the only charged particle going to hit the muons chambers. Thus, the considered decay is :

$$\psi\,' \quad \rightarrow \quad J/\psi\ \pi^+\pi^- \quad \rightarrow \quad \mu^+\mu^-\ \pi^+\pi^-$$

The total branching ratio for this decay is $(1.86 \pm 0.1)\%$. The $\psi\,'$ particle mainly decays to hadrons at $(97.85 \pm 0.13)\%$. However, it can produce also leptons through three leptonic decays :

$$\begin{array}{lcll} \psi\,' & \rightarrow & e^+e^- & (7.55 \pm 0.31) \cdot 10^{-3} \\ \psi\,' & \rightarrow & \mu^+\mu^- & (7.3 \pm 0.8) \cdot 10^{-3} \\ \psi\,' & \rightarrow & \tau^+\tau^- & (2.8 \pm 0.7) \cdot 10^{-3} \end{array}$$

Those leptonic decays cannot produce any noise in this experiment since the leptons cannot be produced along with other charged particle in those cases. Thus, taking two muons along with two other charged particles protect us from taking muons coming from the $\psi\,'$.

Given the data sample size of 14 millions events and considering the total branching ratio of the total transition, this gives from $2.46 \cdot 10^5$ to $2.74 \cdot 10^5$ events of interest. There are still some potential backgrounds in the hadronic decays of the $\psi\,'$. These backgrounds consist mainly in five channels. Their branching ratio are rather small compared to the signal channel, but it can reduce significantly the purity of the reconstructed events. This problem and how to address it is further detailed in the background part of this section.

4.2 Event selection

In order to study the process $\psi' \rightarrow \pi^+\pi^- J/\psi$, four tracks with a sum of charge equal to zero are asked. After this, other selection cuts are made on the muons and pions. Some basic cuts are made in order to have usable data. Those cuts are :

- $p_{xy} > 70$MeV. This cut on the transverse momentum reject events which have particle cycling in the MDC.
- $|\cos\theta| < 0.75$ The angle θ is the angle between the beam axis and the particle track. It removes events that are not completely contained within the detector.

Events passing this selection will be referred as selected events in the latter.

4.3 Event kinematics

To use right selection cuts, it is important to know what are the momenta and energies of the involved particles. The ψ' is produced at rest in the laboratory, and decays to $J/\psi\,\pi^+\pi^-$. Considering the dipion system like a single particle, one can compute that the maximal momentum of the J/ψ particle is of 477.6 MeV. For the pions, the maximal momentum is of 400.9 MeV, when a first pion stay at rest and the second pion goes back-to-back with the J/ψ. The muon momentum will be of 1.544 GeV in the J/ψ center of mass.

Due its very short lifetime and its low momentum, the displacement of the J/ψ decay vertex from the ψ' decay vertex will be very small, thus it should be compatible with a 4-body decay. For this reason, a 4-body kinematical fit can be used with no risks.

4.4 Monte-Carlo signal study

A sample of $261 \cdot 10^3$ Monte-Carlo (MC) events has been generated for the channel of interest. This corresponds to one time the real data events. The generated events are $\psi' \rightarrow \pi^+\pi^- J/\psi$ with the J/ψ particle decaying into muons. Here are presented the plots corresponding to data events compared MC simulation. The following plots are raw data, which means before any improvement with the kinematical fit program. This allow to better see potential problems and backgrounds.

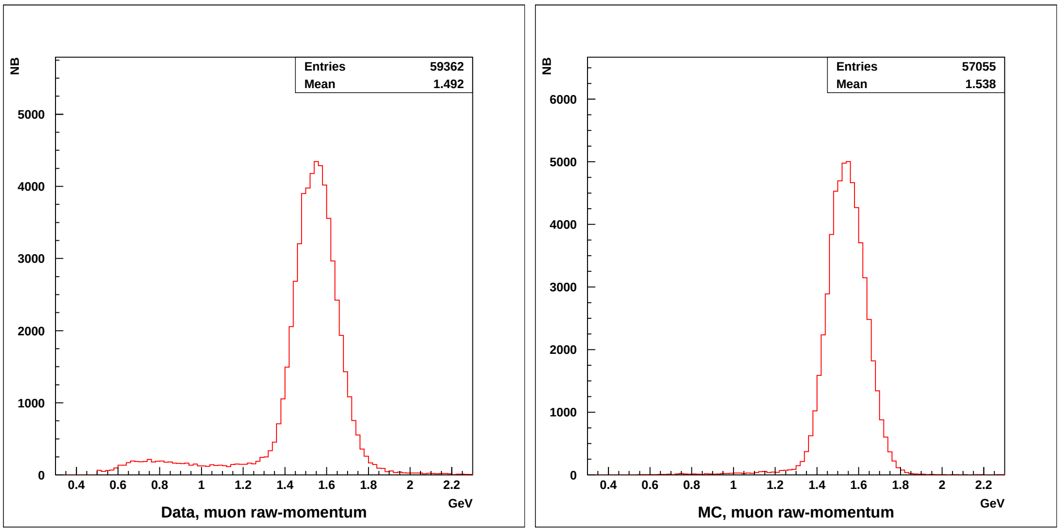

Figure 4: muon raw-momentum. left : data, right : MC

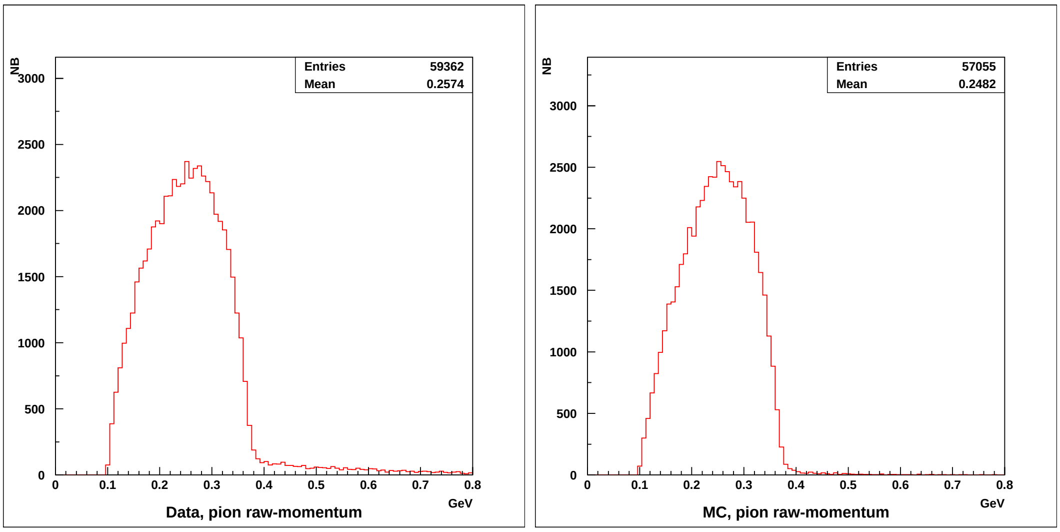

Figure 5: pion raw-momentum. left : data, right : MC

Comparison of the two plots leads to the conclusion that there is some non-resonant background, which adds particle to the signal in a wide range of momentum. This appears to be rather small compared to the resonant signal, and such kind of events are easy to suppress out of the peak with well chosen cuts. An estimate of this background based on this following muon momentum plot gives us $8.0 \cdot 10^3$ events.

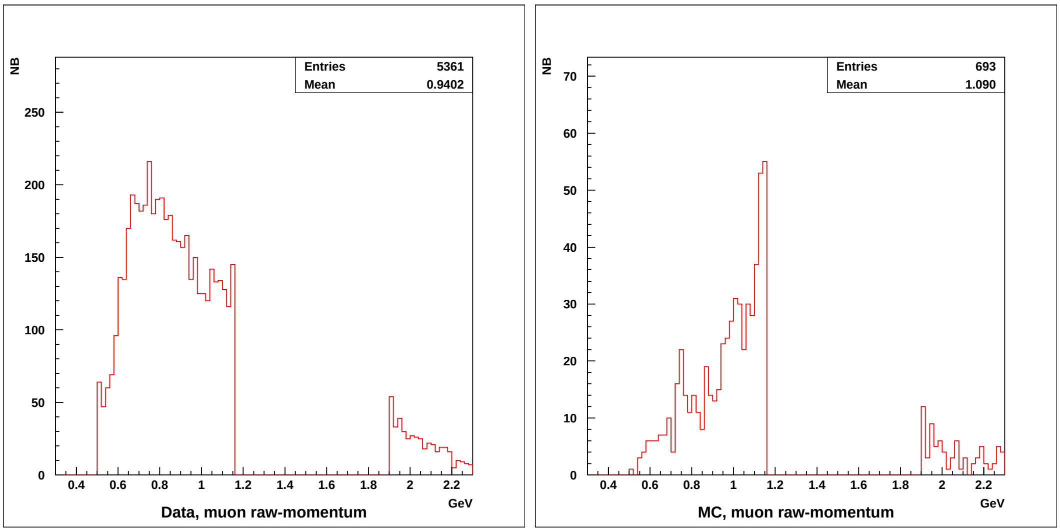

Figure 6: muon raw-momentum without the resonant peak. left : data, right : MC

For the particles momentums, MC events and data events behave the same if one excepts the background. So no problems can be expected on this side and generated events will be usable as reference. MC simulated event will be used after the particle selection cuts to verify that everything works fine.

4.5 Particle selection

Two particles are required to hit the muon chamber and the sum of charges must be zero. These two particles are tagged as muons. The two charged tracks left are tagged as pions. This is justified because there is no other channel producing exactly four particles, two muons and two charged particles. The following selection cuts are based on the former MC study of the signal, kinematical considerations and geometrical cuts linked to the detector coverage.

4.5.1 Muon selection

The muon tracks must satisfy the conditions detailed bellow.

- $0.5 < p_\mu < 2.5$ GeV Where p_μ is the impulsion of the muon track candidate.
- $|\cos\theta_\mu| < 0.60$ This is a verification cut because it only confirms that the muon track candidate is passing through the muon chamber.
- $|\cos\theta_{\mu^+\mu^-}^{CM_{J/\psi}}| < -0.975$ This is the cosine of the angle between the two muons track candidates in the rest frame of the J/ψ . As this is a two bodies decay, the to muon candidates must be nearly back-to-back.
- p_{μ^+} and $p_{\mu^-} > 1.3$GeV and $p_{\mu^+} + p_{\mu^-} > 2.4$GeV This cut selects candidates consistent with a J/ψ decay and thus reject event candidates which have muons coming from light particles, as well as a large part of the non-resonant background events.

4.5.2 Pion selection

The pion tracks are asked only to be opposite in charge. This means there is no particle identification based the measured dE/dx. The following conditions are requested on the pion tracks candidates :

- $p_\pi < 0.5$ Gev with p_π the pion candidate momentum.
- $\cos\theta_{\pi\pi} < 0.9$, where $\theta_{\pi\pi}$ is the angle between the two pion tracks. This angle is taken in the laboratory. This cut eliminates some electron tracks produced by photon conversion into pairs

4.5.3 Additional criteria

One additional criteria is put on m_{recoil} which is the mass recoiling against the dipion system. This m_{recoil} should peak on the jpsi mass, with a little shoulder in the above part. This shoulder is caused by low energy pions decaying in $\pi \rightarrow \mu\nu_\mu$. To reduce this, the following cut is applied :

- $3.07 < m_{recoil} < 3.12$GeV

4.5.4 KFIT4C

As shown in the part concerning the detector, the measurement precision is not always very high. A possibility to improve this is using the additional information that the signal is a 4-body decay. To perform this, a four particle kinematical fit is applied on the charged tracks which correct the observed tracks in new ones. Those new tracks have to obey some constraints. This is done by giving the assumed muons and pions tracks their exact known masses. It is also demanded that the total momentum is consistent with zero and that the a total invariant mass corresponding to the ψ' is reconstructed.

Event if our decay is not exactly a 4-body decay, the error caused by the displaced vertex of the J/ψ is very small due to its small momentum and lifetime. The most interesting thing here concerns background reduction. This fit provides a χ^2 value which tells how good the fit is. In the case of data events, this fit will only correct the tracks to correct the imprecision of the detector. Nearly all potential backgrounds detailed in the latter are characterized by the presence of one or more non-detected particles. This means that in those events a significant part of the momentum is missing, which implies great corrections of the tracks in order to fit with a 4-body decay. As this means a high χ^2 return value, one can use this to discard background events through an appropriate cut, while greatly improving the data quality.

As this is the main background cut, a significance optimization will be made after the noise study trough MC generated events, when possible.

4.6 Monte-carlo signal selection study

An important thing to test is the differences between simulated and real events, after all selection cuts are implemented and the kinematic program run on the events. The first plot presents the plot for the χ^2 value returned by the kinematical fit. A strong difference appears in the number of events

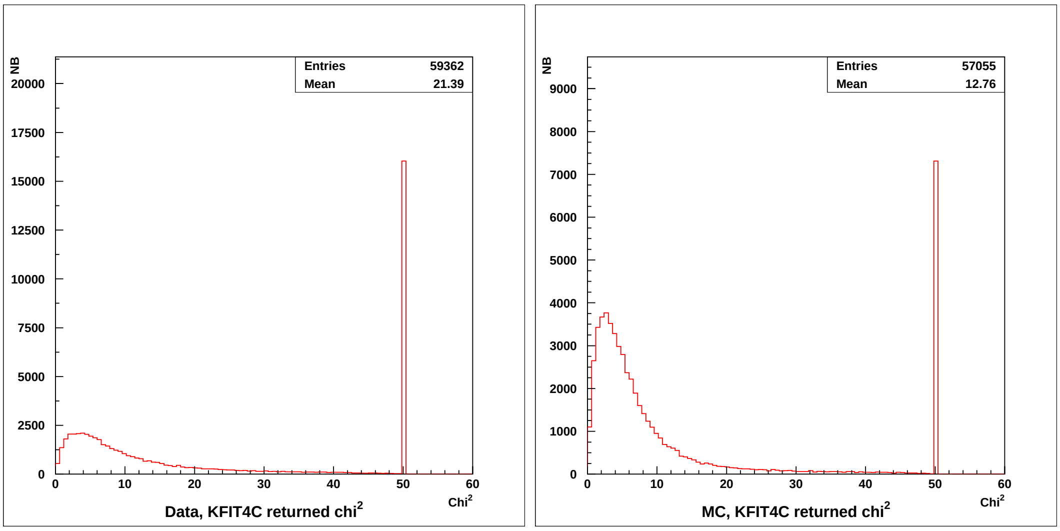

Figure 7: χ^2 returned by KFIT4C. left : data, right : MC

which are tagged with a χ^2 value of 50. This means that a lots of events have bad kinematical fitness in the real data. It can be expected, as there is several noisy channels detailed in the latter able to produce such events. The two following plots present the old non-corrected μ^+ and π^+ momentums after all cuts, including a cut which rejects events with $\chi^2 = 50$. We also look at the momentums before modification by the kinematical fit for all the rejected events, meaning which do not get through the particle selection and/or have a $\chi^2 = 50$. The goal here is to check that the kinematical fit and particle selection are doing a good work.

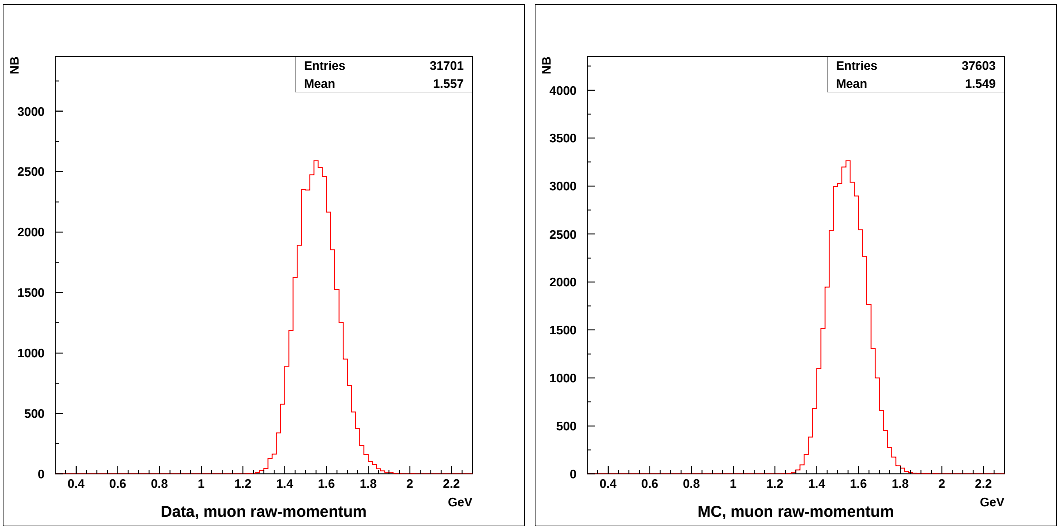

Figure 8: raw-momentum of the muons passing all cuts. left : data, right : MC

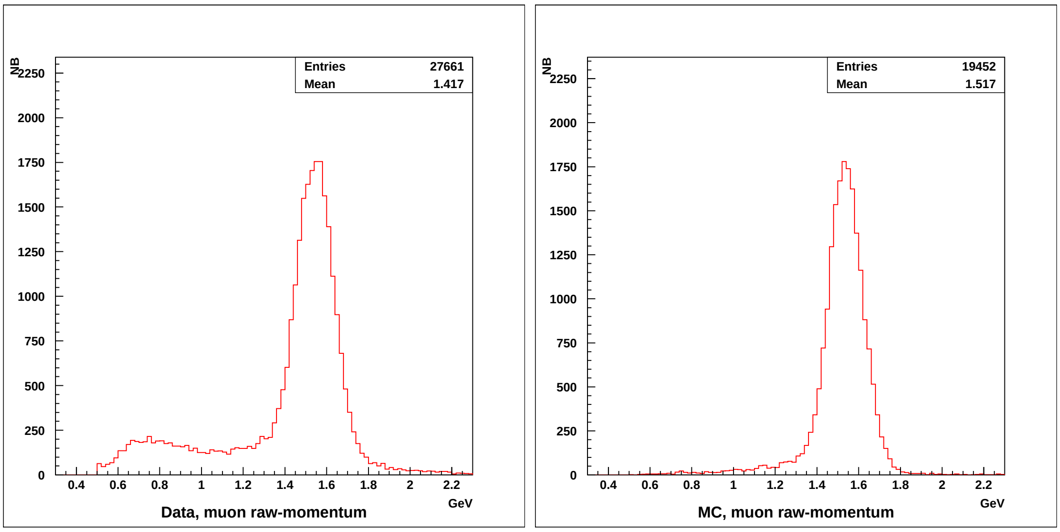

Figure 9: raw-momentum of the rejected muons. left : data, right : MC

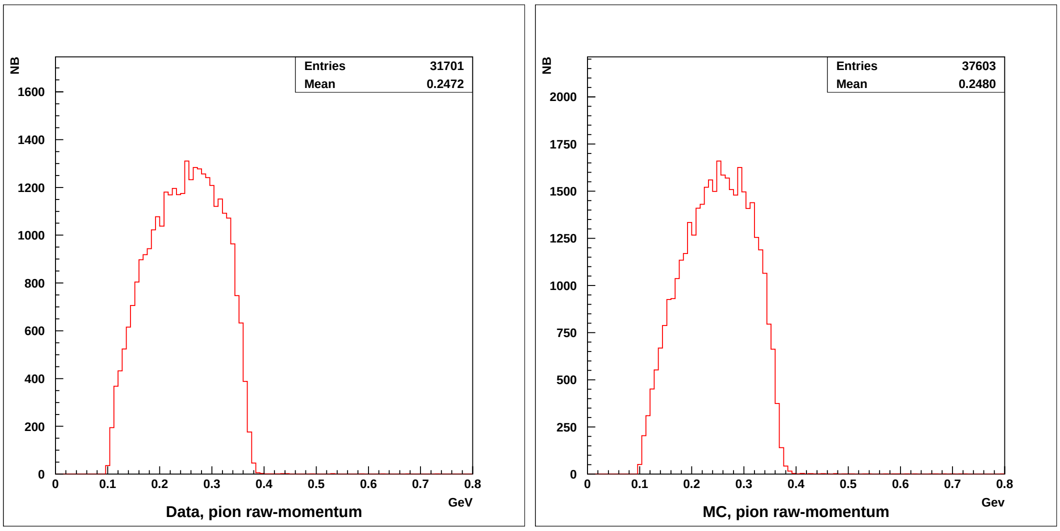

Figure 10: raw-momentum of the pions passing all cuts. left : data, right : MC

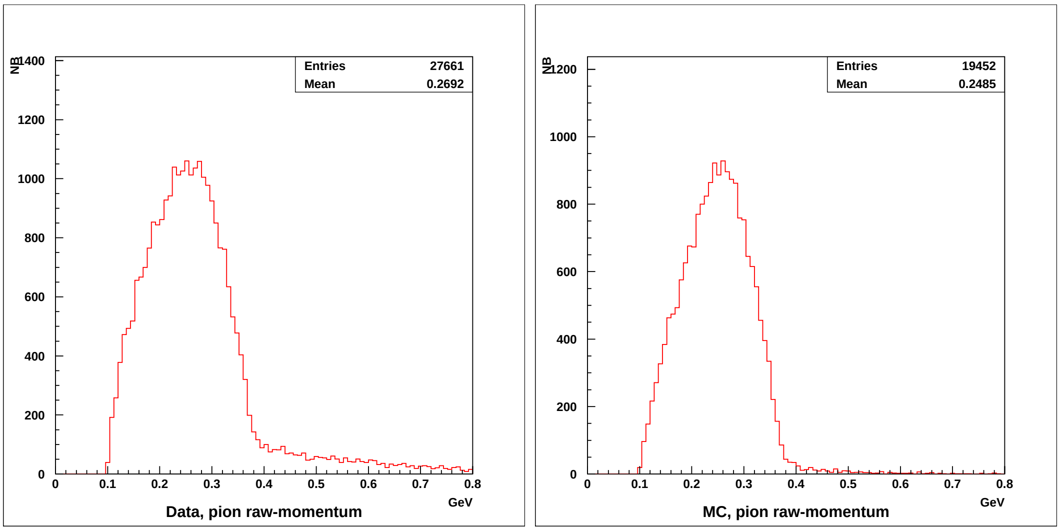

Figure 11: raw-momentum of the rejected pions. left : data, right : MC

Some signal events are lost because of the detector resolution. In the other hand, the quality of the signal is improved while most of the non-resonnant background is rejected.

Here are the pion and muon momentum after the application of the kinematical program to improve the quality of our signal. These are only the events passing through all the particle selection cuts and with a returned value for χ^2 lesser than 50.

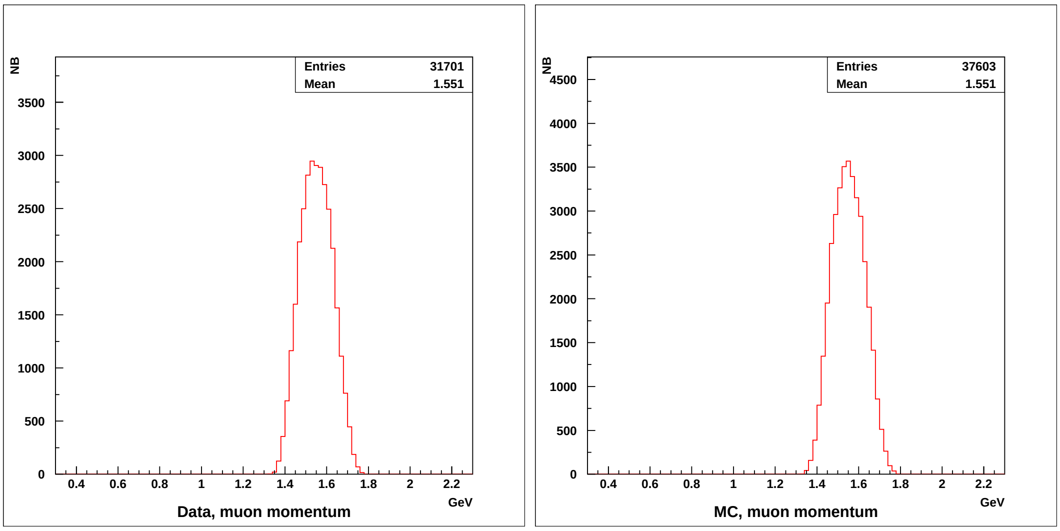

Figure 12: corrected momentum of the muons passing all cuts. left : data, right : MC

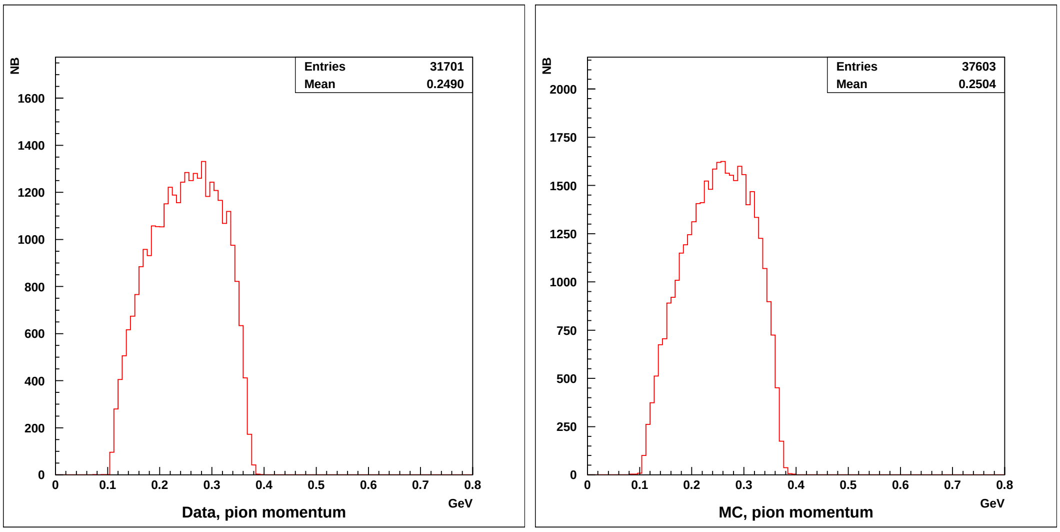

Figure 13: corrected momentum of the pions passing all cuts. left : data, right : MC

As expected, the distributions are a little less widespread after the application of the kinematical fit program. Bellow plots for the angle between the two pions in the laboratory frame, the angle between the two muons in the J/ψ rest frame and the recoil mass against the two pions are presented :

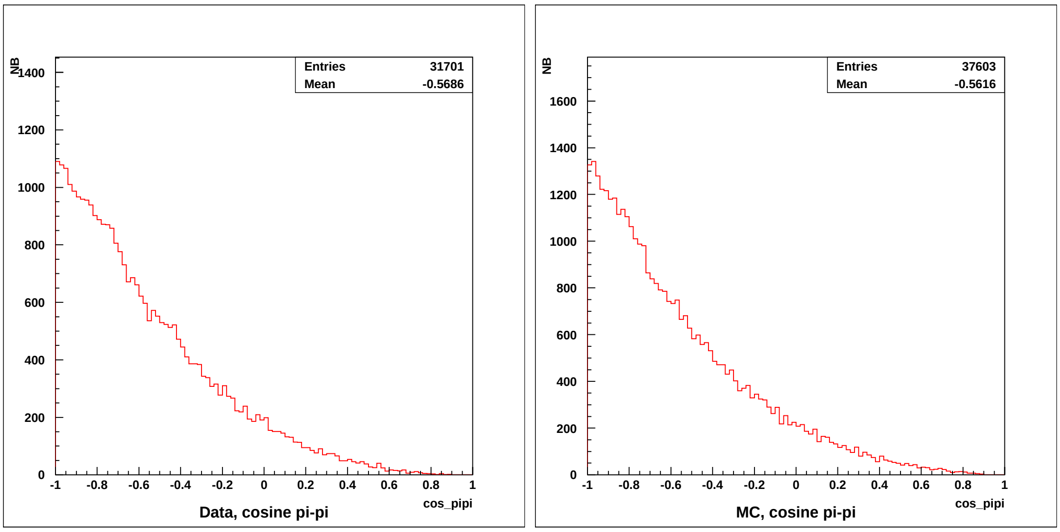

Figure 14: $\cos\theta_{\pi^+\pi^-}$ in selected events. left : data, right : MC

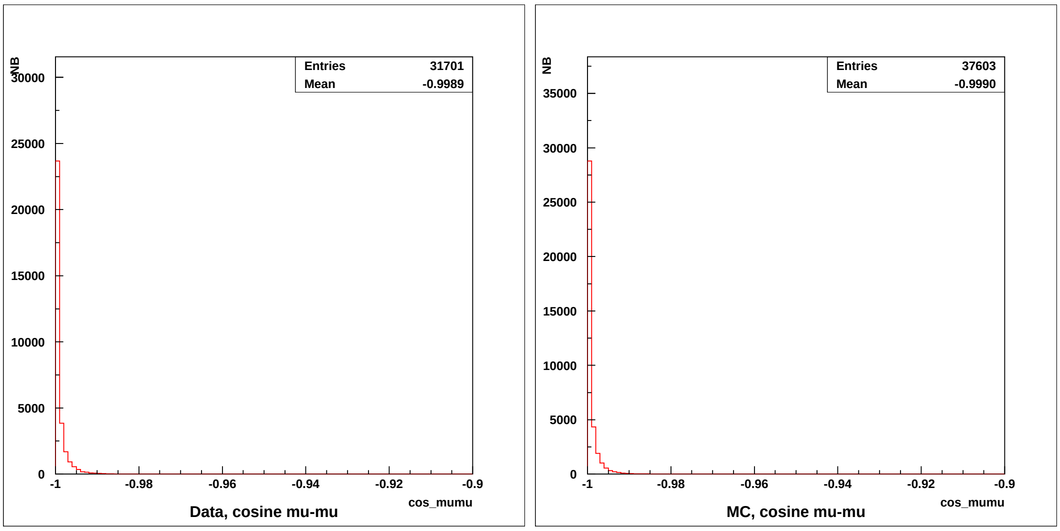

Figure 15: $\cos\theta_{\mu^+\mu^-}$ in selected events. left : data, right : MC

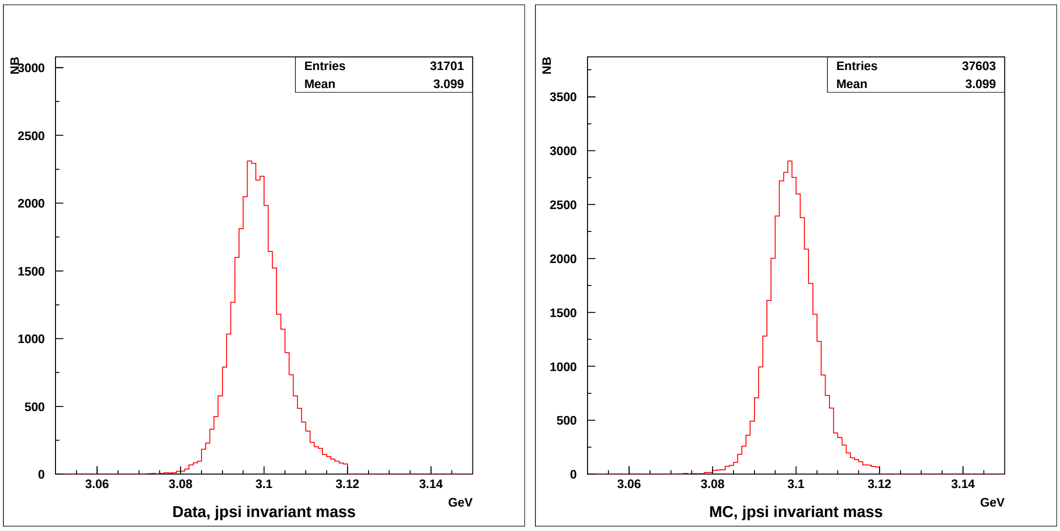

Figure 16: invariant mass of the jpsi in selected events. left : data, right : MC

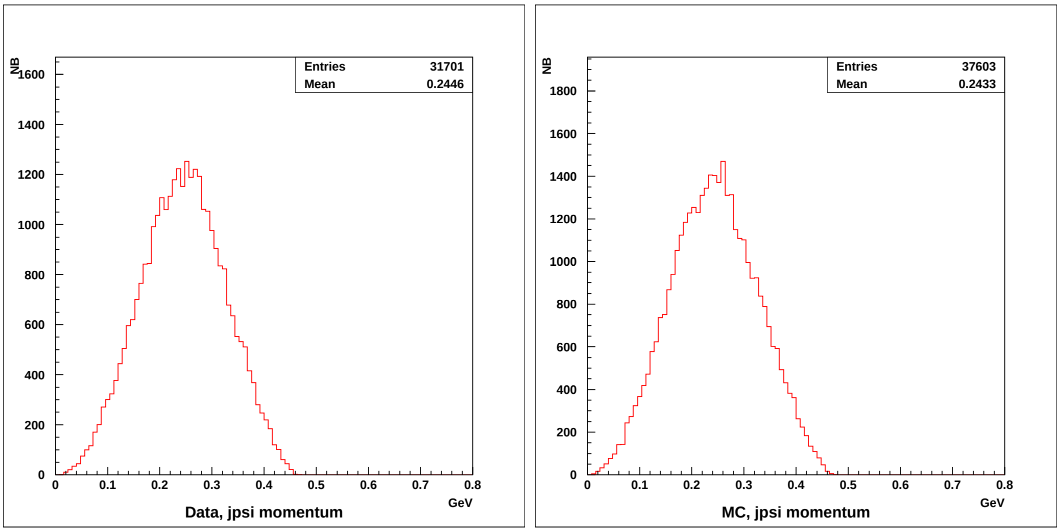

Figure 17: momentum of the jpsi in selected events. left : data, right : MC

An interesting point is the small peak appearing at $\cos\theta_{\pi^+\pi^-} = 1$ in the real data if the cut on the χ^2 value is not applied. This latter effect is presented in a plot in the proper background part. This latter issue is further explained in the background part.

4.6.1 Possible issues

The expected amount of events in the real data, given the total branching ratio and the total 14 millions data, one can expect to have from $2.46 \cdot 10^5$ to $2.74 \cdot 10^5$ events of interest. But in the facts, only 97% of the data are studied. This implies there should be a difference between MC and data events passing all cuts. An other point to consider is the fact that MC events have better track resolution, thus one can expect that compared to data, more events will pass all the cuts. An estimate of this effect is about 6%.

The number of events passing all cuts is $37.6 \cdot 10^3$ for MC, and $31.7 \cdot 10^3$ for the data. With the corrections, the MC events passing all cuts are $34.3 \cdot 10^3$. Accouting these two problems, the difference in the number of events passing all cuts for MC and data is reduced enough to nearly match within the error on the branching ratio which leads to a window going from $32.3 \cdot 10^3$ to $36.0 \cdot 10^3$ events. MC and data are rather well matching, and the observed difference is rather well explained.

4.7 Possible Backgrounds

The possible background candidates must at least have exactly four charged tracks with sum of charge equal to zero. In addition, two of them have to be muons. This puts strong constraints on the possibilities, meaning only a very little number of ψ' decay channels can produce noise in our experiment. These channels can eventually produce some neutral particles, which means a different topology. This can be used through an additional kinematical fit.

4.7.1 The $\psi' \to \eta J/\psi$ background channel

The first possibility is the transition $\psi' \to \eta J/\psi$. The η particle can decay in several charged modes, which can lead to signal contamination. These charged modes are mainly consisting in the following channels :

$$
\begin{array}{lllll}
\psi' & \to & J/\psi\ \eta & \to & \mu^+\mu^-\ \pi^+\pi^-\pi^0 & (7.14 \pm 0.62)\cdot 10^{-3} \\
\psi' & \to & J/\psi\ \eta & \to & \mu^+\mu^-\ \pi^+\pi^-\gamma & (1.48 \pm 0.23)\cdot 10^{-3} \\
\psi' & \to & J/\psi\ \eta & \to & \mu^+\mu^-\ e^+e^-\gamma & (1.91 \pm 0.39)\cdot 10^{-4}
\end{array}
$$

One can notice that there is always a neutral particle emitted with the two charged particles. The missing impulsion will affect the recoiling mass, as well as the supposed pions impulsion. The presence of the neutral particle in those decays will alterate the topology of the event, since the η always makes a three-body decay in the cases of interest. Bellow, the plots presenting the effect of the cuts on this background channel. The three detailed back-

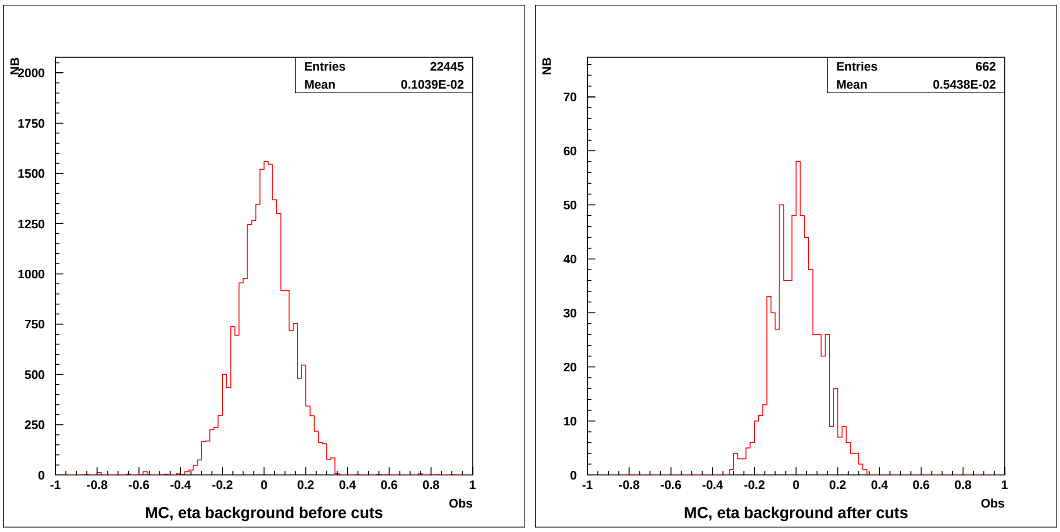

Figure 18: suppression of the eta background

grounds channels effect on the signal has been tested with MC simulation. The amount of data produced to test these noisy channels corresponds to 20 times the real data. While only using the basic selection cuts, this background is already strongly reduced, with only 2428 events passing through the cuts on a total of 22445. The addition of the kinematic cut enhances further this suppression, with 662 remaining. This leads to a maximal contamination of the signal of 33 events or 0.1%.

4.7.2 Indirect muon production by the J/ψ

This potential background is composed of decays where the ψ' particle goes to $\pi^+\pi^- J/\psi$ as in a signal event, but the J/ψ decays in a way which produces muons *indirectly*. For example $J/\psi \to K^+\bar{K}^\star(892)^- + c.c.$ with the two kaons going to two muons and one pion can be problematic. The problem comes from the fact that the topology of such events can be similar to the signal ones. However, there always will be neutrinos emitted along with the muons, and very often other decay products, like the pion in the former example. Due to the complexity of such events, It is not possible to conduct MC simulation, meaning that a precise estimation of the possible contamination is impossible. However, one can expect that such kind of background contamination should be rather low, because of the following :

- The presence of additional particle like neutrinos, neutral pions or undetected charged tracks implies a missing momentum. Thus a strong reduction will occur with the cut on the χ^2 value returned by the kinematical fit.

- Kaon or pion decay into muons produce neutrinos, and maybe other particle. This mean that muons tracks will not be perfectly back-to-back in the J/ψ rest-frame. Thus the cut on $\cos\theta_{\mu\mu}$ should further enhence the suppression.

For these two reasons, a very strong reduction of this background is expected. Taking in account the fact that the probability of producing two indirect muons in the same event is low, the signal contamination should be neglectable.

4.7.3 The $\psi\,' \to \chi_c\gamma$ background channel

This kind of background is made of three different decays. The $\psi\,'$ can decay in χ_{c0}, χ_{c1} or χ_{c2} along with a photon. These latter particles can decay in a J/ψ plus a photon. This means that these decays will have two photons with an energy high enough to produce pairs. This can contaminate the signal, if these e^+e^- are taken as $\pi^+\pi^-$. The respective total branching ratio for these channel, without accounting the pairs creation part, are given below :

$$\begin{array}{lllll}\psi\,' & \to & \chi_{c0}\,\gamma \to & J/\psi\,\gamma\,\gamma & \to \mu^+\mu^-\,\gamma\,\gamma \quad (5.97 \pm 0.32)\cdot 10^{-5} \\ \psi\,' & \to & \chi_{c1}\,\gamma \to & J/\psi\,\gamma\,\gamma & \to \mu^+\mu^-\,\gamma\,\gamma \quad (1.56 \pm 0.34)\cdot 10^{-3} \\ \psi\,' & \to & \chi_{c2}\,\gamma \to & J/\psi\,\gamma\,\gamma & \to \mu^+\mu^-\,\gamma\,\gamma \quad (9.98 \pm 1.94)\cdot 10^{-4}\end{array}$$

Considering the branching ratio, and given that pair creation even lower them, the signal contamination should not be too high. However, it will be important to reduce it as much as possible.

These backgrounds channels signal contamination hasn't been tested with MC simulation. However, one can notice that in the case of pair conversion from the second photon, this leads to really bad χ^2 value from the kinematic fit program, as this implies a missing momentum against the χ_c particle. So, in order to get through the kinematical fit, the electrons have to be generated by the photon coming from the $\psi\,'$ particle. Else the event will be suppressed by the cut on the χ^2 value coming from the kinematical fit.

Therefore, in the case where pair conversion took place with the first photon, the event will produce a good value of χ^2 for the kinematical fit, since there is no missing momentum against the χ_c particle. Hopefully, the photon has an energy of 100 MeV, which means a highly boosted e^+e^- pair. The selection cut on the angle between the two tracks of the assumed pions will remove such kind of events. On the following plot of $\cos\theta_{\pi\pi}$ one can see the small peak at cosine equal 1.

One can conclude that for such background, there is no risks of signal contamination since it is extremely well suppressed. Indeed, when all the cuts are applied, none of these events remain.

4.7.4 non-resonant background

Another kind of background is the non-resonant events. These events are mainly direct decays of the $\psi\,'$ particle into four or more charged tracks. If more than four, then some have to be 'lost', for example due to detector

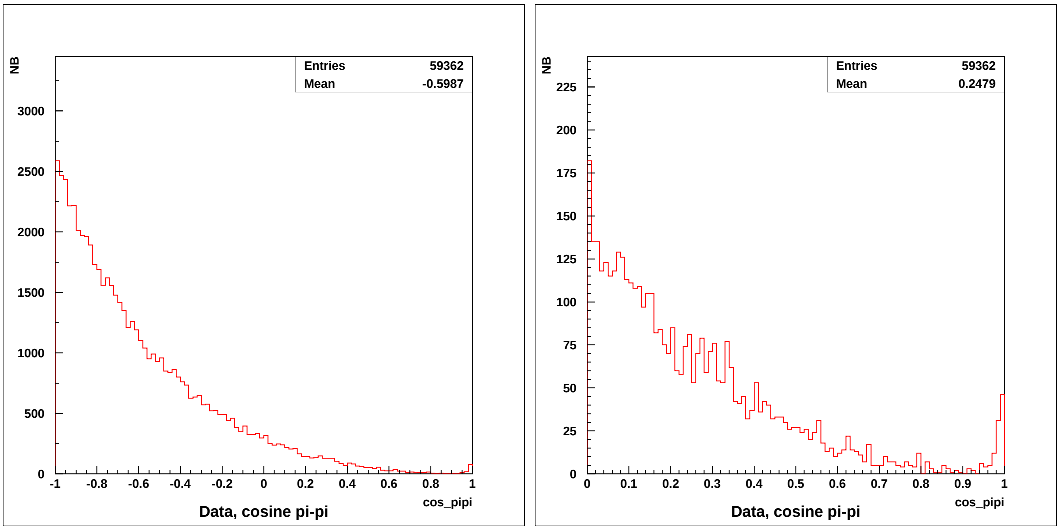

Figure 19: $\cos\theta_{\pi^+\pi^-}$ for all data events, with no cuts

coverage to make the event pass the event selection which requires exactly four charged tracks. Two of these four tracks have to produce a muon, else the event will be discarded. Muons can be produced easily by pions and kaons, and there is a plenty of non-resonant backgrounds producing such particles. Every of these events, are pure 4- or more body decay. This leads to great opportunities to a very effective suppression. Here are detailed the main way this is done through the particle selection cuts.

- More than four particles, but only four are detected. In such case, some momentum is missing, thus the kinematical fit will return a very high χ^2 value for such events.
- Exactly four particles, all well detected. Here the event has no missing momentum, except the one which is lost in the two neutrinos produced along with the muons. This implies that the kinematical fit will maybe return a low χ^2 value. But still, this is a pure 4-body decay, which means it is unlikely that it will pass the selection cuts. This reduce strongly the probability of a contamination of the signal.

The three main non-resonant channels are :

$$\begin{array}{lcll} \psi' & \rightarrow & 3(\pi^+\pi^-)\pi^0 & (3.5 \pm 1.6)\cdot 10^{-3} \\ \psi' & \rightarrow & 2(\pi^+\pi^-)\pi^0 & (3.0 \pm 0.8)\cdot 10^{-3} \\ \psi' & \rightarrow & \pi^+\pi^- K^+ K^- & (1.6 \pm 0.4)\cdot 10^{-3} \end{array}$$

The number of events of such kind is expected to be rather high, of the order $112 \cdot 10^3$ events before. To test the channel, a $50 \cdot 10^3$ events MC sample has been generated. This sample is large enough to decide if there might be a problem or not. The considered channel is $\psi' \to \mu^+\mu^-\pi^+\pi^-$. This channel does not exists it this form, as there is some intermediate kaons and two final neutrinos in the real one. However, this only increase the risks of having events passing the cuts, thus giving a worst case estimation.

Test on MC have shown that none of these 4-body events achieved to pass the selection cut and the kinematical cut. This background will be completely neglected in the latter.

4.8 Significance optimization

Now, as the noise is known, the goal is to choose the best cut on the χ^2 variable returned by the kinematic fit to maximize the number of events, along with having the highest possible purity in the data. As this two objectives are contradictory, one will have to maximize the significance. It is assumed that signal and noise follow normal distributions. The following significance is then used :

$$\mathcal{Z} = \frac{S}{\sqrt{S+N}} \tag{18}$$

The value of $\mathcal{Z}$ is computed for each value of the cut on χ^2, from 5 to 50, incremented by steps of 5. The used MC samples are of 20 times the $\psi' \to \eta J/\psi$ background and one time the full data.

χ^2	Noise	Signal $\cdot 10^3$	$\mathcal{Z}$
5	9.3	12.63	112.3
10	19.3	19.51	139.6
15	25.1	21.59	146.9
20	27.7	22.31	149.3
25	29.6	22.63	150.3
30	31.0	22.83	151.0
35	31.8	22.93	151.3
40	32.4	23.02	151.6
45	32.8	23.07	151.8
50	33.1	23.12	151.9

Given the value of the significance, The choice of a cut on χ^2 lower than 50 is well confirmed. The smallness of noise allow to enlarge this cut at its maximal value since the signal increase much faster than the noise.

5 Results

In a first part, the raw results are presented, and the observable shape distribution is presented for real data and also in the case of Monte-Carlo (MC) simulated events. Then the following explain the way the observable is analyzed, and how the error is computed. Final results are presented and then commented. A last part will conclude with future perspectives and possible amelioration of this experiment.

5.1 Observable

The observable is build with the momenta of the final particles. In order to have the best possible results, the used tracks are the modified ones, meaning the momentum are modified by the 4-body kinematical fitting program. The CP observable, as previously presented is :

$$\mathcal{O} = \hat{p}_{\mu^+} \cdot (\hat{k}_+ - \hat{k}_-)\, \hat{p}_{\mu^+} \cdot \hat{n}$$

Here are the distribution of this CP observable for the signal and for the MC simulated events. The distributions show the same behavior. This is

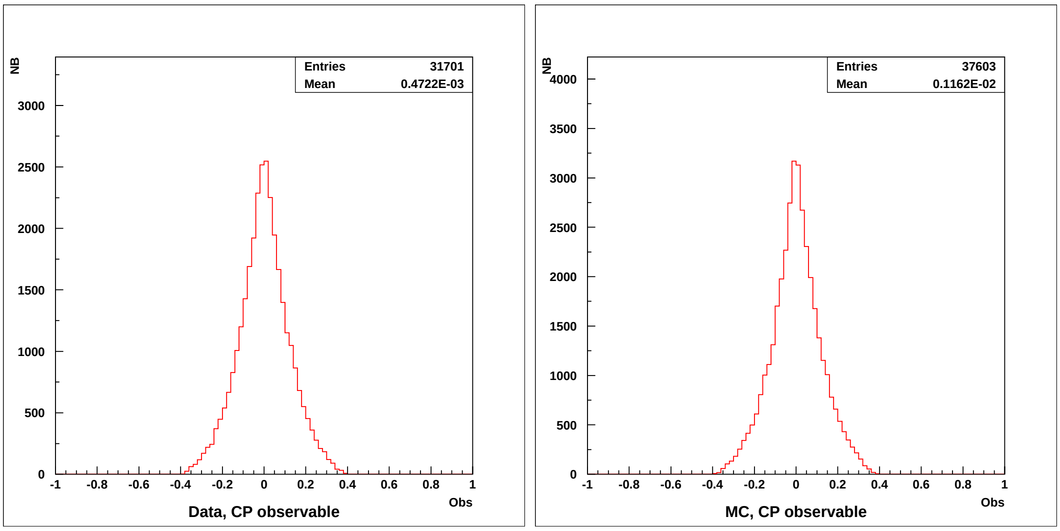

Figure 20: CP Observable for real data and monte carlo sample

nothing unexpected since after the cuts, the two have the same momentum distributions for the final particle. Now, one can compute easily a raw result with the property that our observable can be expressed as :

$$\langle \mathcal{O} \rangle = \frac{N^+ - N^-}{N^+ + N^-} \tag{19}$$

With N^+ and N^- respectively the number of events with $\mathcal{O}$ bigger and lesser than zero. This is the raw value of $\langle\mathcal{O}\rangle$.

5.2 Error calculation and final Observable value

In order to compute the value of $\langle\mathcal{O}\rangle$ in this experiment, the log-likelihood method is used. To do this, first a parameter g is defined as :

$$g = \frac{N^+}{N^+ + N^-} \tag{20}$$

With N^+ and N^- defined in the former. It is now possible to define a binomial probability function depending on the three parameters g, N^+ and N where N is the total number of events.

$$P(N^+) = \frac{N!}{N^+!(N-N^+)!}\, g^{N^+}(1-g)^{N-N^+} \tag{21}$$

The goal is to minimize $-log(P(g)_{N,N^+})$ by varying the parameter g. To do so a simple search of the zero of the derivative is done. This gives a value for the g parameter, which is directly related to the observable by the simple relation :

$$\langle\mathcal{O}\rangle = 1 - 2\cdot g \tag{22}$$

As N^+ and N are rather big integers, it is impossible to perform a direct calculation since it requires taking factorial. Instead, a limited Taylor expansion is used. This expansion is based on the small relative difference between N^+ and N^-, and between g and $\frac{1}{2}$. As long a this differences remain small enough to be neglectable at second order expansion, there is no problem with this methid. The calculations give the following results :

$$\begin{aligned} \langle\mathcal{O}\rangle_{data} &= 1.67 \pm 8.87\cdot 10^{-3} \\ \langle\mathcal{O}\rangle_{MC} &= -6.65 \pm 7.63\cdot 10^{-3} \end{aligned}$$

These are consistent with a zero value, meaning there is no CP violating effects larger than the upper limit given by the error around the value of the observable. Still, it is a useful result as this gives us an upper limit for future measurements. As $\langle\mathcal{O}\rangle$ is related to the electric and chromatic dipole moment of the bound c-quarks, an upper limit can also be given for this value. The calculation allowing to convert the value of the observable into a value for the electric and chromatic dipole moments need some matrix element which is not yet known. Thus it is actually impossible to compute this value.

5.3 Future perspectives and improvements

This experiment can be greatly improved, even with the BESII data sample. The idea is to use other J/ψ decay channels to enhance the total number of $\psi' \to \pi^+\pi^- J/\psi$ transition tested in the experiment. One good possibility for that is given by $J/\psi \to e^+e^-$. As this branching ratio is identical to the muon channel the statistics can be doubled.

In the case of the future BESIII experiment, the gain can be really impressive. Indeed, it is foreseen that it will be able to collect 100 times more data than in BESII. Adding the fact that end-caps are present and that the muon chamber will also cover a larger part of the solid angle, this further increase the data amount by several fold. With such improvements, the value of the CP observable and the electric and chromatic dipole moment of the bounded c-quarks can improved by at least a factor 10. The increase is more likely to be 20 fold given the increased geometrical coverage of the new detector, the better resolution and using the electronic channel.

6 Conclusion

The final conclusion of this work is that if there is any CP violating effects in the studied $\psi' \to \pi^+\pi^- J/\psi$ transition, a lot more data will be needed to probe it. BESIII will provide this opportunity with a statistics of at least two full orders of magnitude better than BESII.

The limit value given by the result of this experiment put an upper limit on the possible amplitude of the CP violating effect, thus is an interesting and useful result.

$$\langle \mathcal{O} \rangle_{data} = \quad 1.67 \pm 8.87 \cdot 10^{-3}$$

The total precision is of the order of $8.87 \cdot 10^{-3}$ which left a rather large place to search deeper. This measurement can be linked to the electric and chromatic dipole moment of the c-quark bound state, and thus is a useful method for getting such kind of informations about very unstable particles. Unlike the neutron, which can be stopped and studied at rest, c-quarks bound states decays cannot be probed directly for EDM. This work shows a possibility to reach a sensitivity in such measurements only limited by data statistics.

Acknowledgment

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