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Polarized radiative Λ_b decays at LHCb

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Abstract

We present a study of decays of the type $\Lambda_b \to \Lambda(X)\gamma$, where $\Lambda(X)$ is a Λ baryon of mass X and spin 1/2 or 3/2. Detailed calculations of decay amplitudes and angular distributions are carried out employing the helicity formalism, and used to derive observables sensitive to new physics. In particular we make use of the initial polarization of the Λ_b baryon to probe the polarization of the photon emitted in $b \to s$ transitions. Such a measurement can be used to probe the chirality of the effective Hamiltonian, and possibly to unveil physics beyond the Standard Model. An estimate of the LHCb sensitivity to this measurement is also given.

Contents

1 Introduction

Polarized radiative Λ_b decays are well suited for probing a large variety of new physics effects, while imposing at the same time important constraints on the Standard Model (SM) at the quantum level. These processes are associated to flavour-changing neutral currents (FCNC) in the $b \to s$ transition.

Up to now, the measured $b \to s\gamma$ decay rate [1, 2] is in good agreement with the SM prediction [3]. However the electromagnetic penguin $b \to s\gamma$ can still be sensitive to new physics effects in the operators and Wilson coefficients that appear in the low energy effective Hamiltonian. The emitted photon is predicted to be mainly left-handed in the SM [4, 5], however right-handed components arise in a variety of new physics models (left-right symmetric models, super-symmetric models with large left-right squark flavor mixing, and models containing new vector-like quarks).

Several methods have been proposed to probe the photon polarization in B meson radiative decays:

- B-$\overline{B}$ interference: CP asymmetries from the interference of mixing and decay in radiative B neutral decays require that both B and $\overline{B}$ decay to a common state, i.e. with the same photon helicity. Therefore if the photon is polarized, as predicted in the SM, the CP asymmetry from the mixing should vanish [4]. The world average for the CP asymmetry (measured from a time dependent analysis of $B \to K_S\pi^0\gamma$ decays) is 0.00 ± 0.28 [6]. The result is consistent with 0, but the errors are still large.

- e^+e^- conversion: conversion electron pairs can be used to probe the photon polarization by measuring the angular correlations between the recoiling K^* and the e^+e^- in $B \to K^*\gamma$ decays, where the photon can be either real (and converts into a e^+e^- pair in the beam pipe) [7] or virtual [8].

- interference of several higher K^* resonances decaying to $K\pi\pi$ gives access to the photon polarization [9]. However from the experimental point of view disentangling the resonance structure is not trivial (see the latest BaBar measurement [10]).

- interference between B decays to the same final state but through different resonances (for instance $B \to K^*(K\gamma)\gamma$, $B \to \eta_c(\gamma\gamma)K$, and $B \to \chi_{c0}(\gamma\gamma)K$ [11].

None of these methods has brought clarifying results up to now, due to the limited statistics at the currently running B factories. However the photon polarization can also be probed in polarized b-baryons decays, which will be produced in large quantities at the LHC. The idea was first proposed in [12, 13], where the decay $\Lambda_b \to \Lambda(1115)\gamma$ has been studied in the framework of Heavy Quark Effective Theory (HQET) [14]. A new observable, the angular asymmetry between the Λ_b spin and the photon momentum, has been proposed. Its measurement, combined with the $\Lambda(1115) \to p\pi$ decay polarization asymmetry, can be used to test the $V - A$ structure of the SM.

However from the experimental point of view the $\Lambda(1115)$ reconstruction is quite delicate since it may traverse a large fraction of the experiment tracking system before decaying to $p\pi$. We will therefore study if decays involving Λ resonances[1], which decay strongly in pK, can be competitive, and if they can be used to improve the experimental sensitivity to the measurement of the photon polarization.

In this note, we focus on radiative Λ_b decays of the type $\Lambda_b \to \Lambda(X)\gamma$, where X is the mass of the Λ baryon. In section 2 we introduce the theoretical framework in which radiative electromagnetic penguins are studied. We use the $\Lambda_b \to \Lambda(1115)\gamma$ decay probability from [13] as a starting point to estimate the branching fractions for decays involving heavier Λ resonances. In section 3 we introduce the photon polarization. In section 4 we extend the results from [13] for the decay $\Lambda_b \to \Lambda(1115)\gamma$ to any decay involving a Λ resonance of spin 1/2 or 3/2. Detailed calculations of angular distributions are carried out in a general way taking advantage of the helicity formalism [15] (see Appendix A). In section 5 we deal with Λ_b production at the LHC, and in section 6 we estimate the LHCb sensitivity to the photon right-handed amplitude based on the measurement of the angular observables previously derived. Detailed studies of the involved errors can be found in Appendix B.

2 Theoretical framework

The decay $\Lambda_b \to \Lambda(X)\gamma$ corresponds at the quark level to the electromagnetic penguin diagram $b \to s\gamma$ (see fig. 1). Long distance contributions, such as W or intermediate meson exchange, have been found to be negligible [12]. The effective Hamiltonian at Leading Order (LO) in α_s can be written as [13]:

$$\mathcal{H}_{\text{eff}} = -4\frac{G_F}{\sqrt{2}}V_{ts}^{*}V_{tb}(C_7 O_7 + C_7' O_7') \tag{1}$$

where G_F is the Fermi constant, C_7 and C_7' are the relevant Wilson coefficients, and the electromagnetic dipole operators take the form:

$$O_7 = \frac{e}{16\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)bF^{\mu\nu}, \quad O_7' = \frac{e}{16\pi^2} m_b \bar{s}\sigma_{\mu\nu}(1-\gamma_5)bF^{\mu\nu}$$

which contain the right- and left-handed projectors $P_R, P_L = \frac{1}{2}(1 \pm \gamma_5)$. The ratio of the corresponding Wilson coefficients $r = \frac{C_7'}{C_7}$ defines the relative strength of the opposite chirality dipole operators. In the SM $\frac{C_7'}{C_7} = \frac{m_s}{m_b}$, hence r is expected to be small [4]. This argument holds as long as $b \to s\gamma$ is a two-body decay, however it cannot be applied to multi-body final states such as $b \to s\gamma$ + gluons. Once QCD corrections are properly included, more operators may contribute to the effective Hamiltonian, and right-handed components contribution to the photon polarization

[1] Status and various parameters of the Λ resonances (above the $\Lambda(1115)$) are given for reference in table 1. These states can be characterized by their spins and decay modes: the ground state $\Lambda(1115)$ has spin 1/2 and mainly decays weakly in $p\pi$ (64% of the times), while heavier Λ baryons decay strongly. The spin ranges from 1/2 to 9/2 for the heaviest known state, while the mass varies from 1405 to 2100 MeV. The first state above the $N\bar{K}$ threshold is the $\Lambda(1520)$.

Table 1: Status of the Λ resonances. The first column contains the name of the state, then its quantum numbers in spectroscopic notation, the width of the resonance, the status, as explained below, and finally the branching fractions for the decay modes $N\overline{K}$ and $\Sigma\pi$. The status and the resonance parameters are estimations from the Particle Data Group [16].

Particle	$L_{I \cdot 2J}$	Γ [MeV]	Status	$N\overline{K}$	$\Sigma\pi$
$\Lambda(1405)$	S_{01}	50	****	-	100%
$\Lambda(1520)$	D_{03}	15.6	****	45%	42%
$\Lambda(1600)$	P_{01}	≈ 150	***	15–30%	10–60%
$\Lambda(1670)$	S_{01}	≈ 35	****	20–30%	25–55%
$\Lambda(1690)$	D_{03}	≈ 60	****	20–30%	20–40%
$\Lambda(1800)$	S_{01}	≈ 300	***	25–40%	seen
$\Lambda(1810)$	P_{01}	≈ 150	***	20–50%	10–40%
$\Lambda(1820)$	F_{05}	≈ 80	****	55–65%	8–14%
$\Lambda(1830)$	D_{05}	≈ 95	****	3–10%	35–75%
$\Lambda(1890)$	P_{03}	≈ 100	****	20–35%	3–10%
$\Lambda(2000)$	???	???	*	???	???
$\Lambda(2020)$	F_{07}	???	*	???	???
$\Lambda(2100)$	G_{07}	≈ 200	****	25–35%	5%
$\Lambda(2110)$	F_{05}	≈ 200	***	5–25%	10–40%
$\Lambda(2325)$	D_{03}	≈ 170	*	???	???
$\Lambda(2350)$	H_{09}	≈ 150	***	$\sim 12\%$	$\sim 10\%$

****	Existence is certain, and properties are at least fairly well explored.
***	Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, BR, *etc.* are not very well determined
**	Evidence of existence is fair.
*	Evidence of existence is poor.

may not be negligible [5]. Unfortunately no explicit calculations exist for Λ_b radiative decays.

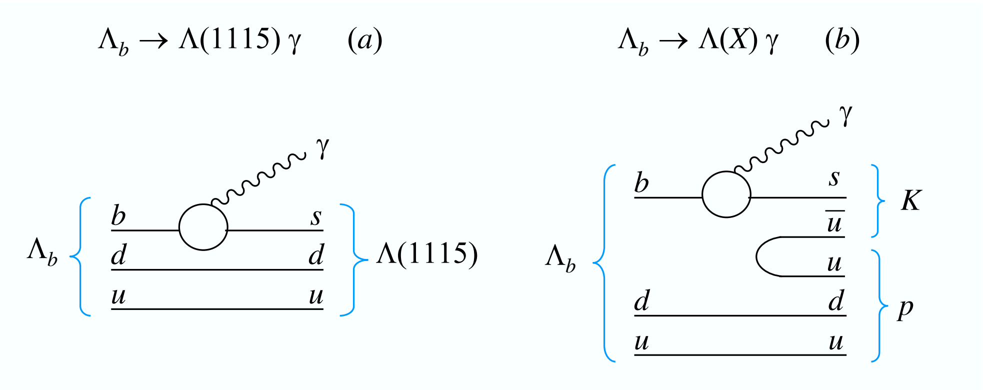

Figure 1: Feynman diagrams for the two decays: $\Lambda_b \to (\Lambda(1115) \to p\pi)\gamma$ (a), $\Lambda_b \to (\Lambda(X) \to pK)\gamma$ (b). In both cases the relevant element is the electromagnetic penguin $b \to s\gamma$.

2.1 Branching ratios

In table 1 we list the properties of the better known $\Lambda(X)$ resonances, many of which can only be crudely estimated at this time, based on data compiled by the Particle Data Group [16]. The table also lists our estimates for the branching fractions for the decays $\Lambda_b \to \Lambda(X)\gamma$, which are based on the kinematic suppression due to the larger mass of the higher resonances given by the factor $(1-m_\Lambda^2/m_{\Lambda_b}^2)^3$ [13]. They do not take into account differences in the form factors, nor a possible spin-dependence of the decay probability, both to be determined by experiment. Judging from recent data on $B \to K^*\gamma$ decays [6] and from dedicated form factor studies on semi-leptonic B and B_s meson decays [17], we may expect these estimates to be correct up to a factor of 2–3 only.

To evaluate the $\Lambda(X) \to pK$ decay probabilities we use the rough $\mathcal{B}(\Lambda(X) \to N\overline{K})$ estimates given in the table and assume equal probabilities for decays to pK^- and $n\overline{K}^0$ from isospin coupling, thereby neglecting possible suppression effects from angular momentum barriers.

Given the above hypothesis, and using the SM estimations for the Wilson coefficients C_7 and C_7', one obtains that the Branching Ratios (BR) for all interesting decay channels (excluding the $\Lambda(1405)$ which is below the $N\overline{K}$ threshold) lie in the range 10^{-5}–10^{-6} (see table 2).

$\Lambda(X)$	$L_{I\cdot 2J}$	Γ (MeV)	$\mathcal{B}_{N\overline{K}}$ (%)	$\mathcal{B}_{\Lambda(X)\gamma}$ (10^{-5})	$\mathcal{B}_{\text{tot}}$ (10^{-5})
$\Lambda(1520)$	D_{03}	15.6	45	5.84	1.31
$\Lambda(1600)$	P_{01}	150	22	5.69	0.65
$\Lambda(1670)$	S_{01}	35	25	5.56	0.69
$\Lambda(1690)$	D_{03}	60	25	5.52	0.69
$\Lambda(1800)$	S_{01}	300	32	5.30	0.84
$\Lambda(1810)$	P_{01}	150	35	5.28	0.92
$\Lambda(1820)$	F_{05}	80	60	5.26	1.57
$\Lambda(1830)$	D_{05}	95	6	5.24	0.15
$\Lambda(1890)$	P_{03}	100	22	5.12	0.56
$\Lambda(2100)$	G_{07}	200	30	4.67	0.70
$\Lambda(2110)$	F_{05}	200	15	4.65	0.34
$\Lambda(2350)$	H_{09}	150	12	4.12	0.28

Table 2: Table of Λ resonances decaying to pK that are established with at least a fair degree of certainty. In the table, $\mathcal{B}_{N\overline{K}} \equiv \mathcal{B}(\Lambda(X) \to N\overline{K})$, $\mathcal{B}_{\Lambda(X)\gamma} \equiv \mathcal{B}(\Lambda_b \to \Lambda(X)\gamma)$, and $\mathcal{B}_{\text{tot}} \equiv \mathcal{B}(\Lambda_b \to \Lambda(X)\gamma \to pK\gamma)$. The values for $\mathcal{B}_{N\overline{K}}$ are estimates based on data compiled in ref. [16], whereas the $\mathcal{B}_{\Lambda(X)\gamma}$ are our estimates derived from simple kinematic suppression (see text).

3 The photon polarization

The aim of the study of polarized radiative Λ_b decays is the measurement of the photon polarization:

$$\alpha_\gamma = \frac{P(\gamma_L) - P(\gamma_R)}{P(\gamma_L) + P(\gamma_R)}$$

where $P(\gamma_L)$ ($P(\gamma_R)$) represents the probability of producing a left(right)-handed photon. At LO in α_s only O_7 and O_7' contribute to the photon polarization:

$$\alpha_\gamma^{\text{LO}} = \frac{1 - |r|^2}{1 + |r|^2} \tag{2}$$

However more operators can give significant contribution to α_γ, if QCD corrections are included [5], such that an experimental determination of the photon asymmetry would only yield an effective ratio r_{eff}. In the following we will set $r_{\text{eff}} = r$ to simplify the notation, but will keep in mind that the relationship between α_γ and r may be more complicated. We furthermore neglect CP violating effects that could appear at next-to-leading order in α_s [13]. We stress the fact that, if a deviation from the naïve SM prediction for the photon polarization is measured, additional theoretical efforts are needed to see if it can still be accommodated in the SM, or if it is a sign of new physics.

It has been shown that the photon polarization can be tested by measuring the proton angular distribution in the decay $\Lambda_b \to (\Lambda(1115) \to p\pi)\gamma$ [12]. If the initial Λ_b is polarized, as it is predicted at the LHC (see section 5), the photon polarization can also be extracted from the photon angular distribution [13]. We will now extend this result to any decay of the type $\Lambda_b \to \Lambda(X)\gamma$ where $\Lambda(X)$ is a Λ resonance of spin 1/2 or 3/2.

4 Angular observables for $\Lambda_b \to \Lambda(X)\gamma$

To work out the final states angular distributions for polarized radiative Λ_b decays we employ the helicity formalism [15]. Detailed calculations can be found in appendix A, where a pedagogical approach to the formalism is given and decays involving a $\Lambda(X)$ resonance with spin 1/2 or 3/2 (which already include the first 8 Λ baryons, as one can see from table 1) are studied. The results are given in terms of the helicity amplitudes characteristic of each decay and of the initial Λ_b polarization. In the first case we will retrieve the angular distributions given in [13]. The same framework can be used to study any $\Lambda_b \to \Lambda(X)\gamma$ decay in a general way. However, if the $\Lambda(X)$ spin is $> 3/2$, the number of the relevant helicity amplitudes is greater than the number of observables, therefore no indication on the photon polarization can be derived, unless new theoretical predictions become available.

4.1 $\Lambda(X)$ spin = 1/2

By integrating the decay probability (App. A, eq. 18) one obtains the following angular distributions for the final states:

$$\frac{d\Gamma}{d\cos\theta_\gamma} \propto 1 - \alpha_\gamma P_{\Lambda_b} \cos\theta_\gamma \tag{3}$$

$$\frac{d\Gamma}{d\cos\theta_p} \propto 1 - \alpha_\gamma \alpha_{p,1/2} \cos\theta_p \tag{4}$$

We remind here that θ_γ is the angle between the Λ_b polarization vector and the photon momentum (measured in the Λ_b rest frame), and θ_p is the angle between the Λ flight direction and the proton momentum (measured in the Λ rest frame). P_{Λ_b} is the initial Λ_b polarization. $\alpha_{p,1/2}$ is the proton asymmetry parameter as defined in appendix A. For the ground state $\Lambda(1115)$, the weak decay parameter $\alpha_{p,1/2}$ is known to 2%, and its value is 0.642 ± 0.013 [16]. For heavier Λ baryons the decay is parity conserving ($\alpha_{p,1/2} = 0$) and the angular distribution of the proton is flat. We indeed find the same results as in [13].

Thus, assuming the Λ_b polarization to be known, one can probe the ratio $|r|$ by measuring the photon and proton angular distributions. The measurement of $|r|$ in two independent ways allows for good sensitivities even if the initial Λ_b polarization is not very well measured. Otherwise combining the two measurements one can obtain the Λ_b polarization, which could also be sensitive to physics beyond the SM. In fact if a discrepancy is found with the value measured in decays of the type $\Lambda_b \to \Lambda_c \ell \nu_\ell X$, non-standard right handed $b \to c$ currents could be present [13].

We remark that for $\Lambda(X)$ resonances that decay into pK, the proton angular distribution is flat, and the photon polarization can only be measured with the photon angular distribution.

4.2 $\boldsymbol{\Lambda(X)}$ spin $= 3/2$

Integrating the decay probability (App. A, eq. 21) over the appropriate solid angle elements, one obtains the photon angular distribution:

$$\frac{d\Gamma}{d\cos\theta_\gamma} \propto 1 - \alpha_{\gamma,3/2} P_{\Lambda_b} \cos\theta_\gamma \tag{5}$$

where:

$$\alpha_{\gamma,3/2} = \frac{\overbrace{|C_{\frac{3}{2},1}|^2 + |C_{-\frac{1}{2},-1}|^2}^{\lambda_\Lambda - \lambda_\gamma = 1/2} - \overbrace{|C_{-\frac{3}{2},-1}|^2 - |C_{\frac{1}{2},1}|^2}^{\lambda_\Lambda - \lambda_\gamma = -1/2}}{|C_{\frac{3}{2},1}|^2 + |C_{-\frac{1}{2},-1}|^2 + |C_{-\frac{3}{2},-1}|^2 + |C_{\frac{1}{2},1}|^2} \tag{6}$$

which defines the asymmetry of Λ_b baryons with different spin projections ($\pm 1/2$) along the decay axis[2]. The C coefficients are the helicity amplitudes characteristic of the decay (see appendix A for the necessary definitions).

We will now disentangle the contributions due to the photon emission and the formation of the $\Lambda(X)$ resonance. We assume that the photon production is independent from the $\Lambda(X)$ state formed in the hadronization process:

$$\alpha_\gamma = \frac{P(\gamma_L) - P(\gamma_R)}{P(\gamma_L) + P(\gamma_R)} = \frac{|C_{-\frac{1}{2},-1}|^2 - |C_{\frac{1}{2},1}|^2}{|C_{-\frac{1}{2},-1}|^2 + |C_{\frac{1}{2},1}|^2} = \frac{|C_{-\frac{3}{2},-1}|^2 - |C_{\frac{3}{2},1}|^2}{|C_{-\frac{3}{2},-1}|^2 + |C_{\frac{3}{2},1}|^2}$$

We furthermore define the ratio of the $\Lambda(X)$ helicity amplitudes:

$$\eta = \frac{|C_{\frac{3}{2},1}|^2}{|C_{\frac{1}{2},1}|^2} = \frac{|C_{-\frac{3}{2},-1}|^2}{|C_{-\frac{1}{2},-1}|^2} \tag{7}$$

Since parity is conserved in strong interactions, the same ratio η can be assumed for positive and negative helicities.

Then (6) becomes

$$\alpha_{\gamma,3/2} = \frac{1-\eta}{1+\eta}\,\alpha_\gamma \tag{8}$$

The photon angular distribution has the same form as (3), with the additional factor $\frac{1-\eta}{1+\eta}$ which takes into account the fact that the $\Lambda(X)$ can now access more helicity states.

The proton angular distribution can be written as:

$$\frac{d\Gamma}{d\cos\theta_p} \propto 1 - \alpha_{p,3/2} \cos^2\theta_p \tag{9}$$

where:

$$\alpha_{p,3/2} = \frac{\overbrace{|C_{\frac{3}{2},1}|^2 + |C_{-\frac{3}{2},-1}|^2}^{|\lambda_\Lambda| = 3/2} - \overbrace{|C_{\frac{1}{2},1}|^2 - |C_{-\frac{1}{2},-1}|^2}^{|\lambda_\Lambda| = 1/2}}{|C_{\frac{3}{2},1}|^2 + |C_{-\frac{3}{2},-1}|^2 + \frac{1}{3}(|C_{-\frac{1}{2},-1}|^2 + |C_{\frac{1}{2},1}|^2)} = \frac{\eta - 1}{\eta + \frac{1}{3}} \tag{10}$$

[2]In a 2-body decay, the decay axis can be defined as the direction of the daughter particles in the mother rest frame.

The proton polar angle distribution is symmetric around $\cos\theta_p = 0$, as expected in a strong decay, but it can still be used to extract the value of η. Since no theoretical predictions exist for η, we will now study the three cases (see also fig. 2):

$\eta \ll 1$ The fraction of $\Lambda(X)$ with helicity 1/2 dominates. In this case $\alpha_{p,3/2} \simeq -3$ and $\alpha_{\gamma,3/2} \simeq \alpha_{\gamma,1/2}$.

$\eta \simeq 1$ About the same amount of $\Lambda(X)$ with helicity 3/2 and 1/2 are produced. The parameters $\alpha_{p,3/2} \simeq \alpha_{\gamma,3/2} \simeq 0$, and the angular distributions (5) and (9) are flat.

$\eta \gg 1$ The fraction of $\Lambda(X)$ with helicity 3/2 dominates. In this case $\alpha_{p,3/2} \simeq 1$ and $\alpha_{\gamma,3/2} \simeq -\alpha_{\gamma,1/2}$.

To summarize, if $\eta \simeq 1$ no asymmetry will be found in the photon angular distribution, and no measurement of $|r|$ is possible. On the other hand, if a photon asymmetry is measured, combining the measurement of $\alpha_{\gamma,3/2}$ and the proton parameter $\alpha_{p,3/2}$ one can obtain the photon polarization :

$$\alpha_\gamma = \frac{1}{2}\,\alpha_{\gamma,3/2}\left(1 - \frac{3}{\alpha_{p,3/2}}\right) \tag{11}$$

and the ratio $|r|$ can be probed as in spin 1/2 $\Lambda(X)$ decays. Note that an independent measurement of the Λ_b polarization is necessary for the extraction of $\alpha_{\gamma,3/2}$ from the measured photon angular distribution.

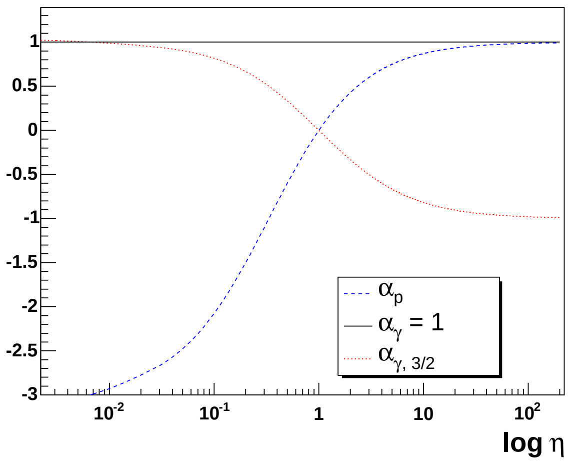

Figure 2: The asymmetry parameters $\alpha_{\gamma,3/2}$ (dotted red) and $\alpha_{p,3/2}$ (dashed blue) as a function of $\log\eta$, for the SM expectation of α_γ (solid black) $= 1$.

5 Λ_b production at the LHC

In the production reaction $pp \to \Lambda_b X$, the Λ_b is expected to be polarized in a similar way to that of strange hyperons [19]. Since parity is conserved in strong interactions, the longitudinal polarization is indeed suppressed and the Λ_b are transversally polarized with respect to their production plane because of a QCD mechanism at the parton level [20].

The sensitivity of the $|r|$ measurement to the value of the Λ_b polarization will be estimated to be of the order of a few percent (see section 6.2). We also remind that $|r|$ can still be extracted from the proton angular distribution in the $\Lambda_b \to \Lambda(1115)\gamma$ decay even in case of unpolarized Λ_b. In the following, we will fix, unless otherwise specified, the Λ_b polarization to be 20%, as predicted in [20]. The ATLAS experiment plans to measure it with a statistical precision of 1% [21] in the decay $\Lambda_b \to \Lambda(1115)J/\Psi$. This decay is also under study at LHCb [22].

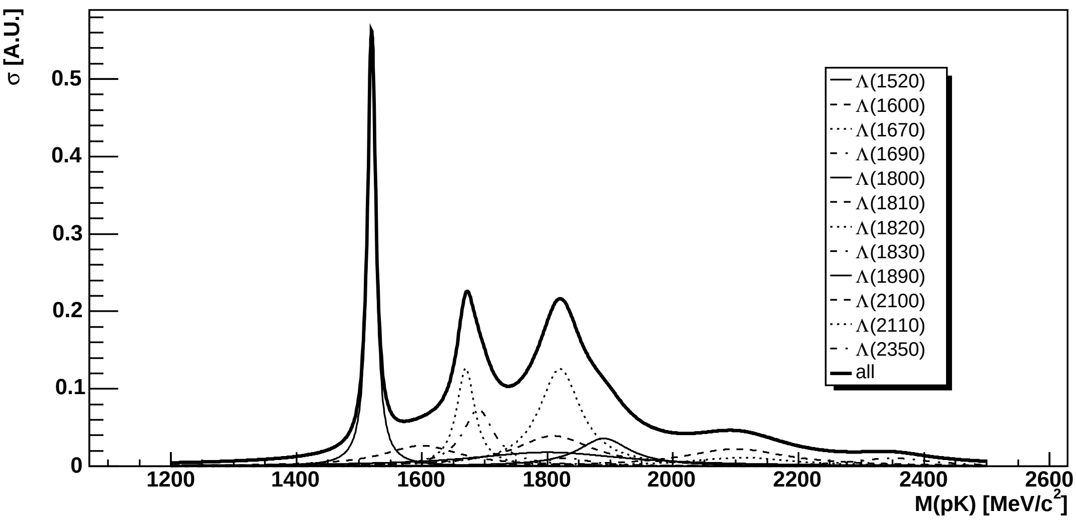

Figure 3: Invariant pK mass spectrum from $\Lambda_b \to \Lambda(X)\gamma$ decays. The contribution of each resonance and the overall cross section are shown. Only resonances with an experimental status of at least *** (with reference to table 1) are shown. The Breit Wigner distribution has been used for the decays $\Lambda(X) \to pK$. $\Lambda(X)$ (averaged) parameters from [16]. The branching fractions used to produce this spectrum are listed in table. 2.The $\Lambda_b \to \Lambda(X)\gamma$ decays branching ratios are assumed to be independent of the pK mass (see section 2.1).

6 The decay $\Lambda_b \to \Lambda(X)\gamma$ at LHCb

We will now consider in the LHCb environment the following decays:

$$\Lambda_b \to (\Lambda(1115) \to p\pi)\gamma, \qquad \Lambda_b \to (\Lambda(X) \to pK)\gamma$$

where the $\Lambda(X)$ has spin 1/2 or 3/2. From the experimental point of view, to observe these decays one has to disentangle the relevant $\Lambda(X)$ resonance from the others. The pK invariant mass spectrum from radiative Λ_b decays presents three peaks in the region of the Λ resonances (see fig. 3). The first peak belongs to the $\Lambda(1520)$ (spin 3/2) alone, therefore this resonance should be the easiest to observe. The second peak is given by both the $\Lambda(1670)$ (spin 1/2) and the $\Lambda(1690)$ (spin 3/2). We assume that the different angular distributions will allow us to distinguish the two resonances. The third peak is formed by the contribution of the $\Lambda(1810)$ (spin 1/2), the $\Lambda(1820)$ (spin 5/2), the $\Lambda(1830)$ (spin 5/2) and the $\Lambda(1890)$ (spin 3/2). The dominant contribution to this peak is given by the $\Lambda(1820)$, which has spin 5/2, therefore we will not consider it further.

6.1 Event yields estimation

The study of the reconstruction of radiative Λ_b decays at LHCb is beyond the scope of this note and has been addressed in [24]. We give here the expected annual event yields for the decays of interest:

$$S_{\text{year}}(\Lambda_b \to (\Lambda(1115) \to p\pi)\gamma) \sim 675$$

$$S_{\text{year}}(\Lambda_b \to (\Lambda(1520) \to pK)\gamma) \sim 4270$$

$$S_{\text{year}}(\Lambda_b \to (\Lambda(1670) \to pK)\gamma) \sim 2250$$

$$S_{\text{year}}(\Lambda_b \to (\Lambda(1690) \to pK)\gamma) \sim 2250$$

6.2 Sensitivity to the photon polarization measurement

To estimate LHCb's statistical sensitivity to the ratio $|r|$ we need to consider the various contributions to its measurement. The procedure applied to estimate the statistical error used throughout this section is given in appendix B. Detector resolution effects have been dealt with in [24] and found to be negligible.

We will use here the expected event yields given above, and furthermore fix the polarization of the Λ_b to a conservative estimation, $P_{\Lambda_b} = 0.20 \pm 0.01$.

The values of $|r|$ that can be probed at 3 sigma significance after 1 and 5 years at LHCb can be read off from fig. 4 (left plots) for various radiative Λ_b decay modes. We remind here that in $\Lambda_b \to \Lambda(1115)\gamma$ decays, $|r|$ can be measured independently from both the photon and the proton asymmetries. The statistical uncertainties on the two asymmetries A_γ and A_p being similar (the values of P_{Λ_b} and α_p and their errors are of the same order of magnitude), one can largely compensate the lower annual yield with the measurements of both asymmetries. As a result, this decay gives in fact the best sensitivity to probe $|r|$. The right-handed component of the photon polarization can be probed down to 20% at 3 sigma significance after 1 year of running. Increasing the statistics by a factor 5 gives an additional 5% improvement.

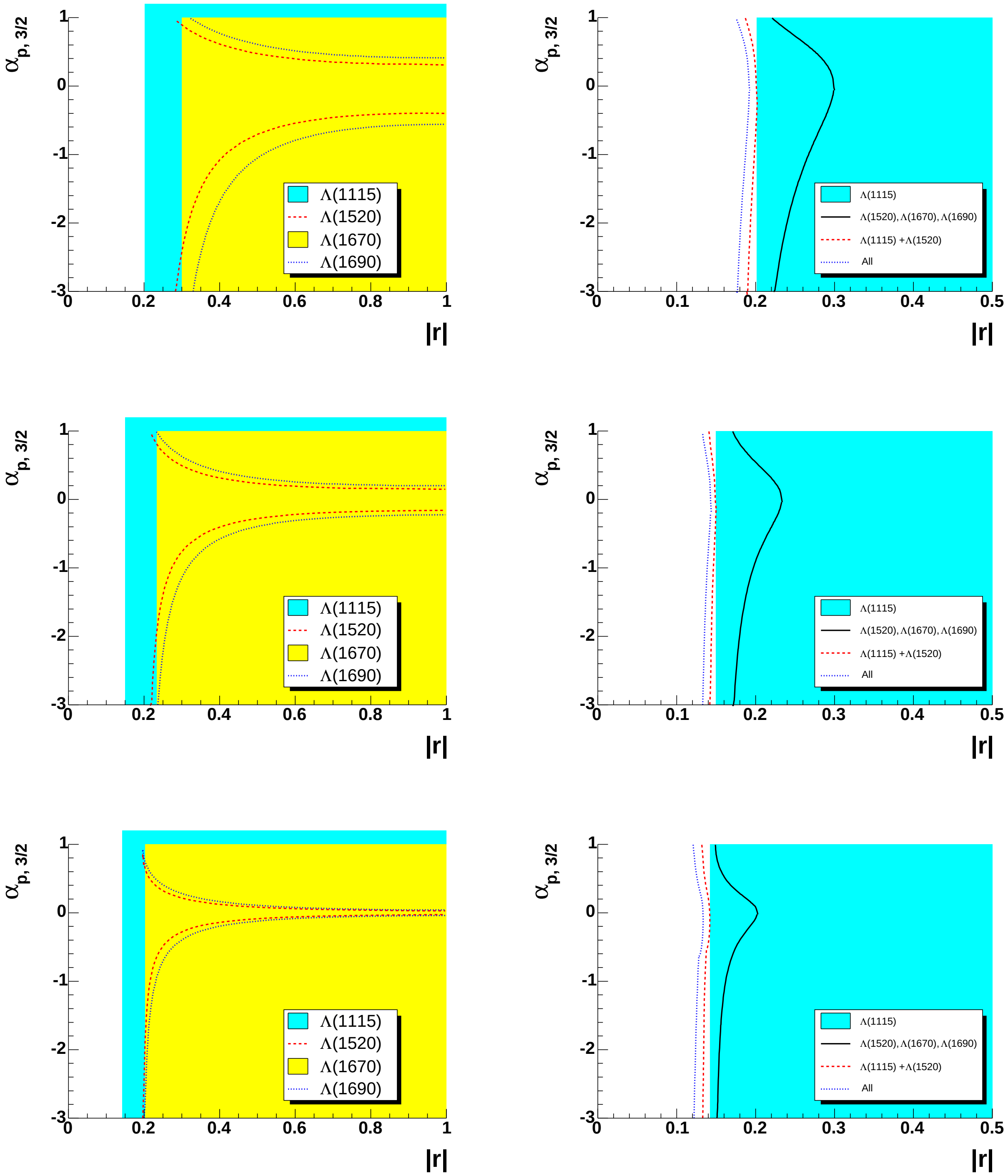

Figure 4: Experimental reach for $|r|$ (as a function of $\alpha_{p,3/2}$ for decays involving a 3/2 spin $\Lambda(X)$), obtained by averaging $|r|$ over CP conjugate decays in the limit of no CP violation. The plots show the values of $|r|$ that can be probed at 3 sigma significance (ranges are to be read starting from the curves and ending at 1) in single (left) and combined (right) measurements of several decay modes. The sensitivities are calculated for 1 (top) and 5 (middle and bottom) years of data taking at LHCb. The Λ_b polarization has been fixed to $(20 \pm 1)\%$ for the top and middle plots, and to $(100 \pm 5)\%$ in the bottom plots.

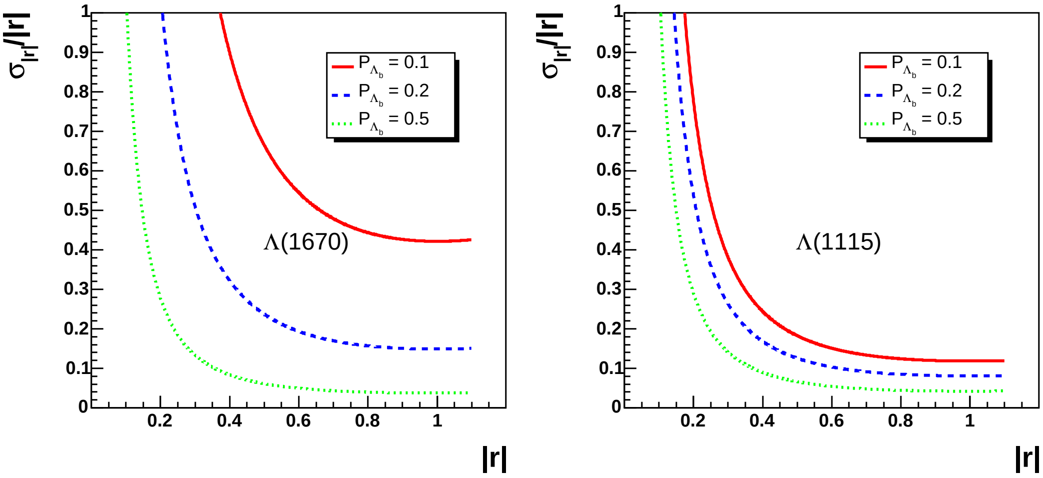

Figure 5: Relative statistical error in $|r|$ as a function of $|r|$ obtained by averaging over CP conjugate decays in the limit of no CP violation, for different values of the Λ_b polarization P_{Λ_b}: 0.1 (solid), 0.2 (dashed), 0.5 (dotted) for 1 year of data taking at LHCb for the decays $\Lambda_b \to \Lambda(1115)\gamma$ (right) and $\Lambda_b \to \Lambda(1670)\gamma$ (left).

Decays involving the $\Lambda(X)$ resonances can become competitive if the proton asymmetry $\alpha_{p,3/2}$ is large enough to allow a clean measurement of the angular distribution of interest. Also increasing the statistics helps to sharpen the significance curves, as can be seen by comparing the two curves after 1 and 5 years of running. In this case the sensitivity to the $|r|$ measurement improves by 5–10 % (according to the resonance) after 5 years.

The possible scenarios obtained by combining[3] the various measurements are (see right plots in fig. 4):

- Only the decay $\Lambda_b \to \Lambda(1115)\gamma$ is measured. The range of $|r|$ that can be probed is given as a reference.

- Only decays involving a strong $\Lambda(X)$ resonances are measured. This is the worst case. With respect to the measurement of the $\Lambda(1115)$ alone, the range of $|r|$ that can be probed is worse by a few percent, depending on the value of $\alpha_{p,3/2}$.

- Decays involving the $\Lambda(1115)$ and $\Lambda(1520)$ are measured (disentangling the $\Lambda(1670)$ and $\Lambda(1690)$ not possible). The sensitivity on the $|r|$ measurement can be improved by 1% at the most.

- All decays are measured and disentangled. The improvement on the $|r|$ sensitivity is of the order of 2% at most.

[3]We assume here that $\alpha_{p,3/2}$ has the same value for the $\Lambda(1520)$ and the $\Lambda(1690)$ resonances.

The sensitivity of the measurement of $|r|$ to the value of the Λ_b polarization is shown in fig. 5 for $\Lambda_b \to \Lambda(1115)\gamma$ and $\Lambda_b \to \Lambda(1670)\gamma$ decays. Combining the measurement of A_p and A_γ one can reduce quite well the dependence on P_{Λ_b}, while if only A_γ is measured, a polarization of at least 20% is needed to have a good sensitivity already after 1 year of data taking. On the other hand a large value of the Λ_b polarization (see bottom plots in fig. 4 where $P_{\Lambda_b} = 1.00 \pm 0.05$) can improve the range of $|r|$ which can be probed by at most a few percent, but reduces the dependence on $\alpha_{p,3/2}$.

7 Conclusions

The sensitivity to NP of polarized radiative decays of the type $\Lambda_b \to \Lambda(X)\gamma$ has been studied. In particular, the results obtained in [13] for decays involving a $\Lambda(1115)$ have been extended to any $\Lambda(X)$ resonance of spin 1/2 or 3/2. A pedagogical approach to the study of angular distributions employing the helicity formalism has been given, and the same framework can be used to study any 2-body radiative decay in a general way. The obtained results have been used to study LHCb performances for these decays.

The main conclusion is that, assuming a Λ_b polarization of at least 20%, LHCb can measure the right-handed component of the photon polarization down to 15% at 3σ significance after 5 years of running from $\Lambda_b \to \Lambda(1115)\gamma$ decays. The additional contribution from the $\Lambda(X)$ resonances to the measurable range has been estimated to be 2% at most. However if the decay involving the $\Lambda(1115)$ is not experimentally accessible (due for instance to failures of the tracking system or to trigger inefficiencies), the right-handed component of the photon polarization can still be measured from the decays involving the $\Lambda(1520)$, $\Lambda(1670)$, $\Lambda(1690)$ (assuming that the last two can actually be disentangled). In this case, the sensitivity is worse by 5% with respect to the measurement of the $\Lambda(1115)$ alone.

The dependence of the photon polarization sensitivity on the initial Λ_b polarization (in the range P_{Λ_b}= 20–100%) has been found to be of the order of a few percent.

To conclude, we find that the decay involving the $\Lambda(1115)$ is the most promising, despite the lower event yield for this channel. This is especially due to the fact that in this case one can measure the photon polarization in two independent ways (which also makes it more robust against our naïve estimate of the Λ_b polarization).

A Helicity formalism for $\Lambda_b \to \Lambda(X)\gamma$ decays

In the helicity formalism, the relevant quantum numbers are the spin of the particles involved, and the possible symmetries of the decay. Hence we start from the consideration that the decays $\Lambda_b \to (\Lambda(1115) \to p\pi)\gamma$ and $\Lambda_b \to (\Lambda(X) \to pK)\gamma$ can be treated as a single decay of the type $\Lambda_b \to (\Lambda(X) \to ph)\gamma$, since the pion and the kaon have both spin 0 (so that both helicities are equal to 0).

The difference between the two decays consists in the fact that the decay $\Lambda(1115) \to p\pi$ is parity violating, while the decay $\Lambda(X) \to pK$ conserves parity. Also cases with different spins of the Λ baryon will be treated separately.

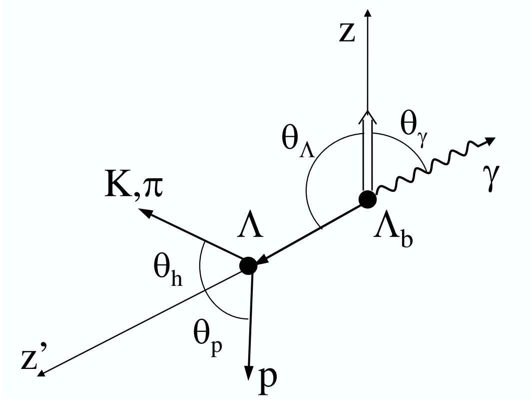

Figure 6: Angles definition for the decay $\Lambda_b \to \Lambda(X)\gamma$. The angles θ_Λ and θ_γ are measured in the Λ_b rest frame, while the angles θ_h and θ_p are measured in the $\Lambda(X)$ rest frame.

As a common framework, and referring to fig. 6, we can define:

- the z axis: the arbitrarily defined spin quantization axis (which we shall take parallel to the Λ_b spin);
- the z' axis: the direction of the $\Lambda(X)$ momentum;
- the angles (θ_Λ,ϕ_Λ) and (θ_γ,ϕ_γ) of the $\Lambda(X)$ and photon momenta with respect to the z axis, measured in the rest frame of the Λ_b;
- the angles (θ_p,ϕ_p) and (θ_h,ϕ_h) of the proton and the pion/kaon momenta with respect to the z' axis, measured in the rest frame[4] of the $\Lambda(X)$.

Thus:

- the Λ_b has angular momentum $J = 1/2$ and spin projection $M = \pm 1/2$ along the z axis;
- the $\Lambda(X)$ has angular momentum J_Λ and helicity $\lambda_\Lambda = M_\Lambda$, defined as its spin projection along z';

[4]We note that the azimuthal angles ϕ_i are the same in both rest frames (of the Λ_b and $\Lambda(X)$).

- the final state particles have helicities λ_γ, λ_p and $\lambda_h = 0$ (where h is π or K according to the decay).

The amplitude probability can be written as:

$$A = \sum_{\lambda_\Lambda} D^{s_\Lambda *}_{\lambda_\Lambda,\lambda_p}(\phi_p, \theta_p, -\phi_p) D^{J*}_{M,\lambda_\Lambda-\lambda_\gamma}(\phi_\Lambda, \theta_\Lambda, -\phi_\Lambda) C_{\lambda_\Lambda,\lambda_\gamma} E_{\lambda_p} \tag{12}$$

where s_Λ is the $\Lambda(X)$ spin and the quantities C and E are the helicity amplitudes of the two decays (respectively $\Lambda_b \to \Lambda(X)\gamma$ and $\Lambda(X) \to ph$). They are independent of the decay angles $(\theta_\Lambda, \phi_\Lambda, \theta_p, \phi_p)$, and of the spin projections M_λ and M.

For the strong decay $\Lambda(X) \to pK$ we can use parity to simplify eq. 12:

$$E_{\lambda_p} = \eta_\lambda \eta_p \eta_k (-1)^{J_\lambda - J_k - J_p} E_{-\lambda_p}$$

where η and J are respectively the parity and spin of the particles [25]. The relevant quantum numbers are $J^P(p) = 1/2^+$ and $J^P(K) = 0^-$, whereas the Λ spin and parity can be found in table 1. According to the Λ parity, we therefore obtain,

$$E_{\lambda_p} = -\eta_\lambda (-1)^{J_\lambda - \frac{1}{2}} E_{-\lambda_p} = \pm E_{-\lambda_p}$$

To write the decay probability we still need to take into account the Λ_b polarization. We then introduce the polarization density matrix ρ:

$$\begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix}$$

where the index $+$ is related to the positive value of the spin projection $m = +1/2$ and the index $-$ indicates the negative one, $m = -1/2$; i.e. ρ_{++}=$\rho_{\frac{1}{2},\frac{1}{2}}$, etc.

It can be shown that if we are not interested in correlations between the production mechanism and the decays, the density matrix non-diagonal components are averaged out [26]:

$$\rho_{mm'} = \delta_{mm'} \rho_{mm'}$$

Since $\mathrm{Tr}\rho = \rho_{++} + \rho_{--} = 1$ the Λ_b polarization can be written as:

$$P_{\Lambda_b} = \rho_{++} - \rho_{--} \tag{13}$$

The decay probability finally becomes:

$$w = \sum_{M,\lambda_\gamma,\lambda_p} \rho_{MM} |A|^2 \tag{14}$$

where we summed over the final helicities since the experiment does not measure them. The sum over λ_h has been dropped since $\lambda_h = 0$.

Detailed calculations of the final-state angular distributions will be given in the following, separately for $\Lambda(X)$ baryons of spin J_Λ equal to 1/2 and 3/2.

A.1 The case $J_\Lambda = \frac{1}{2}$

Let us consider first the decay $\Lambda_b \to \Lambda(X)\gamma$. The relevant quantum numbers are:

	Λ_b	$\Lambda(X)$	γ
J	$\frac{1}{2}$	$\frac{1}{2}$	1
λ	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	± 1

The allowed helicity combinations are:

λ_Λ	λ_γ	$\lambda_\Lambda - \lambda_\gamma$
$+1/2$	$+1$	$-1/2$
$-1/2$	-1	$+1/2$

The helicity amplitudes are then given by the following 2×2 matrix:

$$C_{\lambda_\Lambda,\lambda_\gamma} = \begin{pmatrix} C_{\frac{1}{2},1} & C_{\frac{1}{2},-1} \\ C_{-\frac{1}{2},1} & C_{-\frac{1}{2},-1} \end{pmatrix} = C_{\lambda_\Lambda-\lambda_\gamma} = \begin{pmatrix} C_{-\frac{1}{2}} & 0 \\ 0 & C_{\frac{1}{2}} \end{pmatrix}$$

The amplitude probability (12) can be rewritten as:

$$A = \sum_\lambda D^{\frac{1}{2}*}_{\lambda_\Lambda,\lambda_p}(\phi_p,\theta_p,-\phi_p) D^{\frac{1}{2}*}_{M,\lambda}(\phi_\Lambda,\theta_\Lambda,-\phi_\Lambda) C_\lambda E_{\lambda_p} \tag{15}$$

where $\lambda = \lambda_\Lambda - \lambda_\gamma$.

We can use the fact that $C_{\lambda_\Lambda,\lambda_\gamma} C^*_{\lambda'_\Lambda,\lambda_\gamma} = \delta_{\lambda_\Lambda,\lambda'_\Lambda}|C_\lambda|^2$ to evaluate:

$$\begin{aligned} |A|^2 &= \sum_\lambda \sum_{\lambda'} D^{\frac{1}{2}*}_{\lambda_\Lambda,\lambda_p} D^{\frac{1}{2}*}_{M,\lambda} C_\lambda E_{\lambda_p} D^{\frac{1}{2}}_{\lambda'_\Lambda,\lambda_p} D^{\frac{1}{2}}_{M,\lambda'} C^*_{\lambda'} E^*_{\lambda_p} \\ &= \sum_\lambda |D^{\frac{1}{2}}_{\lambda_\Lambda,\lambda_p}|^2 |D^{\frac{1}{2}}_{M,\lambda}|^2 |C_\lambda|^2 |E_{\lambda_p}|^2 \end{aligned}$$

The decay probability (14) becomes (the sum over λ_γ has been dropped since it is related to λ_Λ):

$$\begin{aligned} w_{\frac{1}{2}} &= \sum_{M,\lambda_p} \rho_{MM}|A|^2 = \sum_{M,\lambda_p,\lambda} \rho_{MM} |D^{\frac{1}{2}}_{\lambda_\Lambda,\lambda_p}|^2 |D^{\frac{1}{2}}_{M,\lambda}|^2 |C_\lambda|^2 |E_{\lambda_p}|^2 \\ &= \sum_{\lambda_p,\lambda} |C_\lambda|^2 |E_{\lambda_p}|^2 |D^{\frac{1}{2}}_{\lambda_\Lambda,\lambda_p}|^2 \left[\rho_{++}|D^{\frac{1}{2}}_{\frac{1}{2},\lambda}|^2 + \rho_{--}|D^{\frac{1}{2}}_{-\frac{1}{2},\lambda}|^2\right] \end{aligned}$$

Let us now add some considerations on the second decay $\Lambda(X) \to p\,h$, where we have:

	$\Lambda(X)$	p	x
J	$\frac{1}{2}$	$\frac{1}{2}$	0
λ	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	0

In this case the helicity matrix takes the simple form:

$$E_{\lambda_p} = \left(\begin{array}{c} E_{\frac{1}{2}} \\ E_{-\frac{1}{2}} \end{array} \right)$$

Adding in, we obtain for the decay probability:

$$\begin{aligned} w_{\frac{1}{2}} &= \sum_{\lambda} |C_\lambda|^2 \left[|E_{\frac{1}{2}}|^2 |D^{\frac{1}{2}}_{\lambda_\Lambda,\frac{1}{2}}|^2 + |E_{-\frac{1}{2}}|^2 |D^{\frac{1}{2}}_{\lambda_\Lambda,-\frac{1}{2}}|^2 \right] \left[\rho_{++} |D^{\frac{1}{2}}_{\frac{1}{2},\lambda}|^2 + \rho_{--} |D^{\frac{1}{2}}_{-\frac{1}{2},\lambda}|^2 \right] \\ &= |C_{\frac{1}{2}}|^2 \left[|E_{\frac{1}{2}}|^2 |d^{\frac{1}{2}}_{-\frac{1}{2},\frac{1}{2}}(\theta_p)|^2 + |E_{-\frac{1}{2}}|^2 |d^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta_p)|^2 \right] \left[\rho_{++} |d^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta_\Lambda)|^2 + \rho_{--} |d^{\frac{1}{2}}_{-\frac{1}{2},\frac{1}{2}}(\theta_\Lambda)|^2 \right] \\ &\quad + |C_{-\frac{1}{2}}|^2 \left[|E_{\frac{1}{2}}|^2 |d^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta_p)|^2 + |E_{-\frac{1}{2}}|^2 |d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_p)|^2 \right] \left[\rho_{++} |d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_\Lambda)|^2 + \rho_{--} |d^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta_\Lambda)|^2 \right] \end{aligned}$$

where we used the fact that:

$$D^j_{m,m'}(\alpha,\beta,\gamma) = e^{i\alpha m'} d^j_{m,m'}(\beta) e^{-i\gamma m} \tag{16}$$

Using:

$$d^J_{M',M}(\theta) = (-1)^{M-M'} d^J_{M,M'}(\theta) = d^J_{-M,-M'}(\theta)$$

and

$$\begin{aligned} d^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta) = d^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta) &= \cos\frac{\theta}{2} \\ d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta) = d^{\frac{1}{2}}_{-\frac{1}{2},\frac{1}{2}}(\theta) &= -\sin\frac{\theta}{2} \end{aligned}$$

we can further simplify:

$$\begin{aligned} w_{\frac{1}{2}} &= |C_{\frac{1}{2}}|^2 \left(\rho_{++} \cos^2\frac{\theta_\Lambda}{2} + \rho_{--} \sin^2\frac{\theta_\Lambda}{2} \right) \left(|E_{\frac{1}{2}}|^2 \sin^2\frac{\theta_p}{2} + |E_{-\frac{1}{2}}|^2 \cos^2\frac{\theta_p}{2} \right) \\ &\quad + |C_{-\frac{1}{2}}|^2 \left(\rho_{++} \sin^2\frac{\theta_\Lambda}{2} + \rho_{--} \cos^2\frac{\theta_\Lambda}{2} \right) \left(|E_{\frac{1}{2}}|^2 \cos^2\frac{\theta_p}{2} + |E_{-\frac{1}{2}}|^2 \sin^2\frac{\theta_p}{2} \right) \end{aligned}$$

We now remark that the expression:

$$A\cos^2\frac{\theta}{2} + B\sin^2\frac{\theta}{2} = A\frac{1+\cos\theta}{2} + B\frac{1-\cos\theta}{2} \propto \frac{1}{2}\left(1 + \frac{A-B}{A+B}\cos\theta\right)$$

then the transition probability becomes:

$$\begin{aligned} w_{\frac{1}{2}} &\propto |C_{\frac{1}{2}}|^2 \left(1 + \frac{\rho_{++} - \rho_{--}}{\rho_{++} + \rho_{--}} \cos\theta_\Lambda \right) \left(1 - \frac{|E_{\frac{1}{2}}|^2 - |E_{-\frac{1}{2}}|^2}{|E_{\frac{1}{2}}|^2 + |E_{-\frac{1}{2}}|^2} \cos\theta_p \right) \\ &\quad + |C_{-\frac{1}{2}}|^2 \left(1 - \frac{\rho_{++} - \rho_{--}}{\rho_{++} + \rho_{--}} \cos\theta_\Lambda \right) \left(1 + \frac{|E_{\frac{1}{2}}|^2 - |E_{-\frac{1}{2}}|^2}{|E_{\frac{1}{2}}|^2 + |E_{-\frac{1}{2}}|^2} \cos\theta_p \right) \end{aligned} \tag{17}$$

We can now define the following quantities, which depend on the helicity amplitudes:

$$\text{Photon asymmetry:} \quad \alpha_{\gamma,1/2} = \frac{|C_{\frac{1}{2}}|^2 - |C_{-\frac{1}{2}}|^2}{|C_{\frac{1}{2}}|^2 + |C_{-\frac{1}{2}}|^2}$$

$$\text{Proton asymmetry:} \quad \alpha_{p,1/2} = \frac{|E_{\frac{1}{2}}|^2 - |E_{-\frac{1}{2}}|^2}{|E_{\frac{1}{2}}|^2 + |E_{-\frac{1}{2}}|^2}$$

We remark that $\alpha_{\gamma,1/2}$ represents the photon polarization which we defined as:

$$\alpha_\gamma = \frac{P(\gamma_L) - P(\gamma_R)}{P(\gamma_L) + P(\gamma_R)} = \alpha_{\gamma,1/2}$$

In case of parity conservation in the decay $\Lambda(X) \to p\ K$, $\alpha_{p,1/2} = 0$.

Inserting the asymmetries defined above, and the Λ_b polarization (13), into the transition probability, one finally obtains:

$$\begin{aligned} w_{\frac{1}{2}} &\propto |C_{\frac{1}{2}}|^2 (1 + P_{\Lambda_b} \cos\theta_\Lambda) \left(1 - \alpha_{p,1/2} \cos\theta_p\right) + |C_{-\frac{1}{2}}|^2 (1 - P_{\Lambda_b} \cos\theta_\Lambda) \left(1 + \alpha_{p,1/2} \cos\theta_p\right) \\ &\propto (1 - \alpha_{p,1/2} P_{\Lambda_b} \cos\theta_p \cos\theta_\Lambda)(|C_{\frac{1}{2}}|^2 + |C_{-\frac{1}{2}}|^2) + (P_{\Lambda_b} \cos\theta_\Lambda - \alpha_{p,1/2} \cos\theta_p)(|C_{\frac{1}{2}}|^2 - |C_{-\frac{1}{2}}|^2) \\ &\propto 1 - \alpha_{p,1/2} P_{\Lambda_b} \cos\theta_p \cos\theta_\Lambda - \alpha_\gamma(\alpha_{p,1/2} \cos\theta_p - P_{\Lambda_b} \cos\theta_\Lambda) \qquad (18) \end{aligned}$$

A.2 The case $J_\Lambda = \frac{3}{2}$

Increasing the spin of the $\Lambda(X)$ baryon means increasing the number of possible helicity combinations. For $J_\Lambda = 3/2$, the $\Lambda(X)$ helicity can assume the values $\pm 1/2, \pm 3/2$. The allowed helicity combinations become:

λ_Λ	λ_γ	$\lambda_\Lambda - \lambda_\gamma$
$+3/2$	$+1$	$+1/2$
$+1/2$	$+1$	$-1/2$
$-1/2$	-1	$+1/2$
$-3/2$	-1	$-1/2$

The helicity amplitudes are given by the following 4×2 matrix:

$$C_{\lambda_\Lambda,\lambda_\gamma} = \begin{pmatrix} C_{\frac{3}{2},1} & 0 \\ C_{\frac{1}{2},1} & 0 \\ 0 & C_{-\frac{1}{2},-1} \\ 0 & C_{-\frac{3}{2},-1} \end{pmatrix}$$

The decay amplitude (12) becomes:

$$A = \sum_{\lambda_\Lambda} D^{\frac{3}{2}*}_{\lambda_\Lambda,\lambda_p}(\phi_p, \theta_p, -\phi_p) D^{\frac{1}{2}*}_{M,\lambda_\Lambda-\lambda_\gamma}(\phi_\Lambda, \theta_\Lambda, -\phi_\Lambda) C_{\lambda_\Lambda,\lambda_\gamma} E_{\lambda_p} \qquad (19)$$

The squared amplitude is then:

$$\begin{aligned} |A|^2 &= \sum_{\lambda_\Lambda} \sum_{\lambda'_\Lambda} D^{\frac{3}{2}*}_{\lambda_\Lambda,\lambda_p} D^{\frac{3}{2}}_{\lambda'_\Lambda,\lambda_p} D^{\frac{1}{2}*}_{M,\lambda_\Lambda-\lambda_\gamma} D^{\frac{1}{2}}_{M,\lambda'_\Lambda-\lambda_\gamma} C_{\lambda_\Lambda,\lambda_\gamma} C^*_{\lambda'_\Lambda,\lambda_\gamma} |E_{\lambda_p}|^2 \\ &= \sum_{\lambda_\Lambda,\lambda'_\Lambda} d^{\frac{3}{2}}_{\lambda_\Lambda,\lambda_p}(\theta_p) d^{\frac{3}{2}}_{\lambda'_\Lambda,\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,\lambda_\Lambda-\lambda_\gamma}(\theta_\Lambda) d^{\frac{1}{2}}_{M,\lambda'_\Lambda-\lambda_\gamma}(\theta_\Lambda) e^{i(\phi_\Lambda+\phi_p)(\lambda'_\Lambda-\lambda_\Lambda)} C_{\lambda_\Lambda,\lambda_\gamma} C^*_{\lambda'_\Lambda,\lambda_\gamma} \end{aligned}$$

where (16) has been used. The factor $|E_{\lambda_p}|^2$ has been dropped because of parity conservation ($|E_{-\frac{1}{2}}| = |E_{\frac{1}{2}}|$).
With the helicity matrix above we can evaluate the following product separately for $\lambda_\Lambda, \lambda_\gamma > 0$:

$$\begin{aligned}\sum_{\lambda_\Lambda,\lambda'_\Lambda} C_{\lambda_\Lambda,\lambda_\gamma} e^{-i\alpha\lambda_\Lambda} C^*_{\lambda'_\Lambda,\lambda_\gamma} e^{i\alpha\lambda'_\Lambda} &= (C_{\frac{3}{2},1} e^{-i\frac{3}{2}\alpha} + C_{\frac{1}{2},1} e^{-i\frac{1}{2}\alpha})(C^*_{\frac{3}{2},1} e^{i\frac{3}{2}\alpha} + C^*_{\frac{1}{2},1} e^{i\frac{1}{2}\alpha}) \\ &= |C_{\frac{3}{2},1}|^2 + |C_{\frac{1}{2},1}|^2 + 2\ \mathrm{Re}\{C^*_{\frac{3}{2},1} C_{\frac{1}{2},1} e^{i\alpha}\}\end{aligned}$$

For symmetry reasons, if $\lambda_\Lambda, \lambda_\gamma < 0$, we have:

$$\sum_{\lambda_\Lambda,\lambda'_\Lambda} C_{\lambda_\Lambda,\lambda_\gamma} e^{-i\alpha\lambda_\Lambda} C^*_{\lambda'_\Lambda,\lambda_\gamma} e^{i\alpha\lambda'_\Lambda} = |C_{-\frac{3}{2},-1}|^2 + |C_{-\frac{1}{2},-1}|^2 + 2\ \mathrm{Re}\{C^*_{-\frac{3}{2},-1} C_{-\frac{1}{2},-1} e^{-i\alpha}\}$$

These results can be used to evaluate the squared amplitude:

$$\begin{aligned}|A|^2 \propto\ & \underbrace{|C_{\frac{3}{2},1}|^2 |d^{\frac{3}{2}}_{\frac{3}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda)|^2}_{|A_1|^2} + \underbrace{|C_{-\frac{3}{2},-1}|^2 |d^{\frac{3}{2}}_{-\frac{3}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda)|^2}_{|A_2|^2} \\ & + \underbrace{|C_{\frac{1}{2},1}|^2 |d^{\frac{3}{2}}_{\frac{1}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda)|^2}_{|A_3|^2} + \underbrace{|C_{-\frac{1}{2},-1}|^2 |d^{\frac{3}{2}}_{-\frac{1}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda)|^2}_{|A_4|^2} \\ & + \underbrace{2\ \mathrm{Re}\{C^*_{\frac{3}{2},1} C_{\frac{1}{2},1} e^{i(\phi_\Lambda+\phi_p)}\} d^{\frac{3}{2}}_{\frac{3}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda) d^{\frac{3}{2}}_{\frac{1}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda)}_{|A_5|^2} \\ & + \underbrace{2\ \mathrm{Re}\{C^*_{-\frac{3}{2},-1} C_{-\frac{1}{2},-1} e^{-i(\phi_\Lambda+\phi_p)}\} d^{\frac{3}{2}}_{-\frac{3}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda) d^{\frac{3}{2}}_{-\frac{1}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda)}_{|A_6|^2}\end{aligned}$$

The decay probability (14) can be written as:

$$w_{\frac{3}{2}} = \sum_{i=1}^{6} \sum_{M,\lambda_\Lambda,\lambda_p} \rho_{MM} |A_i|^2 = \sum_{i=1}^{6} w_i \qquad (20)$$

We will need the following d-functions:

$$\begin{aligned} d^{\frac{3}{2}}_{\frac{3}{2},\frac{1}{2}}(\theta) &= -\sqrt{3}\ \frac{1+\cos\theta}{2} \sin\frac{\theta}{2} = -\sqrt{3}\ \cos^2\frac{\theta}{2} \sin\frac{\theta}{2} \\ d^{\frac{3}{2}}_{\frac{3}{2},-\frac{1}{2}}(\theta) &= \sqrt{3}\ \frac{1-\cos\theta}{2} \cos\frac{\theta}{2} = \sqrt{3}\ \sin^2\frac{\theta}{2} \cos\frac{\theta}{2} \\ d^{\frac{3}{2}}_{-\frac{3}{2},-\frac{1}{2}}(\theta) &= d^{\frac{3}{2}}_{\frac{1}{2},\frac{3}{2}}(\theta) = d^{\frac{3}{2}}_{\frac{3}{2},\frac{1}{2}}(-\theta) = \sqrt{3}\ \cos^2\frac{\theta}{2} \sin\frac{\theta}{2} \\ d^{\frac{3}{2}}_{-\frac{3}{2},\frac{1}{2}}(\theta) &= d^{\frac{3}{2}}_{-\frac{1}{2},\frac{3}{2}}(\theta) = d^{\frac{3}{2}}_{\frac{3}{2},-\frac{1}{2}}(-\theta) = -\sqrt{3}\ \sin^2\frac{\theta}{2} \cos\frac{\theta}{2} \\ d^{\frac{3}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta) &= d^{\frac{3}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta) = \frac{3\cos\theta - 1}{2} \cos\frac{\theta}{2} \end{aligned}$$

$$d^{\frac{3}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta) = d^{\frac{3}{2}}_{-\frac{1}{2},\frac{1}{2}}(\theta) = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2}$$

where we used the fact that

$$d^j_{mm'}(\theta) = d^j_{m'm}(-\theta)$$

Let us now calculate each contribution to the decay probability (20) separately:

$$\begin{aligned}
w_1 &= \sum_{M,\lambda_p} \rho_{MM}|A_1|^2 = \sum_M \rho_{MM}|C_{\frac{3}{2},1}|^2 \left[|d^{\frac{3}{2}}_{\frac{3}{2},\frac{1}{2}}(\theta_p)|^2 + |d^{\frac{3}{2}}_{\frac{3}{2},-\frac{1}{2}}(\theta_p)|^2\right] |d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda)|^2 \\
&= 3\ |C_{\frac{3}{2},1}|^2 \cos^2\frac{\theta_p}{2}\sin^2\frac{\theta_p}{2}\sum_M \rho_{MM}|d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda)|^2 \\
&= 3\ |C_{\frac{3}{2},1}|^2 \cos^2\frac{\theta_p}{2}\sin^2\frac{\theta_p}{2}\left[\rho_{++}\cos^2\frac{\theta_\Lambda}{2} + \rho_{--}\sin^2\frac{\theta_\Lambda}{2}\right] \\
&= \frac{3}{8}\ \det\rho\ |C_{\frac{3}{2},1}|^2 \sin^2\theta_p\ (1 + P_{\Lambda_b}\cos\theta_\Lambda)
\end{aligned}$$

where $\det\rho = \rho_{++} + \rho_{--} = 1$.

Similarly we have:

$$\begin{aligned}
w_2 &= \sum_{M,\lambda_p} \rho_{MM}|A_2|^2 = \sum_M \rho_{MM}|C_{-\frac{3}{2},-1}|^2 \left[|d^{\frac{3}{2}}_{-\frac{3}{2},\frac{1}{2}}(\theta_p)|^2 + |d^{\frac{3}{2}}_{-\frac{3}{2},-\frac{1}{2}}(\theta_p)|^2\right] |d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda)|^2 \\
&= 3\ |C_{-\frac{3}{2},-1}|^2 \cos^2\frac{\theta_p}{2}\sin^2\frac{\theta_p}{2}\left[\rho_{++}\sin^2\frac{\theta_\Lambda}{2} + \rho_{--}\cos^2\frac{\theta_\Lambda}{2}\right] \\
&= \frac{3}{8}\ |C_{-\frac{3}{2},-1}|^2 \sin^2\theta_p\ (1 - P_{\Lambda_b}\cos\theta_\Lambda)
\end{aligned}$$

The third and fourth term become:

$$\begin{aligned}
w_3 &= \sum_{M,\lambda_p} \rho_{MM}|A_3|^2 = \sum_M \rho_{MM}|C_{\frac{1}{2},1}|^2 \left[|d^{\frac{3}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta_p)|^2 + |d^{\frac{3}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_p)|^2\right] |d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda)|^2 \\
&= \frac{1}{2}\ |C_{\frac{1}{2},1}|^2 \left[|d^{\frac{3}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta_p)|^2 + |d^{\frac{3}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_p)|^2\right] (1 - P_{\Lambda_b}\cos\theta_\Lambda)
\end{aligned}$$

and

$$\begin{aligned}
w_4 &= \sum_{M,\lambda_p} \rho_{MM}|A_4|^2 = \sum_M \rho_{MM}|C_{-\frac{1}{2},-1}|^2 \left[|d^{\frac{3}{2}}_{-\frac{1}{2},\frac{1}{2}}(\theta_p)|^2 + |d^{\frac{3}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta_p)|^2\right] |d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda)|^2 \\
&= \frac{1}{2}\ |C_{-\frac{1}{2},-1}|^2 \left[|d^{\frac{3}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\theta_p)|^2 + |d^{\frac{3}{2}}_{-\frac{1}{2},\frac{1}{2}}(\theta_p)|^2\right] (1 + P_{\Lambda_b}\cos\theta_\Lambda)
\end{aligned}$$

where

$$|d^{\frac{3}{2}}_{\frac{1}{2},\frac{1}{2}}(\theta_p)|^2 + |d^{\frac{3}{2}}_{\frac{1}{2},-\frac{1}{2}}(\theta_p)|^2 = \frac{1}{4}\left[\cos^2\frac{\theta_p}{2}(3\cos\theta_p - 1)^2 + \sin^2\frac{\theta_p}{2}(3\cos\theta_p + 1)^2\right] = \frac{1}{4}\ (3\cos^2\theta_p + 1)$$

For the interference terms:

$$w_5 = 2\ \mathrm{Re}\{C^*_{\frac{3}{2},1} C_{\frac{1}{2},1} e^{i(\phi_\Lambda+\phi_p)}\} \sum_{M,\lambda_p} \rho_{MM} d^{\frac{3}{2}}_{\frac{3}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda) d^{\frac{3}{2}}_{\frac{1}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda)$$

$$w_6 = 2\ \mathrm{Re}\{C^*_{-\frac{3}{2},1} C_{-\frac{1}{2},1} e^{-i(\phi_\Lambda+\phi_p)}\} \sum_{M,\lambda_p} \rho_{MM} d^{\frac{3}{2}}_{-\frac{3}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta_\Lambda) d^{\frac{3}{2}}_{-\frac{1}{2},\lambda_p}(\theta_p) d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta_\Lambda)$$

But

$$d^{\frac{1}{2}}_{M,\frac{1}{2}}(\theta) d^{\frac{1}{2}}_{M,-\frac{1}{2}}(\theta) = -\cos\frac{\theta}{2}\sin\frac{\theta}{2} = -\frac{1}{2}\sin\theta, \quad \text{for } M = \pm\frac{1}{2}$$

$$d^{\frac{3}{2}}_{\frac{3}{2},\lambda_p}(\theta) d^{\frac{3}{2}}_{\frac{1}{2},\lambda_p}(\theta) = d^{\frac{3}{2}}_{-\frac{3}{2},\lambda_p}(\theta) d^{\frac{3}{2}}_{-\frac{1}{2},\lambda_p}(\theta) = -\frac{\sqrt{3}}{8}\sin\theta\,(3\cos\theta\mp 1)\,(1\pm\cos\theta), \quad \text{for } \lambda_p = \pm\frac{1}{2}$$

Therefore

$$w_5 = \mathrm{Re}\{C^*_{\frac{3}{2},1} C_{\frac{1}{2},1} e^{i(\phi_\Lambda+\phi_p)}\}\ \frac{\sqrt{3}}{2}\ \cos\theta_p\ \sin\theta_p\ \sin\theta_\Lambda$$

$$w_6 = \mathrm{Re}\{C^*_{-\frac{3}{2},1} C_{-\frac{1}{2},1} e^{-i(\phi_\Lambda+\phi_p)}\}\ \frac{\sqrt{3}}{2}\ \cos\theta_p\ \sin\theta_p\ \sin\theta_\Lambda$$

We can now factorize the various contributions to the decay probability (20) as follows:

$$w_{\frac{3}{2}} = \sum_{i=1}^{6} w_i = \sum_{i=1}^{6} C_i(\phi_\Lambda, \phi_p) \cdot f_i(\theta_p) \cdot g_i(\theta_\Lambda, P_{\Lambda_b}) \qquad (21)$$

where:

$C_i(\phi_\Lambda, \phi_p)$ are functions of the relevant helicity amplitudes and the azimuthal angles ϕ_Λ and ϕ_p;

$f_i(\theta_p)$ are functions of the proton polar angle θ_p;

$g_i(\theta_\Lambda, P_{\Lambda_b})$ are functions depending on the Λ polar angle θ_Λ and the Λ_b polarization P_{Λ_b}.

The C_i, f_i and g_i functions are reported in table 3.

Table 3: Summary of the terms that make up the decay probability $w_{\frac{3}{2}} = \sum_i C_i \cdot f_i(p) \cdot g_i(\Lambda, P_{\Lambda_b})$, in the case $J_\Lambda = 3/2$. For explanation see text.

i	$C_i(\phi_\Lambda, \phi_p)$	$f_i(\theta_p)$	$g_i(\theta_\Lambda, P_{\Lambda_b})$
1	$\lvert C_{\frac{3}{2},1}\rvert^2$	$\frac{3}{8}\ \sin^2\theta_p$	$1 + P_{\Lambda_b}\cos\theta_\Lambda$
2	$\lvert C_{-\frac{3}{2},-1}\rvert^2$	$\frac{3}{8}\ \sin^2\theta_p$	$1 - P_{\Lambda_b}\cos\theta_\Lambda$
3	$\lvert C_{\frac{1}{2},1}\rvert^2$	$\frac{1}{8}\ (3\cos^2\theta_p + 1)$	$1 - P_{\Lambda_b}\cos\theta_\Lambda$
4	$\lvert C_{-\frac{1}{2},-1}\rvert^2$	$\frac{1}{8}\ (3\cos^2\theta_p + 1)$	$1 + P_{\Lambda_b}\cos\theta_\Lambda$
5	$\mathrm{Re}\{C^*_{\frac{3}{2},1} C_{\frac{1}{2},1}\ e^{i(\phi_\Lambda+\phi_p)}\}$	$\frac{\sqrt{3}}{2}\ \cos\theta_p\ \sin\theta_p$	$\sin\theta_\Lambda$
6	$\mathrm{Re}\{C^*_{-\frac{3}{2},-1} C_{-\frac{1}{2},-1}\ e^{-i(\phi_\Lambda+\phi_p)}\}$	$\frac{\sqrt{3}}{2}\ \cos\theta_p\ \sin\theta_p$	$\sin\theta_\Lambda$

B Statistical error on the measurement of $|r|$

The ratio r depends on the photon polarization α_γ:

$$|r| = \sqrt{\frac{1-\alpha_\gamma}{1+\alpha_\gamma}}$$

and its relative statistical error is given by:

$$\frac{\sigma_{|r|}}{|r|} = \frac{1}{|r|^2}\frac{\sigma_{\alpha_\gamma}}{(1+\alpha_\gamma)^2}$$

The photon helicity parameter α_γ can be extracted from a linear fit of the measured angular distributions (3), (4) and (8). To estimate the statistical error on the measurement of the slope we used a fast toy Monte Carlo (RooFit [27]). We generated 1000 experiments of 1000 events each using as p.d.f. a linear distribution of the type $1 - s_\gamma \cos\theta$ for various values of the slope s_γ in the interval [0,1]. The results are shown in fig. 7, where the error on the slope is shown as a function of s_γ. We can therefore parameterize the error on the measurement of s_γ from a linear fit (for N events)as:

$$\sigma_{s_\gamma} = 0.0554 \cdot \sqrt{1 - 0.71 \cdot s_\gamma^2}\sqrt{1000/N}$$

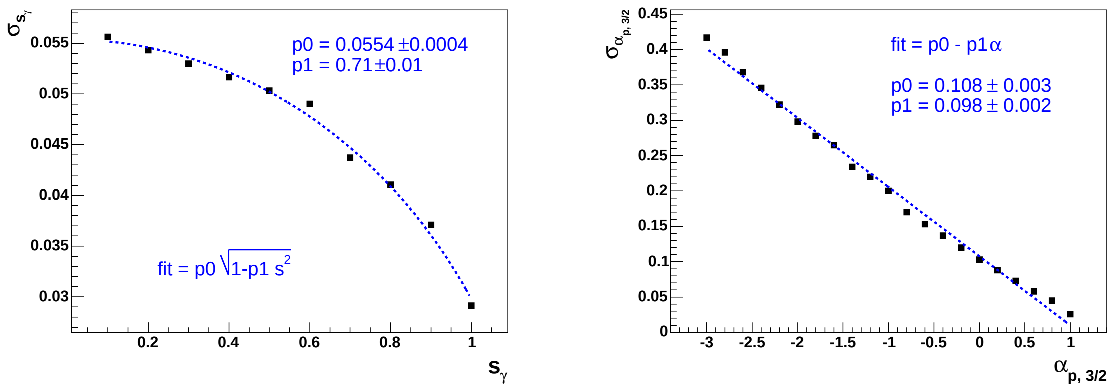

Figure 7: Fast toy MC studies: statistical error in the measurement of the slope s_γ (left) and the proton parameter α_p from decays involving a spin-3/2 Λ resonance (right). The dashed lines represent a fit of the statistical error distributions.

For $\Lambda(X)$ baryons of spin 1/2, the photon polarization is then given by:

$$\alpha_\gamma = \frac{s_\gamma}{a}$$

where a is either the Λ_b polarization P_{Λ_b} or the weak decay parameter $\alpha_{p,1/2}$. The (absolute) statistical error on the photon polarization is therefore:

$$\sigma_{\alpha_\gamma} = \frac{1}{a}\sqrt{\alpha_\gamma^2\sigma_a^2 + \sigma_{s_\gamma}^2}$$

For $\Lambda(X)$ baryons of spin 3/2, the photon polarization is extracted from the measurement of the slope s_γ and the proton parameter $\alpha_{p,3/2}$ (see section 4.2):

$$\alpha_\gamma = \frac{s_\gamma}{2P_{\Lambda_b}}\left(1 - \frac{3}{\alpha_{p,3/2}}\right)$$

and its statistical error:

$$\sigma_{\alpha_\gamma} = \sqrt{\alpha_\gamma^2\left(\frac{\sigma_{s_\gamma}^2}{s_\gamma^2} + \frac{\sigma_{P_{\Lambda_b}}^2}{P_{\Lambda_b}^2}\right) + \frac{9}{4}\,\frac{s_\gamma^2}{P_{\Lambda_b}^2}\,\frac{\sigma_{\alpha_{p,3/2}}^2}{\alpha_{p,3/2}^4}}$$

The proton asymmetry parameter $\alpha_{p,3/2}$ can be extracted from a fit of the angular distribution (9). To estimate its statistical error we generated with a fast toy Monte Carlo (MC) 1000 experiments of 1000 events each (using as p.d.f. the distribution (9)). The results are shown in fig. 7, where $\sigma_{\alpha_{p,3/2}}$ is shown for the range of $\alpha_{p,3/2}$ of interest. We assume a linear dependance of the type (for N events):

$$\sigma_{\alpha_{p,3/2}} = (0.11 - 0.10 \cdot \alpha_{p,3/2})\sqrt{1000/N}$$

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