

# Heavy Flavour Averages

Summary of present activities

... what, why and how

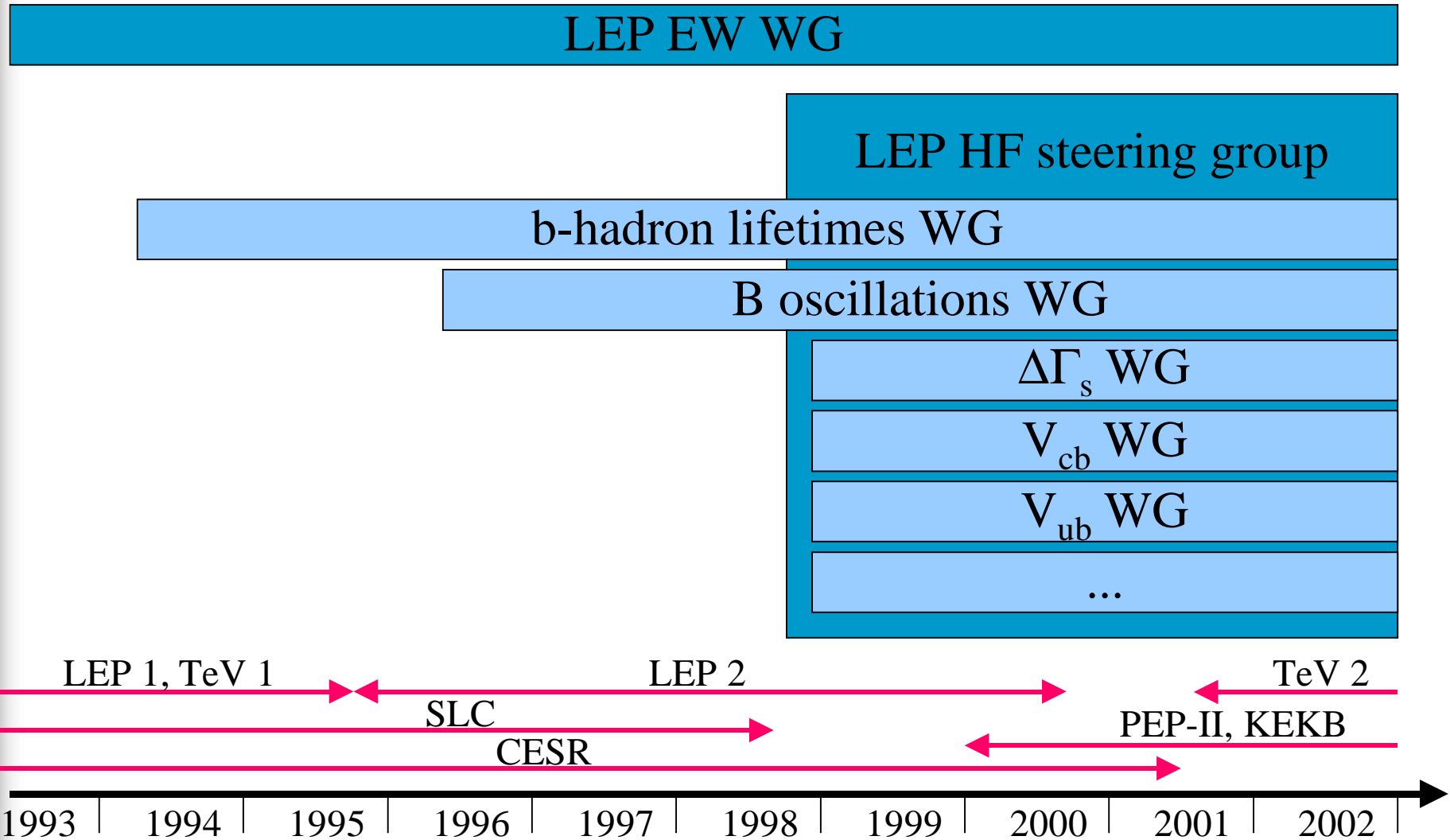


Olivier Schneider

on behalf of  
the LEP Heavy Flavour Steering group  
and various working groups

CKM workshop, CERN, February 15, 2002

# CERN-based working groups performing Heavy Flavour averages





# Why these averaging WGs ?

- LEP has 4 experiments
  - Common experimental methods
  - Close in space (CERN)
- Many “inclusive” measurements (hence with non-negligible systematics)
- Some measurements with significant systematics
  - important to understand correlations
- Quite a few measurements not statistically limited
  - can decrease the error by combining



# b-hadron lifetime WG

- Started at LEP in 1994
- Combination methods quickly established (in 1994)
- Contact people from CDF and SLD joined

- **Products:**

$\tau(B^+)$ ,  $\tau(B^0)$ ,  $\tau(B^+)/\tau(B^0)$ ,  $\tau(B_s)$

$\tau(\text{b-baryon})$ ,  $\tau(\Lambda_b)$ ,  $\tau(\Xi_b)$

weighted averages of above at high energy:  $\tau^{\text{incl}}$ ,  $\tau^{\text{sl}}$ , ...

- World averages in PDG since 1996

<http://home.cern.ch/~claires/blife.html>



# Lifetime averages at 1994 winter conf.: an interesting anecdote

- Two  $B_s$  lifetime averages based on same data:
  - ① weighted average using weights  $= (1/\sigma_{\tau_i})^2$   
 $\tau(B_s) = 1.38 \pm 0.17$  ps (La Thuile, 1994)
  - ② weighted average using weights  $= (\tau_i/\sigma_{\tau_i})^2$   
 $\tau(B_s) = 1.66 \pm 0.22$  ps (Moriond, 1994)
- Exponential distributions have  $\sigma \propto \tau$ :
  - ① leads to negative bias,  
especially for small sample and high resolution
  - ② leads to positive bias,  
especially for small sample and low resolution
  - Confirmed and quantified by toy MC studies  
[ R.Forty, CERN-PPE/94-144]

# Averaging methods for lifetimes

## ■ COMBINE:

- based on a Best Linear Unbiased Estimator (BLUE) technique using a “relative error matrix E”

$$\chi^2 = \sum_{i,j} (\bar{\tau} - \tau_i)(E^{-1})_{ij}(\bar{\tau} - \tau_j) \quad \bar{\tau} = \sum_{i,j} \tau_i (E^{-1})_{ij} / \sum_{i,j} (E^{-1})_{ij} \quad (\sigma/\bar{\tau})^2 = 1 / \sum_{i,j} (E^{-1})_{ij}$$

- can take statistical and systematic correlations into account
- can handle symmetric uncertainties only

## ■ COMBY:

- based on a model for the likelihood (knowing pdf is exponential convoluted with gaussian resolution)
- reconstruct an approximate likelihood for each measurement given its asymmetric errors and maximize total likelihood
- treat systematic errors in an absolute way

- In practice, very small differences for sufficiently large statistics

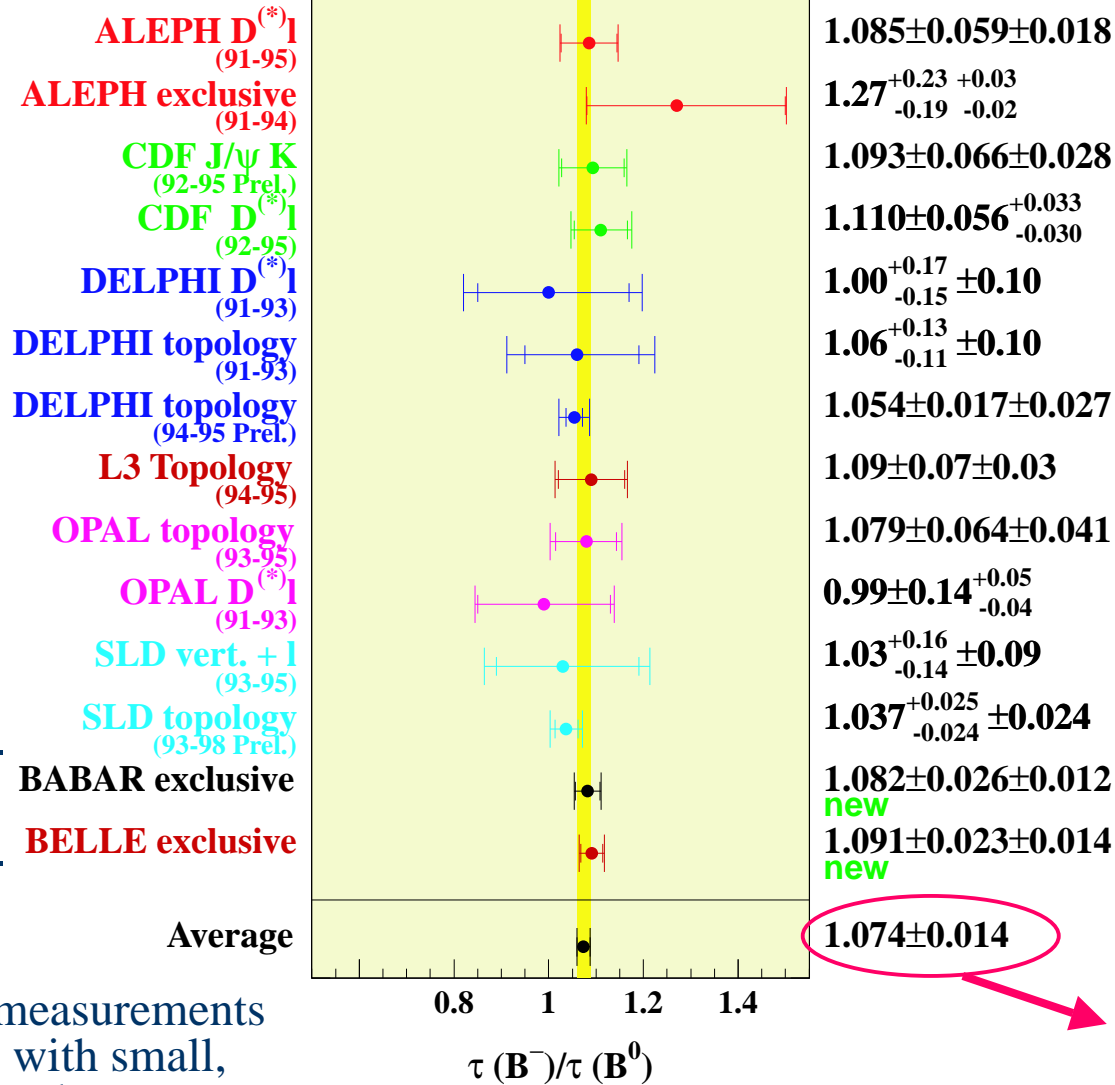


# Correlated systematics between lifetime analyses

- **detector effects**
  - vertex reconstruction, ...
- **physics backgrounds**
  - $\text{BR}(B \rightarrow D^{**}l\nu)$
  - b-hadron fractions, ...
- **momentum estimate**
  - b fragmentation
  - decay model, multiplicities
  - BRs of b- and c-hadrons
  - b-baryon polarization and mass, ...

(most important for (semi-)inclusive analyses)

# New $B^+/B^0$ lifetime ratio (Feb 2002)



New  $B^+$  and  $B^0$  measurements from B factories with small, almost uncorrelated errors

$\tau(B^+) \neq \tau(B^0)$





# B oscillations WG

- EPS-HEP 1995 (Brussels):
  - ALEPH proposes amplitude method for  $\Delta m_s$  analyses
- WG setup in 1996 by LEP collaborations
  - main goal and motivation: **combine  $\Delta m_s$  limits**
  - other goals quickly added:  **$\Delta m_d$  average + b-hadron fractions**
- SLD+CDF joined in 1997

- **Products:**

$f(B_s)$ ,  $f(B_d)$ ,  $f(B_u)$ ,  $f(\text{b-baryon})$  and correlations  
 $\Delta m_d$ ,  $\chi_d$   
 $\Delta m_s$  limit, combined amplitude spectrum

- World averages in PDG since 1998

<http://www.cern.ch/LEPBOSC/>

# Determination of b-hadron fractions

Fractions of different species in an unbiased sample of weakly-decaying b-hadrons produced at high energy

## ■ Assumptions:

$$f(\mathbf{B}_u) + f(\mathbf{B}_d) + f(\mathbf{B}_s) + f(\mathbf{b}\text{-baryon}) = 1$$

$$f(\mathbf{B}_u) = f(\mathbf{B}_d)$$

**fractions identical in Z decays and at Tevatron**

## ■ Values from BR measurements, compiled and averaged by WG:

$$f(\mathbf{B}_s) = (9.2 \pm 2.4)\%, \quad f(\mathbf{b}\text{-baryon}) = (10.6 \pm 2.0)\%$$

## ■ Improve fractions using:

- average  $\chi = f'(\mathbf{B}_s) \chi_s + f'(\mathbf{B}_d) \chi_d = 0.1197 \pm 0.0040$  from LEP EW fit
- world average  $\chi_d = 0.184 \pm 0.004$  by WG
- $\chi_s = 1/2$

$$f(\mathbf{B}_s) = (9.7 \pm 1.1)\%, \quad f(\mathbf{b}\text{-baryon}) = (10.3 \pm 1.8)\%$$

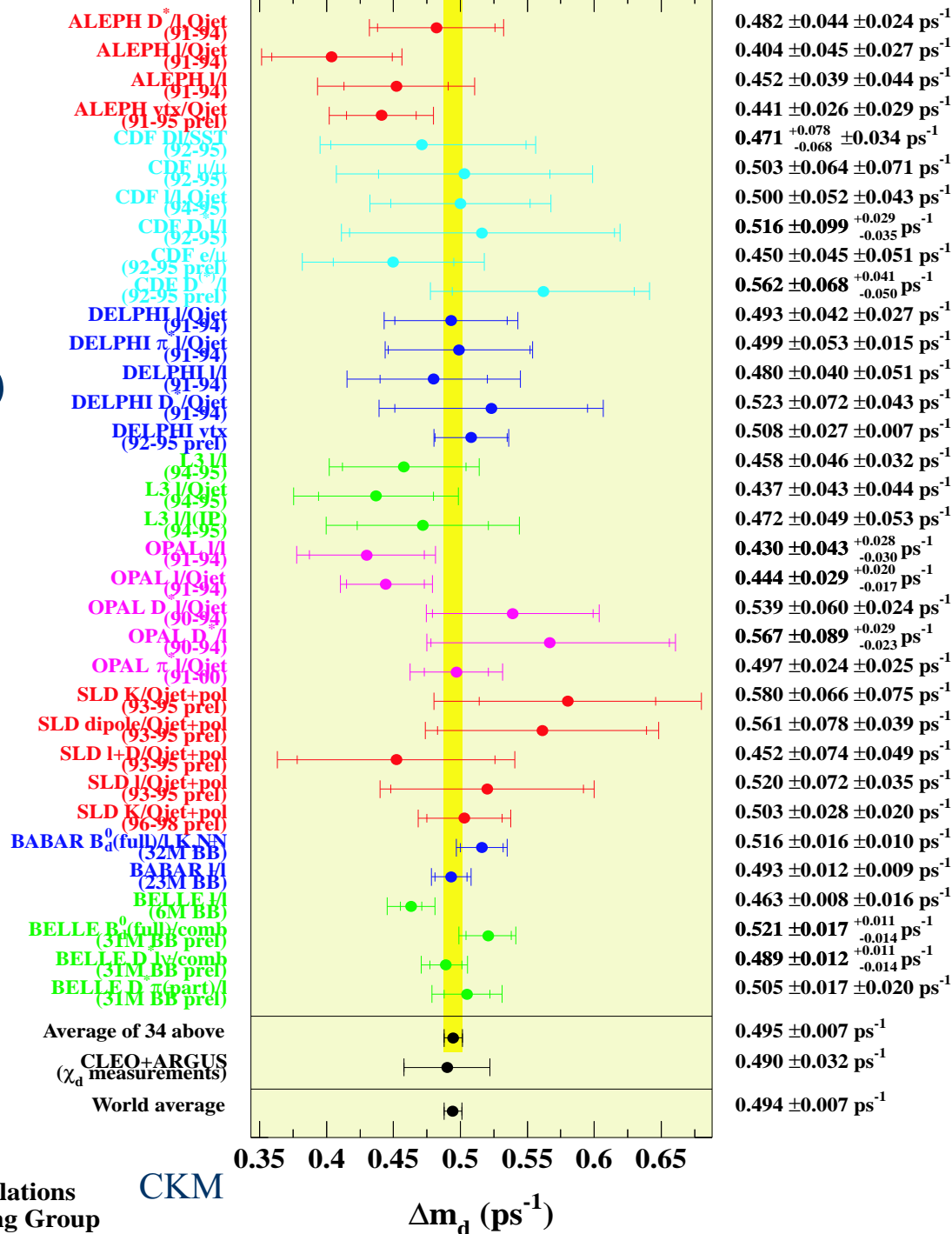
**best estimates**

factor 2 improvement

# 34 measurements of $\Delta m_d$ by 8 experiments !

(22 published + 12 prelim.)

new B factory  
measurements



Olivier Schneider

B Oscillations  
Working Group

CKM

$\Delta m_d$  ( $\text{ps}^{-1}$ )

# Main issues for $\Delta m_d$ combination

- Several correlated systematic (and statistical) uncertainties
- Several systematics depend on physics parameters
  - b-hadron lifetimes
  - b-hadron fractions,
  - integrated mixing (!), ...
- Dependence on physics parameters not always explicit
- Results are not “homogeneous”
  - different sets of physics parameters used (e.g. lifetime ratios instead of lifetimes) } too late to avoid this !
  - different central values and uncertainties assumed for these physics parameters
- Average  $\Delta m_d$  is used to determine the fractions



# Combination method for $\Delta m_d$

- **Adjust individual measurements** (both central values and appropriate systematic uncertainties) to a common set of physics parameters
  - b-hadron lifetimes
  - b-hadron fractions
  - ...
  
- Establish list of **correlated systematics**
  - for each such systematics assume full (anti-) correlation between analyses:
    - b-hadron fractions, experimental effects, ...
  - if not possible, add certain systematic errors in quadrature to form “**lumps**” assumed to be fully correlated across analyses:
    - uncertainties due to b-hadron lifetimes
    - uncertainties due to b-fragmentation
    - uncertainties due to cascade decays ( $b \rightarrow l$ ,  $b \rightarrow c \rightarrow l, \dots$ )
    - ...
  
- **Symmetrize** all uncertainties (using quadratic average)

# Combination method for $\Delta m_d$ (cont.)

- MINUITize (a modified version\* of) the following  $\chi^2$  with respect to  $X$  and  $Y_\alpha$ :

$$\chi^2 = \sum_{i,j=1}^n \left( X_i + \sum_{\alpha=1}^m \Delta_{\alpha i} Y_\alpha - X \right) (E^{-1})_{ij} \left( X_j + \sum_{\beta=1}^m \Delta_{\beta j} Y_\beta - X \right) + \sum_{\gamma=1}^m Y_\gamma^2$$

$$E_{ij} = \rho_{ij} \sigma_i^{stat} \sigma_j^{stat} + \delta_{ij} \sigma_i^{syst} \sigma_j^{syst}$$

$X_i \pm \sigma_i^{stat} \pm \sigma_i^{syst}$  = central values, stat. err., uncorrelated syst. err.

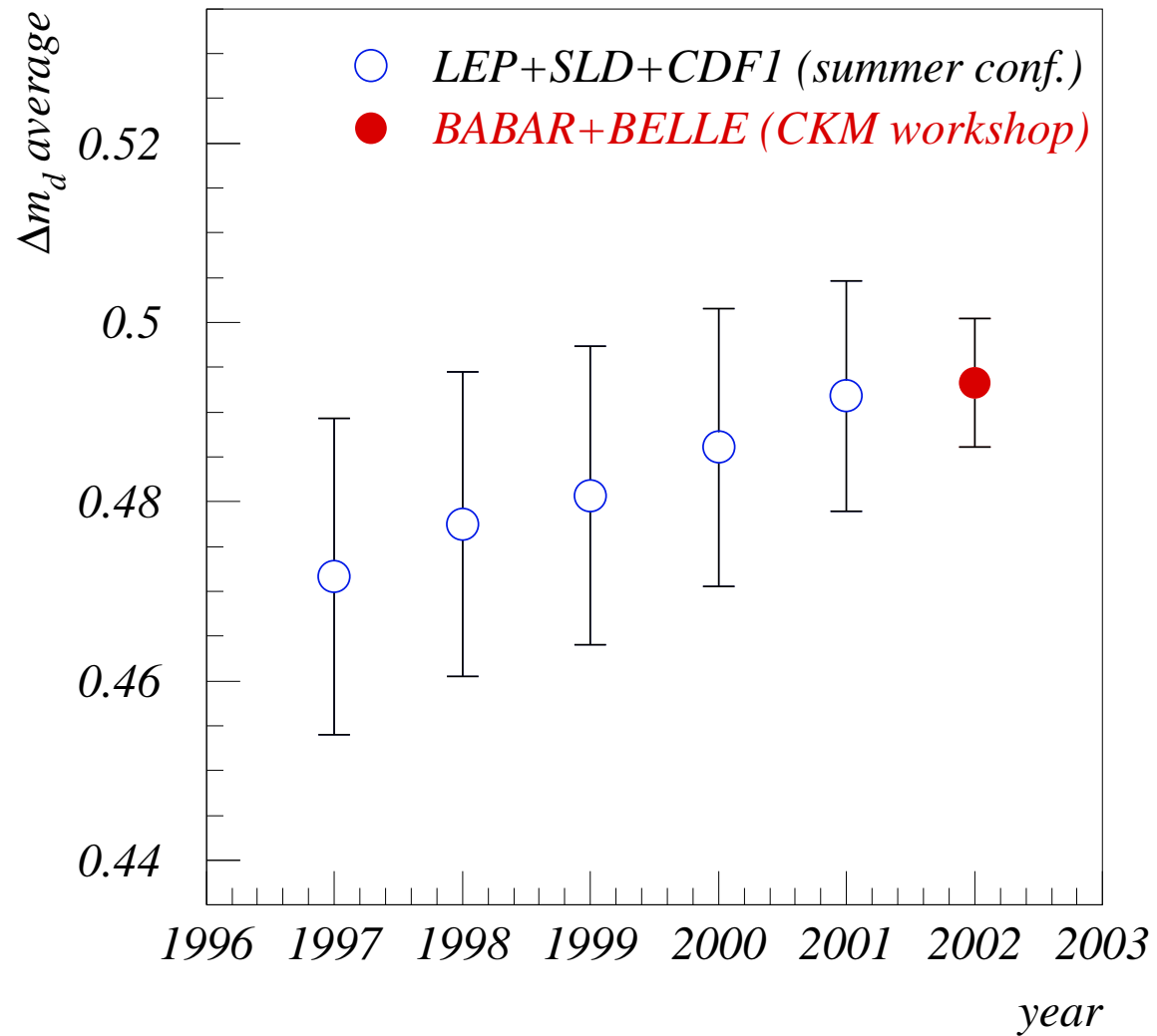
$\rho_{ij}$  = statistical correlations

$\Delta_{\alpha i}$  = correlated systematic errors

- Quote fitted  $X$  as the  $\Delta m_d$  average and ignore fitted  $Y_\alpha$

- \* Modification for each systematics related to a b-hadron fraction:
  - remove the  $Y_\alpha^2$  constraints
  - redefine  $Y_\alpha$  as a function of  $X$  and appropriate physics parameters (e.g.  $\chi$ ,  $\tau$ 's)
  - add gaussian constraints on these parameters (if not already in)

# $\Delta m_d$ averages: “old vs new”





# $\Delta m_d$ averages (Feb 2002)

- Standard procedure:

$$0.4923 \pm 0.0128 \text{ ps}^{-1} \text{ (LEP+SLD+CDF1)}$$

$$0.4947 \pm 0.0076 \text{ ps}^{-1} \text{ (BABAR+BELLE)}$$

$$0.4947 \pm 0.0067 \text{ ps}^{-1} \text{ (LEP+SLD+CDF1+BABAR+BELLE)}$$

$$0.4944 \pm 0.0066 \text{ ps}^{-1} \text{ (same + time-integrated meas.)}$$

- Without lumping b-lifetime systematics

(after asking BABAR to use same input parameters as BELLE):

$$\Delta m_d = 0.4933 \pm 0.0072 \text{ (BABAR+BELLE)}$$

← takes advantage of anti-correlations in the systematics due to  $\tau(B^0)$  and  $\tau(B^+)/\tau(B^0)$

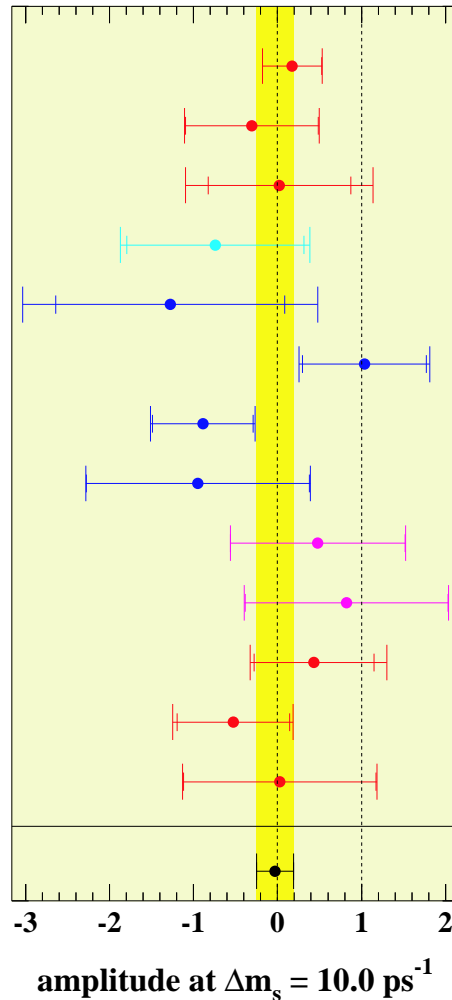
- Lessons:

- lumping is “conservative”
- important to have experiments quote their results in the same way
- B-factory average might become better than world average (!), if treatment of lifetime systematics not improved



# Combination of $B_s$ oscillation amplitudes

**ALEPH I**  
 (91-95, no  $D_s1$ , adjusted)  
**ALEPH D I**  
 (91-95)  
**ALEPH B**  
 (91-00)  
**CDF  $1\phi/1$**   
 (92-95)  
**DELPHI B +D h**  
 (92-95)  
**DELPHI D I+ $\phi$ 1**  
 (92-95, prel)  
**DELPHI I**  
 (92-95, prel)  
**DELPHI vtx**  
 (92-95, prel)  
**OPAL I**  
 (91-95)  
**OPAL D I**  
 (91-95)  
**SLD I+D**  
 (96-98, prel.)  
**SLD dipole**  
 (96-98, prel.)  
**SLD D**  
 (96-98, prel.)



amplitude	(sensitivity)
$0.17 \pm 0.35 \pm 0.06$	$(13.7 \text{ ps}^{-1})$
$-0.31 \pm 0.79 \pm 0.15$	$(7.5 \text{ ps}^{-1})$
$0.02 \pm 0.85 \pm 0.73$	$(0.4 \text{ ps}^{-1})$
$-0.74 \pm 1.05 \pm 0.40$	$(5.1 \text{ ps}^{-1})$
$-1.28 \pm 1.36 \pm 1.11$	$(3.2 \text{ ps}^{-1})$
$1.04 \pm 0.74 \pm 0.25$	$(8.7 \text{ ps}^{-1})$
$-0.89 \pm 0.60 \pm 0.16$	$(9.9 \text{ ps}^{-1})$
$-0.95 \pm 1.33 \pm 0.16$	$(6.1 \text{ ps}^{-1})$
$0.48 \pm 1.04 \pm 0.09$	$(7.2 \text{ ps}^{-1})$
$0.82 \pm 1.21^{+0.14}_{-0.15}$	$(4.2 \text{ ps}^{-1})$
$0.43 \pm 0.71^{+0.50}_{-0.25}$	$(6.3 \text{ ps}^{-1})$
$-0.53 \pm 0.67^{+0.25}_{-0.26}$	$(8.6 \text{ ps}^{-1})$
$0.03 \pm 1.15^{+0.15}_{-0.13}$	$(1.7 \text{ ps}^{-1})$
$-0.03 \pm 0.22$	$(19.3 \text{ ps}^{-1})$

Use similar  $\chi^2$  method as for  $\Delta m_d$  averaging, with adjustments and lumping, as each value of the frequency

In addition, adjust statistical errors:

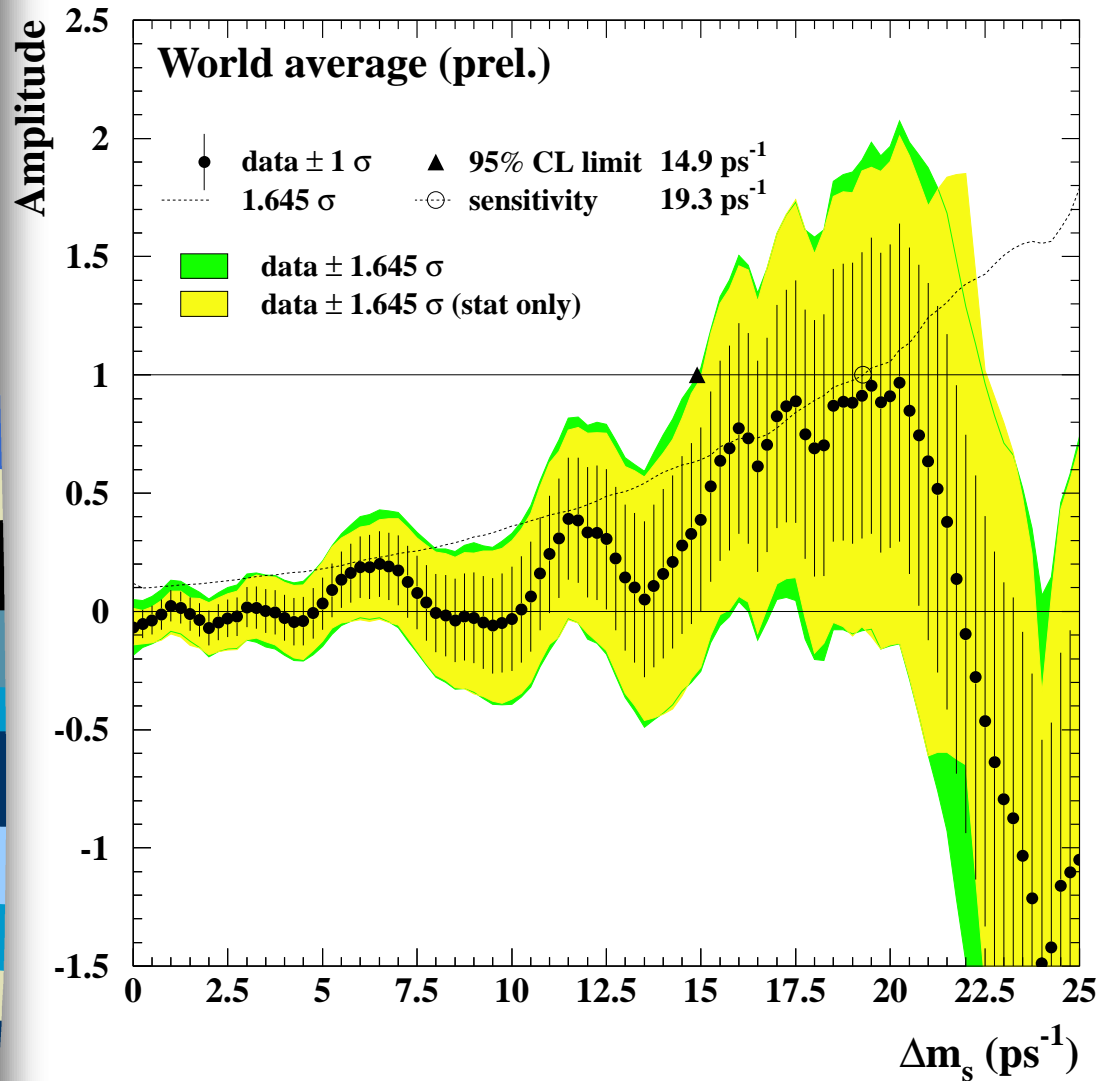
- new ALEPH analyses\*  
→ use quoted  $\Delta\sigma$ 's
- other inclusive analyses  
→ assume  $\sigma \propto 1/f(B_s)$
- other analyses  
→ no adjustment of  $\sigma$

B Oscillations Working Group

Olivier Schneider

Cl \* use adjusted  $\Delta A$ 's as systematic errors, ignoring  $\Delta\sigma$ 's

# Combined amplitude spectrum (Feb 2002)



$\Delta m_s > 14.9 \text{ ps}^{-1}$  @ 95% CL  
 expected limit = 19.3 ps<sup>-1</sup>

(stat. only: 15.0 / 19.6 ps<sup>-1</sup>)

With this sensitivity,  
 we would expect:

- 5 $\sigma$  signal  
 if  $\Delta m_s = 9.5 \text{ ps}^{-1}$
- 3 $\sigma$  signal  
 if  $\Delta m_s = 13.5 \text{ ps}^{-1}$



# LEP HF steering group

- **Setup by the LEP collaborations** at the end of 1998
  - 17 members, including representatives from **SLD & CDF**
- **Goals:**
  - coordinate HF averaging activities (production and decays)
  - encourage collaborations to present results in a way which will ease combination
  - identify areas where combined results are useful, and ensure that important HF physics opportunities are not missed

**Effectively:**

**3 years after the end of LEP1 data-taking,  
join remaining forces to complete HF analyses at LEP**



# LEP HF steering group (cont.)

## ■ Coordinate WG activities:

- old WGs (b lifetime, B oscillations)
- new WGs ( $\Delta\Gamma_s$ ,  $V_{cb}$ ,  $V_{ub}$ )
- other activities  
(charm counting, b fragmentation, CKM fits, ...)

## ■ Results summarized in CERN preprints:

- ALEPH,CDF,DELPHI,L3,OPAL,SLD; CERN-EP/2000-096
- ALEPH,CDF,DELPHI,L3,OPAL,SLD; CERN-EP/2001-050

<http://www.cern.ch/LEPHFS>

# $\Delta\Gamma_s$ WG

## LEP+CDF effort:

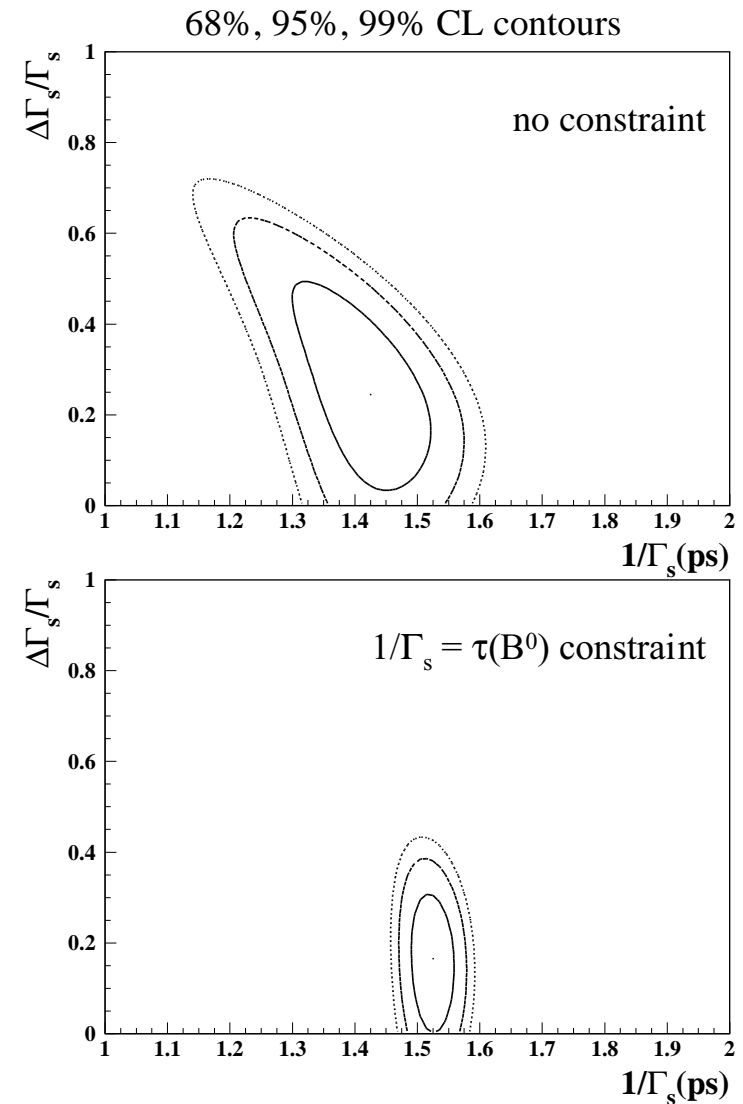
- Convert various  $B_s$  results (based on different mixture of CP-odd and CP-even) into a 2-D likelihood for  $1/\Gamma_s$  and  $\Delta\Gamma_s/\Gamma_s$
- Results for PDG since 2000

$$\Delta\Gamma_s/\Gamma_s < 0.52 \text{ at } 95\% \text{CL}$$

$$\Delta\Gamma_s/\Gamma_s < 0.32 \text{ at } 95\% \text{CL}$$

with  $1/\Gamma_s = \tau(B^0)$  constraint

PDG 2001



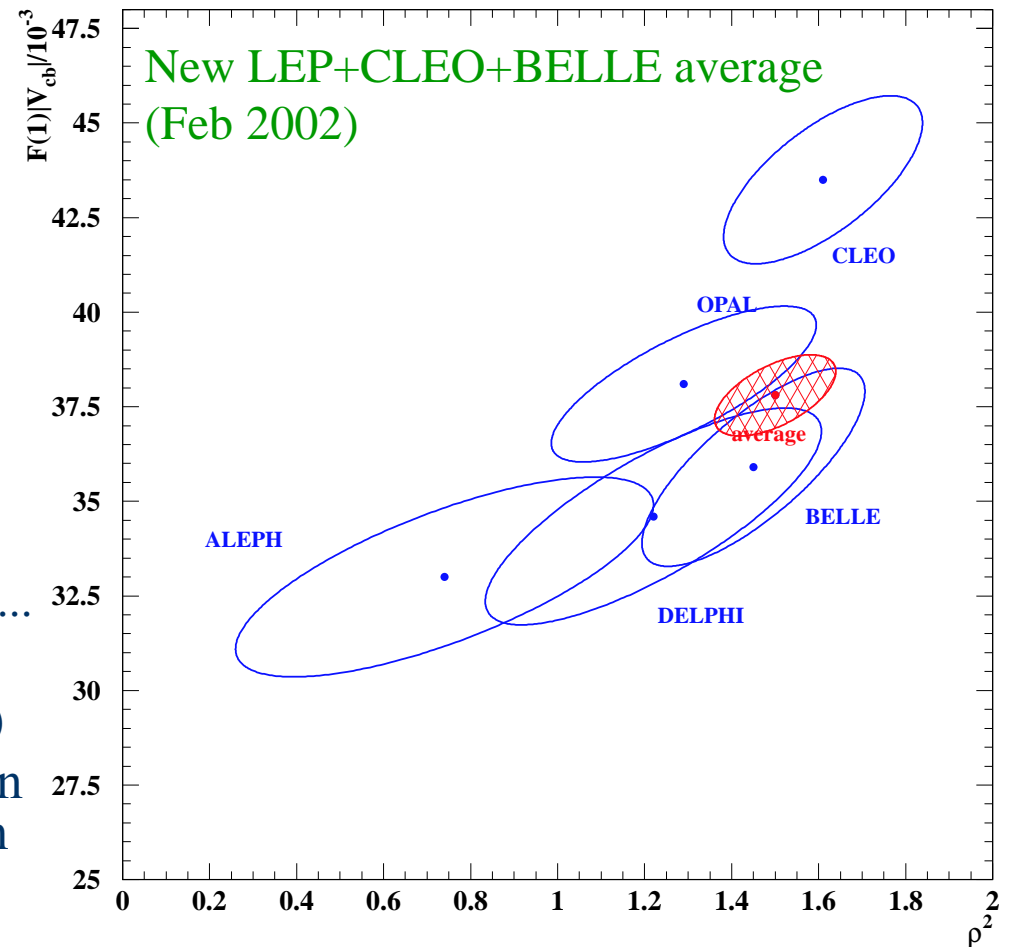
# $V_{cb}$ WG

## ■ $|V_{cb}|$ from $B^0 \rightarrow D^* l \nu$ :

- 2D average of  $F(1) |V_{cb}|$  and  $\rho^2$  (with error matrix)
- generalisation of method used for  $\Delta m_d$
- adjustments & corr. syst:
  - $R_b$ ,  $f(B^0)$ ,  $BR(D \rightarrow \dots)$ ,  $\tau_b$ , ...
  - $B \rightarrow D^{**} l \nu$  back. param.
  - $R_1, R_2$  (form factor ratios)
- succeeded in using common form factor parametrization

## ■ $|V_{cb}|$ inclusive

- $BR(b \rightarrow l)$  from LEP EW fit,  $BR(b \rightarrow ul \nu)$  from  $V_{ub}$  WG
- $\tau_b$  from lifetime WG





## $V_{ub}$ WG

### ■ $BR(b \rightarrow ul\nu)$ average at LEP

- use  $|V_{cb}|$  from  $V_{cb}$  WG to convert DELPHI's  $|V_{ub}|/|V_{cb}|$
- significant correlated systematics:
  - $b \rightarrow u$  modelling,  $b \rightarrow u$  modelling
- use BLUE technique
- in PDG since 2001

### ■ $|V_{ub}|$ from inclusive decays

- $\tau_b$  from lifetime WG

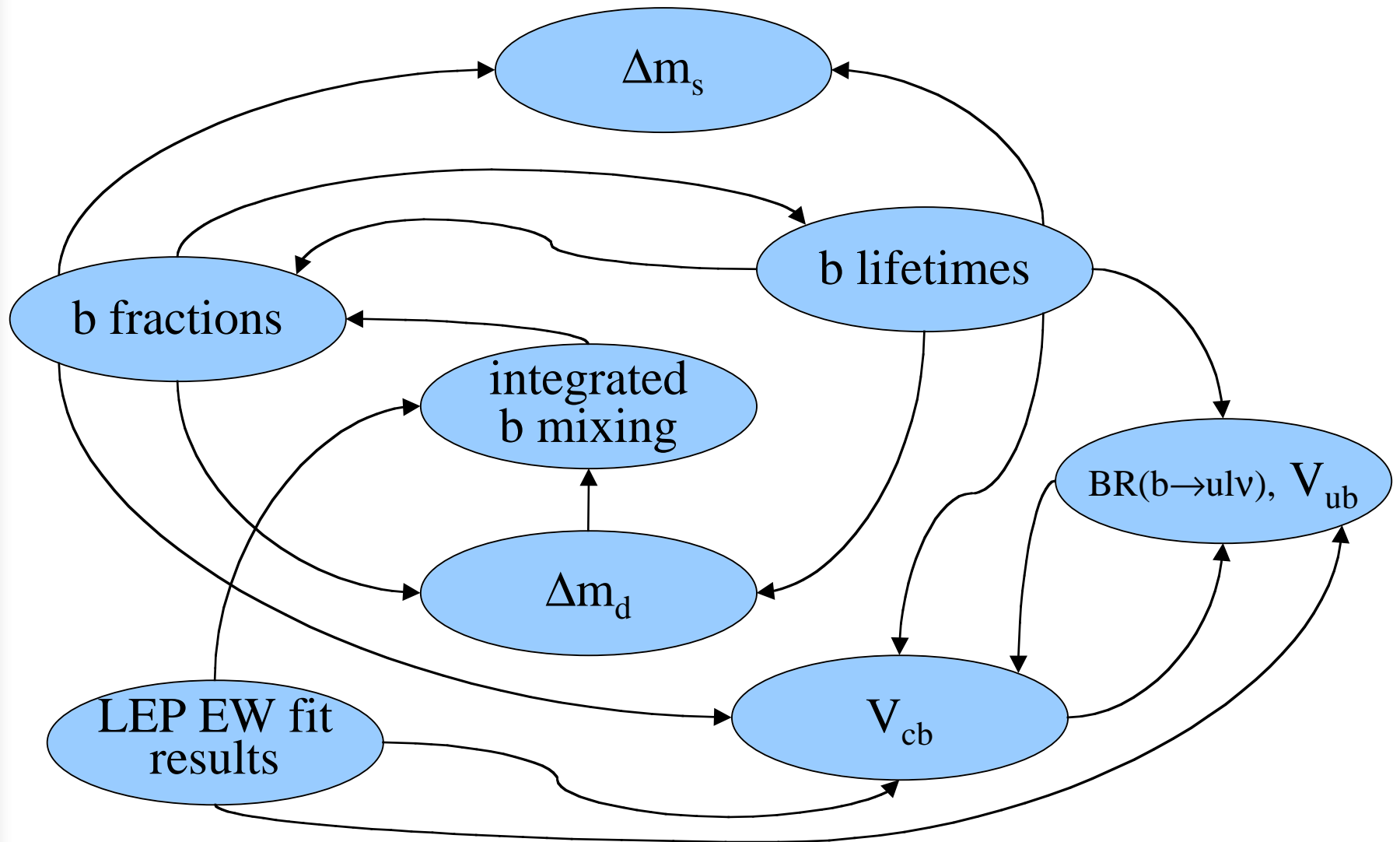


# Software

- **COMBOS program** (<http://www.cern.ch/LEPBOSC/combos>)
  - Framework to perform combinations, developed originally by B oscillations WG
  - Quite general, several averaging methods implemented
  - Input data (individual measurements) are described in files which are read as input by COMBOS
  - Used for  $\Delta m_d$ ,  $A(\Delta m_s)$ ,  $(F(1)|V_{cb}|, \rho^2)$ ,  $BR(b \rightarrow ul\nu)$
- **Other specific code:**
  - for b-hadron lifetimes (could be done by COMBOS)
  - for b-hadron fractions
  - for  $\Delta\Gamma_s$
  - ...



# Dependencies ...





# LEP HF steering group: summary and plan for the future

- HF averaging activities (and their associated structures) have been successful:
  - fulfilled specific needs of LEP experiments to combine data where uncertainties could be reduced
  - stimulated discussions and data (re)analysis
  - good collaboration/partnership with other experiments (including new generation)
  - world averages produced
- Plan:
  - PDG 2002 averages
  - continue until last publications of LEP1/SLD results
  - prepare final publication of LEP/SLD/CDF1 HF averages
  - ... then retire LEP HF steering group (~ end 2002)